Coexistence in the 2.4 GHz ISM Band

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Abstract - The 2.4 GHz ISM band provides an attractive medium for developing tethered free applications based on current and proposed wireless standards. Since the ISM band is a shared spectrum allowing unlicensed operation of wireless services, coexistence between the wireless services needs to be considered to ensure performance requirements are maintained. A general method for evaluating coexistence is presented. The method is based on developing a stochastic model to evaluate coexistence in an arbitrary operational environment. To develop the model, a three-stage approach is used where the first two stages use empirical test results to estimate parameters and to substantiate the analytical model. By using the model, coexistence is evaluated based on variations in the operational environment, which takes into account the uncertainty in an installation’s location and in the interference traffic. This evaluation points to potential trouble spots and helps in determining strategies for alleviating them. In addition, the model can be used to evaluate techniques for mitigating coexistence and assist in optimizing their utility. The evaluation methodology is illustrated based on examining the IEEE 802.11b performance in the presence of Bluetooth piconets, however, the methodology is general and can be used to address other coexistence issues involving packet based wireless protocols.

1 Introduction

Operation in the 2.4 GHz industrial, scientific and medical (ISM) band provides the convenience of an unlicensed band with availability almost worldwide. Wireless devices based on IEEE 802.11b standard have been widely deployed. The standard has become the de facto wireless local area network (WLAN) for university campuses and business installations requiring partial or total untethered connectivity. In addition, diverse business, home and industrial applications
are using or are planning to use wireless connectivity to add value in the form of installation ease and/or untethered operation. These application enhancements use wireless technologies typically based on the 802.11b standard, the Bluetooth™ specification or the proposed low data rate standard, IEEE 802.15.4. The devices based on these standards and specification operate in the global unlicensed band and offer a potential lower cost due to technology availability, high volumes and limited reengineering.

Unlike other bands where interference is avoided by different wireless services being regulated to operate at separate frequencies or separate physical locations, in the ISM band access to the medium by different services is typically not coordinated. Therefore coexistence between services in the 2.4 GHz ISM band is a concern. To illustrate, a WLAN installed at an airport terminal provides business travelers access to airport services and the internet. Within the same terminal, travelers and airport personnel are using PDAs, cellular telephone headsets and computer peripherals enabled with Bluetooth. Many similar scenarios can be envisioned where WLANs and wireless personal area networks (WPAN™) such as Bluetooth are providing complimentary services requiring simultaneous operation in close proximity. Since both wireless services operate in the same band without coordination, the potential exists for interference impacting either one or both networks.

Performance requirements for wireless enabled devices need to be evaluated against coexistence issues where performance may be measured in terms of data throughput, transmission latency or both. In evaluating the performance with respect to coexistence issues, variations in the operational environment need to be considered including both the characteristics of the interfering wireless service and the RF propagation characteristics. This ensures the evaluation takes into account the uncertainty in an installation’s location and in the interference traffic. Evaluating the performance requirements in terms of coexistence issues provides a method for quantifying the applications interference susceptibility and assists in establishing usage policies.
Various techniques for addressing coexistence problems are being investigated including 802.11b packet fragmentation, Bluetooth adaptive frequency hopping and adaptive power control. These methods need to be evaluated in order to determine both their effectiveness and their utilization. As an example, an 802.11b can use packet fragmentation to avoid collisions with Bluetooth interference and thereby potentially improve the packet transmission time. Fragmenting a packet incurs an increase in the packet overhead, which can be outweighed by a decrease in the overhead associated with packet retransmission. Issues involving how and when packet fragmentation should be employed require a coexistence evaluation model in order to optimize the strategy for using the technique.

It is therefore important to understand:

- How collocated and cochannel wireless services perform under different operational environments, i.e., coexistence evaluation.
- How methods for mitigating performance degradation due to a collocated and cochannel wireless service can be most effectively utilized.

These two issues are interrelated and have been addressed by various research groups [1-13], including members of the IEEE 802.15 Task Group 2 [14] and the Bluetooth SIG Radio Committee.

This article provides an overview of a methodology for evaluating the coexistence issues outlined above. In the paper, the evaluation methodology is applied to assessing the effect of Bluetooth on the IEEE 802.11b WLAN performance. The method has also been successfully used to examine the effect of IEEE 802.11b on Bluetooth [10] as well as the mutual interference from independent Bluetooth piconets [15].

2 Evaluation Methodology Overview

Performance requirements for a wireless service are application dependent. Measures of performance (MOPs) for a manufacturing sensor array would likely require transmission latency constraints whereas an office WLAN might have throughput requirements. Coexistence issues
need to be evaluated in terms of the factors influencing the MOPs. These factors involve the operational environment for the wireless service, including the interference environment and the signal propagation characteristics. Due to the multiplicity and the uncertainty of these factors, a stochastic model of the underlying process is well suited for coexistence evaluation. Packet transmission with a packet acknowledgement is a typical component of the communication protocol used for data transmission in the ISM band. For these protocols, packet collision is the underlying process determining coexistence. A formal definition of a packet collision, $C$, is the event where one or more interference signals corrupt a desired packet transmission such that the retransmission of the data packet is required. Using this definition, analytical expressions can be determined such that the probability of collision, $\text{Pr}[C]$, can be evaluated in terms of the operational environment and the MOPs can be evaluated in terms of $\text{Pr}[C]$.

The analytical model for evaluating $\text{Pr}[C]$ in terms of the operational environment is developed based on a three-stage process:

1) Characterize the interference under static conditions, i.e., when both interfering and desired signals remain stationary. Empirical test results are used to estimate model parameters and to substantiate the model.

2) Characterize the desired network performance under the influence of a single source of interference. Empirical test results are used to substantiate the model.

3) Characterize the $\text{Pr}[C]$ in an arbitrary operational environment.

Each stage is discussed in more detail in Section 3.

Utilizing the analytical model, the coexistence can be evaluated for a given application as follows:

1) Define the MOPs relevant to the network application requirements and derive an expression for the MOPs in terms of the $\text{Pr}[C]$.

2) Define the variations in the operational environment in terms of the analytical model parameters. This defines a parameter space over which the model is evaluated.
3) Characterize Pr[C] in terms of the parameter space, i.e., determine the fraction of the parameter space that satisfy the constraints specified by the MOP. Each of these steps is discussed in Section 4. To illustrate the model’s ability to facilitate methods for mitigating interference, the results based on using the model to optimize WLAN packet fragmentation in the presence of Bluetooth interference are presented in Section 4.3.

3 Coexistence Model Development

For the coexistence evaluation methodology developed, Pr[C] needs to be evaluated over a wide range of operational environments based on the wireless communication application. A closed form analytical model or a numerical based model is well suited for this requirement. To illustrate the three-stage approach outlined above, an analytical model developed to evaluate the Pr[C] for an 802.11b WLAN in the presence of an arbitrary environment of Bluetooth piconet interferers is now presented. Empirical test results were used both to develop and to substantiate the analytical model.

Figure 1 depicts the temporal and spectral characteristics of the coexistence between Bluetooth and 802.11b. A collision occurs when an 802.11b packet needs to be retransmitted due to Bluetooth interference. A Bluetooth packet causes interference when it is transmitted during the same time interval as an 802.11b packet transmission, with sufficient power to cause an error within the detection of the 802.11b packet. The power required to cause a collision is dependent on the frequency offset between the 802.11b and the Bluetooth carrier frequencies. As illustrated in the figure, Bluetooth’s [16] protocol is based on frequency hopping with time division duplexing between the master and slaves within a piconet. The transmission bandwidth is nominally 1 MHz with 79 hop frequencies uniformly distributed within the ISM band. Typical packet transmissions are 366μs with approximately 250μs required for synthesizer re-tuning. The 802.11b [17] is based on direct sequence spread spectrum with a transmission bandwidth of approximately 20 MHz and packet duration of up to 1210μs.
3.1 Interference Characterization

The initial stage of the analytical model development involves characterizing the interference power to signal power threshold, $\gamma$, at which a packet’s retransmission is likely to be required. That is, if $\Omega_{I/S} \geq \gamma$, then the event $C$ occurs where $\Omega_{I/S}$ is the received interference to signal power ratio at the input to the desired receiver, e.g., an 802.11b receiver. The threshold $\gamma$ is derived under “stationary conditions”, i.e., the interferer signal characteristics are stationary with respect to the desired signal. Based on the authors’ previous work on coexistence analysis [5-10, 13], characterizing $\gamma$ for both cochannel and adjacent channel interference is essential to effectively evaluate $Pr[C]$. Therefore, $\gamma(f_{offset})$ was evaluated over both $\Omega_{I/S}$ and $f_{offset}$ where $f_{offset}$ is the frequency separation between the desired signal’s and the interfering signal’s carrier frequencies.

An empirical study was conducted to evaluate $\gamma(f_{offset})$. The test setup is depicted in Figure 2. An 802.11b compliant transmit (Prism II) signal was attenuated such that the signal at the 802.11b receiver was approximately -40 dBm. The Bluetooth interference signal was based on an Agilent ESG-D 4432 B RF signal generator transmitting a GFSK signal in accordance with the Bluetooth specification. The Bluetooth interference signal was transmitting continuously on the same carrier frequency; i.e., frequency hopping was not enabled. In this way, the uncertainty associated with the timing and frequency coincidence of the two signals is removed; the only element remaining is the jamming suppression capability of the WLAN. Multiple tests were conducted varying the interference signal transmit power and $f_{offset}$. In Figure 3a empirical test results are used to graph curves of equal probability, $Pr[\gamma \leq \Omega_{I/S}|f_{offset}] = 0.05$ and $Pr[\gamma \leq \Omega_{I/S}|f_{offset}] = 0.9$, over $f_{offset}$ and $\Omega_{I/S}$. Based on the empirical test results, $\gamma(f_{offset})$
should be modeled as a bivariate random variable (RV) where the graph in Figure 3a can be viewed as an estimate of the conditional cumulative distribution function (CDF) of $\gamma(f_{\text{offset}})$.

An analytical model of $\gamma_{I/S}(f_{\text{offset}})$ is given by

$$\gamma_{I/S}(f_{\text{offset}}) = \chi_{\gamma} - J_{S}(f_{\text{offset}}) \quad \text{(dB)}$$

where $J_{S}(f_{\text{offset}})$ is the normalized jamming suppression of the 802.11b in the presence of a Bluetooth signal and $\chi_{\gamma}$ is a log-normal RV with mean, $\gamma$, and standard deviation, $\sigma_{\gamma}$, estimated based on the empirical data. The jamming suppression, $J_{S}(f_{\text{offset}})$, was analytically derived based on

$$J_{S}(f_{\text{offset}}) = \Phi_{vv}(f_{\text{offset}}) * J_{CW}(f_{\text{offset}}) \quad \text{(dB)}.$$ (2)

where $J_{CW}(f_{\text{offset}})$ is the normalized jamming suppression of the 802.11b in the presence of a continuous wave (CW) tone derived in [6, 8] and $\Phi_{vv}(f_{\text{offset}})$ is an estimate of the power spectral density (PSD) for a Bluetooth signal. In Figure 3b, contour plots are shown corresponding to the analytical model, (1), with $\gamma = -6.5 \, \text{dB}$ and $\sigma_{\gamma} = 1.5 \, \text{dB}$. The importance of modeling $\gamma(f_{\text{offset}})$ as a bivariate RV can be observed from the graph. If the two signal powers are equal, then, with over a 90% probability, interference will cause a packet retransmission when the frequency offset is less than 6.5 MHz. The probability drops to below 5% over the range $6.5 < f_{\text{offset}} < 8$ MHz.

### 3.2 Coexistence in a Single Interferer Environment

The next stage in the coexistence model development is to formulate a solution for evaluating $\Pr[C]$ in the presence of a single active interferer, e.g., a single Bluetooth node. The underlying assumptions are: there is only one interferer at a fixed $\Omega_{I/S}$ and the interferer is actively transmitting. Therefore, in the case of a Bluetooth interferer, the interference signal is frequency hopping based on a pseudorandom hopping pattern and the packets are transmitted on even time slots assuming a single time slot packet. Using this scenario, it is again straightforward to make
comparisons between empirical and analytical results. The empirical tests are conducted in a similar fashion as outlined in the previous section.

For the analytical model, the results from the previous section are built upon to derive a solution for determining \( \Pr[C_1] \) where \( C_1 \) is a collision event based on a single interferer. The following two assumptions are made to facilitate an analytic solution. First, time coincidence and frequency coincidence are assumed to be independent events. Second, it is assumed if the signals are time coincident, then \( \Pr[C_1] \) depends on the relationship between \( \Omega_{I/S}, f_{\text{offset}} \) and \( \gamma(f_{\text{offset}}) \) and is independent of the time duration. These assumptions are approximations. It is likely high power interference in the pass band of the intended receiver requires less time to cause a collision than the time required by a lower power interferer. The underlying assumption is that the time difference is insignificant when compared to the other mechanisms impacting \( C_1 \). These assumptions can be substantiated based on a comparison with empirical results.

The relative timing between the desired wireless service’s packet transmission time with the interferer’s packet frame timing is used to determine the number of interference packets overlapping the desired packet and the corresponding temporal coincident probabilities. Figure 4 illustrates the relative timing between an 802.11b packet transmission with that of the packet transmission timing for a Bluetooth interferer where each Bluetooth packet is transmitted on independent hopping frequency. The relative timing between the onset of the packet transmissions, \( \tau_{\text{offset}} \), is modeled as a uniform RV and, therefore, the number of Bluetooth packets time coincident with the 802.11b packet transmission is either \( n_\tau \) or \( n_\tau - 1 \) where \( n_\tau = \lceil (T_p + \tau_{BT})/T_{BT} \rceil \) and \( \lceil \cdot \rceil \) is the ceiling function. The corresponding probabilities are \( \Pr[n_\tau] = (\tau_{BT} + T_p - (n_\tau - 1) T_{BT})/T_{BT} \) and \( \Pr[n_\tau - 1] = 1 - \Pr[n_\tau] \). These two events are independent; therefore,

\[
\Pr[C_1] = \Pr[n_\tau] \ Pr[C_1|n_\tau] + \Pr[n_\tau - 1] \ Pr[C_1|n_\tau - 1].
\]
where \( \Pr[C_f | \mu_r] = 1 - (1 - L_{BT} \Pr[C_f])^{\mu_r} \), since the frequency coincidence of each Bluetooth packet is independent. The parameter \( L_{BT} \) models the loading factor for a given Bluetooth piconet. \( L_{BT} = 1 \) indicates every time slot, both the master’s and the slave’s, is being utilized and \( L_{BT} = 0.5 \) indicates that, on average, half of the time slots are used. \( \Pr[C_f] \) is the probability the interfering signal is frequency coincident with sufficient power to cause interference. Based on the Bluetooth hopping sequence uniformly covering the ISM band with bandwidth \( B_{UL} \) and, using the results from the previous section, then

\[
\Pr[C_f] = \frac{2}{B_{UL}} \int_0^{R_u/2} \Pr[\Omega_{I/S} \geq \gamma(f_{\text{offset}}) | f_{\text{offset}}] \, df_{\text{offset}}.
\]

Using the analytical model for the \( \gamma(f_{\text{offset}}) \) given in (1), (4) can be evaluated in a near closed form expression. Then, combining (4) with (3), \( \Pr[C_1] \) can be evaluated. A comparison between the analytical model and empirical test results for \( \Pr[C_1] \) is given in Figure 5.

### 3.3 Coexistence in a Network Interference Environment

The third stage of the coexistence model is to extend the model to provide a method for evaluating \( \Pr[C] \) when the desired wireless service is operating in an arbitrary environment with multiple interferers operating independently. To illustrate, the topology depicted in Figure 6 is used in developing an analytical model for evaluating the effect of independently operating Bluetooth piconets on the downlink of an 802.11b station (STA). The 802.11b access point, AP, is located a distance \( d_S \) from the 802.11b STA. The Bluetooth piconets are uniformly distributed within the same location as the STA with a density of \( D_{BT} \) piconets/m\(^2\). The loading and activity at each piconet is independent and identically distributed with \( \Pr[A] \), indicating the probability a Bluetooth piconet is active. Based on these assumptions,
\[ \Pr[C] \equiv \sum_{l=0}^{\infty} \left( 1 - \Pr[A] \Pr \left[ \frac{C}{\Omega_{I/S}} = \frac{\left( \gamma_l + \gamma_{l+1} \right)}{2} \right] \right)^{N_o} \Pr[\gamma_l < \Omega_{I/S} \leq \gamma_{l+1}] \]  

(5)

where \( \gamma_l \) is from an ordered sequence of threshold values \( \gamma_l < \gamma_{l+1} \) and \( N_0 \) is the expected number of piconets with sufficient power to cause interference, i.e., the piconets with \( \Omega_{I/S} \geq \gamma_0 \).

Unlike the single interferer scenario, for the network analysis \( \Omega_{I/S} \) is a RV and (5) represents the total probability based on the accumulative probability over a set of mutually exclusive events. \( \gamma_l < \Omega_{I/S} \leq \gamma_{l+1} \). The probability of the events are estimated by determining the expected number of interferers with \( \Omega_{I/S} > \gamma_l \), \( N_l \), and then

\[ \Pr[\gamma_l < \Omega_{I/S} \leq \gamma_{l+1}] = \frac{(N_l - N_{l+1})}{N_0} . \]

\( N_l \) is based on determining the effective area of interference, \( A_{eff} \left( \Gamma_l, d_S \right) \), where \( \Gamma_l \) is the normalized interference to signal power ratio threshold, \( \Gamma_l = \gamma_l + \left( \Omega_s \right)_{Tx} - \left( \Omega_I \right)_{Tx} (\text{dB}) \); \( \left( \Omega_s \right)_{Tx} \) and \( \left( \Omega_I \right)_{Tx} \) are the transmit powers at the AP and the Bluetooth interferer, respectively. Using the effective area of interference, the number of interferers satisfying \( \Omega_{I/S} > \gamma_l \) is \( N_l = A_{eff} \left( \Gamma_l, d_S \right) D_I \). Based on a similar approach used by Jake [18] to determine the percentage coverage area of a cell, the author derived a formula for determining the effective interference area [6]:

\[ A_{eff} \left( \Gamma_l, d_S \right) = \pi \left( d_S \right)^2 \exp \left[ \frac{2 \left( \sigma_{I/S}^2 - 10n \Gamma_l \log_{10}(e) \right)}{(10n \log_{10}(e))^2} \right] . \]

(6)

In (6), the signal propagation is based on a lognormal shadowing model with exponential path loss where \( n \) is the path loss exponent and \( \sigma_{I/S} \) is the lognormal shadowing standard deviation.

The \( \Pr[C] \) for the IEEE 802.11b STA can be evaluated by using (5) in conjunction with (6) which is illustrated in Section 4.
4 Evaluation Methodology

As indicated in the Section 2, a set of MOPs can be used to specify the wireless communication requirements for a given application. An expression for the 802.11b packet transmission time in terms of $\Pr[C]$ is given in Section 4.2 in order to illustrate the mapping between $\Pr[C]$ and a MOP. A fundamental question when evaluating coexistence is whether the MOP requirements are compromised by coexistence issues. To assess this question often requires evaluating $\Pr[C]$ based on the uncertainty in the operational environment. A method for addressing this uncertainty is presented first. Finally, an optimal strategy for using 802.11b packet fragmentation to mitigate Bluetooth interference is presented.

4.1 Evaluating Coexistence for a Wireless Application

From Section 3 it is evident $\Pr[C]$ is dependent on a set of parameters associated with the operating conditions of the WLAN and the interference environment associated with the installation. Specifically, there are six parameters that can be grouped into two sets,

1) Bluetooth piconet parameters $V_{BT} \equiv [L_{BT}, D_{BT}, \Pr[A_{BT}]]$,

2) Radio propagation parameters $V_{RF} \equiv [n, \sigma_{L/S}, d_{S}]$.

In order to provide insight into the coexistence issue, $\Pr[C]$ needs to be evaluated over the range of parameters anticipated by the application. E.g., typical installation sites for the application can provide bounds on the RF propagation parameters and usage scenarios will assist in bounding the Bluetooth environment as seen by the 802.11 network.

The ranges for each parameter in $V_{BT}$ and $V_{RF}$ define a parameter space $V = [V_{BT}, V_{RF}]^T$. A specific point in the parameter space, $v^i \in V$, can be used to evaluate the collision probability, $\Pr[C|v^i]$, by evaluating (5) at $v^i$. The points in the parameter space can
then be categorized into intervals on $\Pr[C]$, i.e., the set $v_k = \{v^i\}$ where the elements of $v_k$ result in $\xi_k < \Pr[C|v^i] \leq \xi_{k+1}$ and $\xi_k$ is an element of an ordered sequence such as $\xi_k \in [0,0.1,0.2,\ldots,1]$. The sample mean and sample variance for each parameter can be evaluated over the set $v_k$. The sample mean provides a measure of a parameter’s average value required for $\Pr[C]$ to be on the interval $[\xi_k, \xi_{k+1}]$. A parameter’s sample variance over the set $v_k$ provides a measure of the parameter’s spread within the set.

Based on the parameter ranges given in Table 1, the sample mean and sample variances were determined for the parameters $d_S$ and $D_{BT}$ as depicted in Figure 7a and 7b, respectively. In the figure, the dashed lines represent the mean $\pm$ one standard deviation. From Figures 7a and 7b, if the variance is small, then in order for $\Pr[C] \in [\xi_k, \xi_{k+1}]$, the parameter must take on a value close to the mean. Likewise, if the variance is large, then the specific parameter value is less important on the outcome $\Pr[C] \in [\xi_k, \xi_{k+1}]$. Using this observation, the importance of each parameter can be weighted, $w_{kk}$, relative to the other parameters as a function of $[\xi_k, \xi_{k+1}]$. A formal derivation of $w_{kk}$ is given in [10, 15] based on feature ordering given in [19]. Figure 7c depicts the graph of $w_{kk}$ based on the parameter values specified in Table 1. In determining $w_{kk}$ for each interval $[\xi_k, \xi_{k+1}]$, the weights are constrained by $\prod_{kk=1}^6 w_{kk} = 1$. The larger the weight, the more important the parameter where importance is based on the relative size of the sample variance over the set $v_k$.

The interrelationship between the graphs of Figure 7 provides an important tool for both evaluating coexistence performance as well as identifying methods for alleviating coexistence problems. As an example, if maintaining a $\Pr[C] \leq 40\%$ is important for achieving the desired MOP, then based on the graph in Figure 7c, both $D_{BT}$ and $d_S$ are the most important
parameters. The graphs in Figure 7a and Figure 7b indicate restricting $D_{BT}$ to be less than one Bluetooth piconet per $10m^2$ and restricting the 802.11 range to $d_S \leq 12m$ could assist in achieving the desired collision probability.

The coexistence evaluation over the parameter space is summarized by determining the fraction of the parameter space, $U(\zeta)$, that results in $\Pr[C] \leq \zeta$, i.e.,

$$U(\zeta) = \frac{|\{v' | \Pr[C|v'] \leq \zeta\}|}{|\{v'\}|} \tag{7}$$

where $|\cdot|$ is the cardinality of a set. $U(\zeta)$ is evaluated based on the parameter space defined by the parameter ranges given in Table 1 and the resulting graph is shown in Figure 8. Continuing the example from above, slightly more than 50% of the parameter space corresponds to $\Pr[C] \leq 40\%$ based on the parameter ranges from Table 1. Using the restricted parameter space, $d_S \leq 12m$ and $D_{BT} \leq 1/10$ piconets/m$^2$, the percentage is increased to over 80%. Therefore, a modest change in the usage policy for both the 802.11b and Bluetooth piconets results in a dramatic improvement in the percentage of operational environments achieving the desired performance.

### 4.2 Relating Network Performance to the Probability of Collision

The packet transmission time, $S$, is defined as the time required to successfully transmit a packet to a destination via the medium where the packet contains a message of duration $T_m$. It is a common performance measure, MOP, and can be used to derive other MOPs such as throughput or transmission latency. For evaluating $S$ in a Bluetooth interference environment, it is assumed the 802.11 transmitter does not delay transmission due to sensing the presence of a Bluetooth signal, i.e., collision avoidance is not initiated due to a Bluetooth signal. Therefore, the transmission timing is impacted by the likelihood of a collision occurring and a collision results in the retransmission of the packet or a packet fragment after a random back-off time interval.
The packet transmission time is expressed in terms of both a single packet and a sequence of packet fragments in order to facilitate the discussion in Section 4.3 concerning optimizing the packet fragmentation for mitigating Bluetooth interference. A fragmented message requires transmitting $N_f$ packets with each packet fragment containing a portion of the message with duration $T_m N_f^{-1}$. The advantages of fragmentation are derived from the shorter transmission time for each packet fragment. The shorter transmission time decreases the likelihood of a packet collision with a collocated transmitter. This advantage needs to be weighed against the increase in overhead time associated with packet fragmentation. The performance can be evaluated by assessing the expected transmission time, $E[S]$, of the IEEE 802.11 as a function of $N_f$ and the factors influencing $Pr[C]$.

Based on the 802.11, timing for a single message, either via a single packet or a sequence of packet fragments, is

$$S = N_f \left( T_m N_f^{-1} + T_{os} \right) + \sum_{i=1}^{N_f} \left[ (T_m N_f^{-1} + T_{of}) K_i + T_{bo}(K_i) \right]$$

(8)

where $T_{os}$ is the overhead time associated with a successful packet fragment transmission, $T_{of}$ is the overhead time associated with a failed packet fragment transmission due to a packet collision. The first term in (8) represents the transmission time required when no collisions occur. The second term encompasses the delay due to collisions where the RV $K_i$ is the number of collisions that incur while transmitting the $i^{th}$ fragment. The 802.11 uses a random back off algorithm dependent on the number of collision. The distribution for the random back off, $T_{bo}(\cdot)$, is based on the number of unsuccessful attempts in transmitting, i.e., the number of collisions.

Under mild assumptions concerning packet collisions due to Bluetooth interferers,

$$E[S] = T_m (1 + E[K]) + N_f E[F]$$

(9)
where \( E[F] = T_{as} + T_{sf} E[K] + E[T_{bs}(K)] \) and the number of collisions, \( K \), can be modeled as an exponentially distributed RV with \( E[K] = \Pr[C]/(1-\Pr[C]) \). Using (9), along with the characteristics of the 802.11 back off algorithm a closed form expression can be obtained for \( E[S] \). Figure 9 illustrates the relationship between the normalized \( E[S] \) with \( \Pr[C] \) over several levels of fragmentation. The graphs indicate the relative cost associated with using packet fragmentation, i.e., the more packet fragments, the higher the overhead. For packet fragmentation to be a viable option for mitigating Bluetooth interference, it must reduce the \( \Pr[C] \) sufficiently. To illustrate this point, given the \( \Pr[C] \) for a single packet transmission is 40%, then the \( \Pr[C] \) needs to be reduced to less then 30% when using \( N_f = 2 \) in order to improve \( E[S] \).

### 4.3 Mitigating Coexistence Problems Using Fragmentation

The evaluation methodology presented in the paper is useful for developing and evaluating optimal strategies for mitigating coexistence interference. In this section, an extension to the coexistence model discussed in Section 3 is used to evaluate \( \Pr[C] \) based on the number of fragments \( N_f \). This allows the optimal number of fragments, \( N_f^* \), to be estimated based on the MOP \( E[S] \) derived in terms of \( \Pr[C] \) and \( N_f \). Details concerning the approach are given in [13].

The approach for estimating \( N_f^* \) is illustrated graphically in Figure 10. In Figure 10(a), \( \Pr[C] \) monotonically decreases as \( N_f \) increases, but as seen in Figure 10(b), \( E[S] \) graphed with respect to \( N_f \) has a unique minimum. To illustrate, for \( d_s = 12m \), the optimal fragmentation occurs at \( N_f^* \approx 2.8 \) with 31% improvement in \( E[S] \) over \( E[S] \big|_{N_f=1} \), i.e., \( E[S] \) evaluated at \( N_f = 1 \). A corresponding 28% improvement in \( \Pr[C] \) is observed between \( \Pr[C] \big|_{N_f^*} \) and
\[ \Pr(C) \bigg|_{N_f=1} \] for the same scenario. Other scenarios exhibit similar results but with different optimal \( N_f \) values and different levels of performance improvement.

Operationally, in order for the IEEE 802.11b to estimate the optimal number of packet fragments, \( \hat{N}_f^* \), the estimate needs to be made on the observed \( \Pr(C) \bigg|_{N_f=1} \), i.e., probability of collision given no packet fragmentation. In order to characterize the relationship between \( \hat{N}_f^* \) and \( \Pr(C) \bigg|_{N_f=1} \), \( \hat{N}_f^* \) was evaluated over a wide range of operational environments. The sample mean and variance of \( \hat{N}_f^* \) is then evaluated as a function of \( \Pr(C) \bigg|_{N_f=1} \) and the results are depicted in Figure 11. The small standard deviation about the mean indicates that \( \Pr(C) \bigg|_{N_f=1} \) provides a reliable method for estimating \( \hat{N}_f^* \).

The performance gain in selecting \( S \bigg|_{N_f^*} \) over \( S \bigg|_{N_f=1} \) and the corresponding impact of \( \Pr(C) \bigg|_{N_f^*} \) over \( \Pr(C) \bigg|_{N_f=1} \) was also evaluated over a wide range of operational environments. The RMS improvement was evaluated using

\[
R_p = \left( E \left[ \left( \left( \Pr(C) \bigg|_{N_f^*} \right) - \left( \Pr(C) \bigg|_{N_f=1} \right) \right)^2 \right] \right)^{\frac{1}{2}}
\]

(10)

\[
R_s = \left( E \left[ \left( S \bigg|_{N_f^*} - \left( S \bigg|_{N_f=1} \right) \right)^2 \right] \right)^{\frac{1}{2}}
\]

(11)

Graphs of \( R_p \) and \( R_s \) versus \( \Pr(C) \bigg|_{N_f=1} \) are shown in Figure 12.

5 Conclusion

The 2.4 GHz ISM band provides an attractive medium for developing tethered free applications based on current and proposed wireless standards. Since the ISM band is a shared spectrum allowing unlicensed operation of wireless services, coexistence between the wireless services needs to be considered to ensure performance requirements are maintained. Techniques for
mitigating interference are being proposed and implemented. These techniques need to be tested to ensure reliability in providing performance improvement as well as determining the best operational strategy for using the technique.

This paper presents a general method for evaluating coexistence. First, a stochastic model for determining the probability of collision in an arbitrary operational environment is developed based on a three-stage approach. Second, a method for using the model is presented for evaluating the wireless service’s MOPs when there is uncertainty in the operational environment. This evaluation can point to potential trouble spots and assist in determining strategies for alleviating them. In addition, the model can be used to evaluate techniques for mitigating coexistence based interference and assist in optimizing their utility. The evaluation methodology is illustrated based on examining the IEEE 802.11b performance in the presence of Bluetooth piconets, however, the methodology is general and can be used to address other coexistence issues involving packet based wireless protocols.


Figure 1. Time and frequency relationship between IEEE 802.11b and Bluetooth signals.

Figure 2. Test setup for empirical measurements
Figure 3: Contour plots of $\gamma(f_{\text{offset}})$ vs. frequency offset and interference to signal ratio, (a) is based on empirical test results, (b) is based on analytical model.
Figure 5  Comparison between the analytical model and the empirical data for the probability of collision based on a single Bluetooth interferer.

Figure 4  Relative Packet Timing for IEEE 802.11b and Bluetooth
Figure 6  Topology and geometry for analyzing the network interference scenario.

\[ N_{BT} = A_{eff}(\Gamma, d_s | D) D_{BT} \]
Figure 7 Graphs (a) and (b) are the sample means, solid line, for two of the six parameters used in evaluating $\Pr[C]$. Dashed lines represent $\pm \sigma$ about the mean. Graph (c), weighting factors used to categorize parameter importance in determining $\Pr[C]$. 

Figure 8 Fraction of the parameter space, $U(\zeta)$, with $\Pr[C] \leq \zeta$. Parameter space based on values in Table 1.
Figure 9  Normalized Expected packet transmission time for 802.11b versus probability of collision due to Bluetooth interference for four different packet fragmentation.
Figure 10 (a) Impact of $N_f$ and $d_S$ on $\Pr[C]$; (b) Impact of $N_f$ and $d_S$ on $E[S]$. 
Figure 11 Sample mean, $E[\hat{N}_f^\ast]$ and $\pm \sigma_{N_f^\ast}$, deviation about the mean as a function of the probability of collision for an unfragmented packet, $p|_{N_f=1}$. 
Figure 12  RMS performance improvement of $p$ and $S$, $R_p$ and $R_S$, respectively, vs. probability of collision given $N_f = 1$ ($p_{|N_f=1}$).
Table 1  Bluetooth interference parameter ranges and radio propagation parameter ranges used in the mutual interference analysis in an arbitrary environment.

<table>
<thead>
<tr>
<th>Parameter Set</th>
<th>Parameter</th>
<th>Range</th>
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<tbody>
<tr>
<td>$V_{BT}$</td>
<td>$L_{BT}$</td>
<td>$[0.1, 0.2, 0.3, \ldots, 1]$</td>
</tr>
<tr>
<td></td>
<td>$\text{Pr}[A_{BT}]$</td>
<td>$[0.1, 0.2, 0.3, \ldots, 1]$</td>
</tr>
<tr>
<td></td>
<td>$D_{BT}$</td>
<td>$[1/10^2, 1/9^2, 1/8^2, \ldots, 1]$ piconets/m$^2$</td>
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<tr>
<td>$V_{RF}$</td>
<td>$n$</td>
<td>$[1.5, 2, 2.5, \ldots, 5]$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{I/S}$</td>
<td>$[5, 6, 7, \ldots, 11]$ dB</td>
</tr>
<tr>
<td></td>
<td>$d_S$</td>
<td>$[1, 2, 3, \ldots, 20]$ m</td>
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