Let's denote the arithmetic for closed real intervals by $\mathbb{I}$ and that for connected real intervals by $\mathbb{J}$. Further let $\boldsymbol{a}=\left[a_{1}, a_{2}\right]$ and $\boldsymbol{b}=\left[b_{1}, b_{2}\right]$ be two floating-point intervals of $\mathbb{I}$ and $\circ$ an operation $\circ \in\{+,-, \cdot, /\}$. Then the lower bound of an interval operation $\boldsymbol{a} \circ \boldsymbol{b}$ is computed by $\nabla\left(a_{i} \circ b_{j}\right)$ and the upper bound by $\triangle\left(a_{\mu} \circ b_{\nu}\right)$ where the $i, j, \mu, \nu$ are to be selected by the usual formulas for interval operations.

In general the results $a_{i} \circ b_{j}$ and $a_{\mu} \circ b_{\nu}$ will not be floating-point numbers so that the roundings have to be applied. Then arithmetic in $\mathbb{I}$ delivers the result $I=\left[\nabla\left(a_{i} \circ b_{j}\right), \Delta\left(a_{\mu} \circ b_{\nu}\right)\right]$ while arithmetic in $\mathbb{J}$ delivers $J=\left(\nabla\left(a_{i} \circ b_{j}\right), \Delta\left(a_{\mu} \circ b_{\nu}\right)\right)$ and we have $I \subset J$.

More drastic examples can be given in case of reasonable fused operations. Consider, for instance, two interval matrices. In $\mathbb{J}$ the dot products are computed exactly with only one rounding at the end of the accumulation while in $\mathbb{I}$ a rounding is applied after each addition and each multiplication in the dot products. In addition to the difference in accuracy there is a difference in computing speed. For details see section 8.6.2 in my book Computer Arithmetic and Validity. The unit discussed there was built in 1993/94. The book was on the market before IEEE P1788 was founded.

