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Title: Code Separation vs. Frequency Reuse

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Abstract
This submission compares an 8 chip code with a frequency reuse of 3 to a 16 chip code with a frequency reuse of 2. Simulation results show that in an average scenario, the 8 chip/3 frequency scheme outperforms the 16 chip/2 frequency scheme.
1. INTRODUCTION

This paper compares an 8 chip system with a frequency reuse of 3 (called an 8/3 scheme) with a 16 chip system with a frequency reuse of 2 (called a 16/2 scheme.) The 8/3 scheme has an important advantage over the 16/2 scheme with respect to frequency reuse because the 16/2 scheme must contend with larger power and more cochannel interferers. However, the 16/2 scheme has the advantage of larger code separation than the 8/3 scheme. For the models used in this paper, the code separation for the 16/2 scheme is 4 chips/bit (16 chips/4 information bits), and the code separation for the 8/3 scheme is 2 chips/bit (8 chips/4 information bits.) Because of these two competing issues, it is very important to compare these two techniques on equal ground.

Although the issue of an 8 chip sequence vs. a 16 chip sequence has been analyzed with respect to the sequences’ tolerance to additive Gaussian noise and CW interference in [1], these types of interference are not the critical issue in a system which is using direct sequence codes to isolate a desired user from a cochannel user. It is well known interference from cochannel interferers in a direct sequence system can devastate detection, and these interferers should not be treated as Gaussian noise or CW interference (see [2], [3], [4]). The Central Limit Theorem does not apply in most of the important cases.

2. OVERVIEW OF FREQUENCY PLANS

The 8/3 scheme that we consider has a frequency reuse of 3 while the 16/2 scheme has a frequency reuse of 2. Figure 1 illustrates these two frequency plans with geometric shapes. Cells which use the same frequency band are shown in the same color.

Figure 1 shows that if the red cells correspond to the frequency band of interest then the K=3 pattern (the 8/3 scheme) has 6 distant cells which contain cochannel interferers. The K=2 pattern (the 16/2 scheme), on the other hand, has 8 cells which contain cochannel interferers (4 near interfering cells and 4 distant interfering cells).
Figure 1: Frequency Reuse 3 vs. Frequency Reuse 2.

The geometrical shapes are convenient for planning purposes; however, Figure 2 illustrates a more accurate frequency plan which uses circles to represent BSA’s. The nearest cochannel interferer on the reverse link is indicated with a triangle.

Figure 2: BSA's are represented more accurately as circles.

It is apparent from Figure 1 and Figure 2 that the 16/2 scheme is bombarded by considerably more cochannel interferers than the 8/3 scheme.

3. Code Separation

As indicated in the previous section, the 16/2 scheme is disadvantaged because of the number and proximity of cochannel interfering cells. However, this scheme does achieve isolation by using encoding the data so that the code separation is 4 while the 8/3 scheme only has a code separation of 2. The comparison between these two schemes is further muddied by the fact that in both the 16/2 and the 8/3 schemes, the users’ codes are asynchronous relative to each other.

Figure 3 illustrates the asynchronous nature of these systems for 3 different users. Notice that the users’ codes are not time aligned; instead, they are offset relative to each other. Thus, in order to quantify the performance of the system in the presence of cochannel interference, it is necessary to analyze the performance at each possible relative delay. Under these circumstances, simulations are an appropriate way to compare these two techniques. This paper compares the two schemes via simulations in Section 4, but first the coding techniques used by the two schemes are described in more detail in sections 3.1 and 3.2.
3.1. 16/2 Coding Scheme

An overview of the 16/2 scheme is shown in Figure 4. This scheme partitions the incoming data into groups of 4 bits. Each group of 4 bits is mapped to one of sixteen possible Walsh functions (which are 16 chips long.) Once 4 Walsh functions (64 chips) have been selected, they are multiplied by a length 64 “cover code.” These codes do not spread but rather scramble the signal. Nearby cochannel cells can be assigned different cover codes to help isolate the desired user.
The 48 possible cover codes are given by the following table (taken from [5]).

<table>
<thead>
<tr>
<th></th>
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<td>C₀C₁C₂C₃</td>
<td>13</td>
<td>C₂C₃C₄C₅</td>
<td>25</td>
<td>C₄C₅C₆C₇</td>
<td>37</td>
<td>C₆C₇C₈C₉</td>
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<tr>
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<td>C₂C₃C₄C₁</td>
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<td>C₄C₅C₆C₆</td>
<td>38</td>
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<td>16</td>
<td>C₂C₄C₅C₀</td>
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<td>C₄C₅C₆C₄</td>
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<td>20</td>
<td>C₂C₅C₆C₁</td>
<td>32</td>
<td>C₄C₆C₇C₆</td>
<td>44</td>
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</tr>
<tr>
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<td>C₂C₄C₅C₃</td>
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<td>22</td>
<td>C₂C₅C₆C₀</td>
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<td>C₅C₆C₇C₄</td>
<td>46</td>
<td>C₇C₈C₉C₄</td>
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<tr>
<td>11</td>
<td>C₃C₀C₆C₂</td>
<td>23</td>
<td>C₃C₆C₇C₁</td>
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<td>C₅C₇C₈C₆</td>
<td>47</td>
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<tr>
<td>12</td>
<td>C₃C₆C₇C₀</td>
<td>24</td>
<td>C₃C₆C₇C₀</td>
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<td>C₅C₇C₈C₄</td>
<td>48</td>
<td>C₇C₈C₉C₄</td>
</tr>
</tbody>
</table>

where \( C₀=0158, C₁=0461, C₂=131F, C₃=1626, C₄=020E, C₅=0737, C₆=1049, C₇=1570. \) (These numbers are given in hex.) In all simulations, the sequences are transmitted in BPSK format.

### 3.2. 8/3 Coding Scheme

Figure 5 shows the 8/3 coding scheme. The 8/3 scheme partitions the incoming data into groups of 4 bits. The first 3 bits are mapped into one of eight possible Walsh functions (of length 8 chips), and the 4th bit determines the polarity of the Walsh function. This 8 bit sequence is then multiplied by the 8 bit code 0₃H. In all simulations, the sequences are transmitted in BPSK format.
4. RESULTS

The first three sections in this paper show that simulations are necessary in order to compare the 8/3 scheme to the 16/2 scheme. This section shows simulation results which directly compare the 8/3 scheme with the 16/2 scheme. In section 4.1, a desired user with a fixed amplitude is subjected to a single interferer whose signal power is varied. In section 4.2, the average probability of error is obtained for both schemes relative to the activity of the interferers. All users’ signals are simulated as BPSK signals.

4.1. Performance with a Single Interferer

In these results, a desired user with a fixed amplitude was simulated in the presence of a single interferer. The signal-to-interference-ratio (SIR) is the ratio of power of the desired user to the signal power of an interfering user.

The relative delays of the users were randomly chosen out 8 possible delays for the 8/3 scheme and 64 possible delays for the 16/2 scheme (see section 3 for more information on the asynchronous nature of the system.) For the 16/2 scheme, the cover codes were randomly chosen for the desired user and the interferer. Both schemes generated 1024 data bits for each Monte Carlo run. Approximately 1000 Monte Carlo runs were performed for each scheme. These ideal simulations do not include any Gaussian noise or fading.

Notice that the probability of error is constant over a period of SIR’s for both schemes. This behavior is a result of the short coding sequences and the fact that there is only one
interferer. At the discontinuities, the 8/3 scheme has the same Walsh word error rate as the 16/2 scheme for an SIR that is about 3 or 4 dB higher.

![Figure 6: Simulation results for a desired user in the presence of a single interferer.](image)

Although the 16/2 scheme has a lower Walsh error rate for the SIR’s shown in the plots in this section, this does not imply anything about the relative performance of the two systems. Recall that the 16/2 scheme is hampered by more large powered interferers than the 8/3 scheme. Because of this interference, the 16/2 scheme will operate at a lower SIR region than the 8/3 scheme. Therefore, simulations which compare performance must include the frequency reuse effect.

4.2. Average Performance

In order to simulate average performance in a system, the realistic scenario shown in Figure 7 is analyzed. The simulations for this scenario include lognormal fading and use a Signal to Interference Ratio (SIR) based on a number of interferers varying from 0 to 6 for the 8/3 scheme and from 0 to 8 interferers for the 16/2 scheme. The SIR is the ratio of signal power (seen by the access point) from a desired user at the edge of the cell to the signal power from a user located at the centroid of the “nearest half” of the interfering cell.
Frequency Reuse: 3

The distance between the access point and the nearest interferer is determined by the equation given in Figure 8.

$$\text{distance} = \text{circle’s radius} + \text{hexagon’s side} + \text{distance to centroid of semicircle}$$

$$= r + r + r(3\pi - 4)/3\pi = 2.5756r$$

Figure 8: Average distance calculations.

Once the distances are calculated, the SIR’s are derived using the parameters given in Table 1 for both the 8/3 and the 16/2 scheme.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency in Hz</td>
<td>2.45E+09</td>
</tr>
<tr>
<td>Wavelength in M</td>
<td>0.12244898</td>
</tr>
<tr>
<td>TX antenna Gain</td>
<td>1</td>
</tr>
<tr>
<td>RX antenna Gain</td>
<td>1</td>
</tr>
<tr>
<td>Free Space Reference Distance</td>
<td>1.224489796</td>
</tr>
<tr>
<td>Transmit Power in watts</td>
<td>0.1</td>
</tr>
<tr>
<td>Sensitivity in dBm</td>
<td>-87</td>
</tr>
<tr>
<td>Sensitivity in Watts</td>
<td>1.99526E-12</td>
</tr>
<tr>
<td>Wavelength dependent expansion</td>
<td>6.33257E-05</td>
</tr>
<tr>
<td>Path Loss Exponent</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1: Simulation parameters.
All users (desired and interferers) have a lognormal fading with a standard deviation of 8 dB. The relative delays of the users were randomly chosen from 8 possible delays for the 8/3 scheme and 64 possible delays for the 16/2 scheme (see section 3 for more information on the asynchronous nature of the system.) For the 16/2 scheme, the cover codes were randomly chosen for the desired user and the interferers. Both schemes generated 256 data bits for each Monte Carlo run. Ten thousand Monte Carlo runs were performed for each scheme.

The 8/3 scheme has been simulated for \( m = [0, ..., 6] \) interferers assuming that the desired user is at the edge of the cell and the interferers are located at the centroid of the interfering cells. The 16/2 scheme was simulated for \( m = [0, ..., 8] \) interferers in which the desired user is at the edge of the cell and the interferers are at the centroid of the interfering cell. However, 24 simulations were run in order to account for all possible combinations of 4 sets of near interferers and the 4 sets of distant interferers. For example, in the case of \( m = 2 \) interferers, there can be either 2 near interferers, 1 near interferer and 1 far interferer, or 2 far interferers. In this analysis, these three possibilities are averaged (assumed equally likely) in order to determine the probability of 2 out of 8 possible interferers.

By using simulations to determine the word error rate given \([0, ..., M]\) interferers, \( \Pr(\text{word error} / \text{interferers}) \), the overall word error rate is given by the following equation

\[
\Pr(\text{word error}) = \sum_{m=0}^{M} \Pr(\text{word error} / \text{interferers}) \cdot \Pr(\text{m interferers})
\]

where \( M \) is the total number of interferers. The \( \Pr(\text{m interferers}) \) is simply a binomial distribution given by

\[
\Pr(\text{m interferers}) = \binom{M}{m} p^m (1-p)^{M-m}
\]

where \( p \) is the probability that a given user is active. This probability will largely depend on the particular network; therefore, the two schemes are compared for a range of values of \( p \).

Simulation results on the probability of Walsh word error are shown in Figure 9. In general, the error rate of the 8/3 scheme is approximately half the error rate of the 16/2 scheme. The probability of an active interferer is the probability that a user in a cochannel cell is transmitting at the same time as the desired user.
In order to make a fair comparison of the two schemes, it is important to examine the probability of packet error as well. If an 8000 bit (1000 byte) packet is transmitted, the packet error rate of the 8/3 scheme is given by \(1 - \text{Pr}(\text{correct word})^{8000/4}\) (since there are 4 bits per word.) Thus, the packet error rate is given by \(1 - (1 - \text{WER}_{8/3})^{2000}\). Similarly, 5 bit words in the 16/2 scheme yield a packet error rate of \(1 - (1 - \text{WER}_{16/2})^{1600}\). Notice that this calculation actually gives an unfair advantage to the 16/2 scheme since the simulation only uses 4 bits/word rather than 5 bits/word.

Using the results from Figure 9 and the packet error rate equations above, the packet error rate curve is shown in Figure 10. Notice that the 8/3 scheme consistently outperforms the 16/2 scheme.
5. CONCLUSION

Although the 8/3 scheme has half the processing gain of the 16/2 scheme, the 8/3 frequency plan provides much more isolation than the 16/2 scheme. From the simulations in this paper, it is clear that the 8/3 scheme performs better than the 16/2 scheme in the average case scenario with lognormal fading.

REFERENCES


