Status of models for UWB propagation channels

SG4a channel model group

Abstract

This is a discussion document for the IEEE document of the IEEE 802.15.4a channel modeling subgroup. It gives the current status of the generic channel model for UWB that has been discussed. Feedback from all participants is requested.

Index Terms

UWB, channel model, propagation

List of contributors: Chia-Chin Chong, Kannan Balakrishnan, Ulrich Schuster, Johan Karedal, Andrew Fort, Andreas F. Molisch, Norbert Daniele, WHO DID I FORGET? Anybody providing text or parameterizations should be included here.

I. INTRODUCTION

A. Background and goals of the model

This document summarizes the activities and recommendations of the channel modeling subgroup of IEEE 802.15.4a. The Task Group 802.15.4a has the mandate to develop an alternative physical layer for sensor networks and similar devices, working with the IEEE 802.15.4 MAC layer. The main goals for this new standard are energy-efficient data communications with data rates between 1kbit/s and several Mbit/s; additionally, the capability for geolocation plays an important role. More details about the goals of the task group can be found in [?]. In order to evaluate different forthcoming proposals, channel models are required. Their main goal of those channel models is a fair comparison of different proposals. They are not intended to provide information of absolute performance in different environments. Though great efforts have been made to make the models as realistic as possible, the number of available measurements is insufficient for that purpose; furthermore, it was acceptable to do some (over)simplifications that affect the absolute performance, but not the relative behavior of the different proposals.

A major challenge for the channel modeling activities derived from the fact that the PAR and call for proposals does not mandate a specific technology, and not even a specific frequency range. For this reason, this document contains three different models:

• an ultrawideband (UWB) model, spanning the frequency range from 2 to 10 GHz. Models for any narrowband system within that frequency range can be derived by a simple bandpass filtering operation.
• an ultrawideband model for the frequency range from 100-900 MHz. Again, narrowband systems located within that frequency range can obtain their specific model by filtering.
• a narrowband model for the frequency range around 1 MHz.

The generic structure of the UWB models for the two considered frequency ranges is identical, only the parameterizations are different. The model structure for the 1MHz model is fundamentally different. All the models are time-continuous; the temporal discretization (which is required for any simulation) is left to the implementer. To further facilitate the use of the model, this document also includes a MATLAB program for the generation of impulse responses, as well as Excel tables of impulse responses. The use of these stored impulse responses are mandatory for the simulations of systems submitted to 802.15.4a.

The main goals of the model were the modeling of attenuation and delay dispersion. The former subsumes both shadowing and average pathloss, while the latter describes the power delay profile and the small-scale fading statistics; from this, other parameters such as rms delay spread, number of multipath components carrying
The channel modeling subgroup started its activities at the meeting in September 2003 (Singapore), and is submitting this final report in September 2004 (Berlin) for vote by the full group. During the course of this year, progress was made mainly through bi-weekly phone conferences as well as at the IEEE 802 meetings; the minutes of all these meetings have been presented in documents [?], [?], [?]. A large number of documents on specific topics has been presented to the subgroup at the IEEE 802 meetings; they can be found on the www.802wirelessworld.com server, and are cited where appropriate in this document. Appreciation is extended to all the participants from academia and industry, whose efforts made this model possible.

The remainder of the document is organized the following way: Section II gives an overview of the considered environments, as well as the definitions of the channel parameters that will be used in later sections. Section III describes general procedures for the measurement and the evaluation of the data, as recommended by the modeling subgroup, and used for the extraction of the data. Section IV contains the generic channel model for UWB. Sections V and VI contain the parameterizations for the 2-10 GHz and the 100-900MHz range, respectively. Next, we describe the narrowband model for 1MHz. A summary and conclusion wrap up the report. Appendices contain a summary of all measurement documents and proposals presented to the group; a MATLAB program for the generation of impulse responses, and some example realizations.

B. Environments

>From the "call for applications", we derived a number of environments in which 802.15.4a devices should be operating. This list is not comprehensive, and cannot cover all possible future applications; however, it should be sufficient for the evaluation of the model:

1) Indoor residential: these environments are critical for "home networking", linking different appliances, as well as danger (fire, smoke) sensors over a relatively small area. The building structures of residential environments are characterized by small units, with indoor walls of reasonable thickness.

2) Indoor office: for office environments, some of the rooms are comparable in size to residential, but other rooms (especially cubicle areas, laboratories, etc.) are considerably larger. Areas with many small offices are typically linked by long corridors. Each of the offices typically contains furniture, bookshelves on the walls, etc., which adds to the attenuation given by the (typically thin) office partitionings.

3) Industrial environments: are characterized by larger enclosures (factory halls), filled with a large number of metallic reflectors. This is anticipated to lead to severe multipath.

4) Body-worn devices: communication between devices located on the body, e.g., for medical sensor communications, "wearable" cellphones, etc. Due to the fact that the main scatterers is in the nearfield of the antenna, and the generally short distances, the channel model can be anticipated to be quite different from the other environments.

5) Outdoor. While a large number of different outdoor scenarios exist, the current model consists only a suburban-like microcell scenario, with a range of less than 300m.

6) Agricultural areas/farms: for those areas, few propagation obstacles (silos, animal pens), with large distances in between, are present. Delay spread can thus be anticipated to be smaller than in other environments.

7) Disaster areas: for this model, we include mainly propagation through avalanches in the model, for the recovery of victims. Related important applications would include propagation through rubble (e.g., after an earthquake), again for victim recovery and communications between emergency personnel. However, no measurement data were available for the latter case.

II. GENERIC CHANNEL MODEL

In this chapter, we describe the generic channel model that is used for both the 100-900MHz and the 2-10 GHz model. An exception to this case is the "body-area network", which shows a different generic structure, and thus will be treated in a separate chapter. Also, the structure for the 1MHz model is different, and will be treated in a separate chapter.
A. Pathloss

The pathloss is defined as

\[ PL(d) = \frac{E\{P_{RX}(d)\}}{P_{TX}} \]  

where \( P_{TX} \) and \( P_{RX} \) are transmit and receive power, respectively, \( d \) is the distance between transmitter and receiver, \( f_c \) is the center frequency, and the expectation \( E\{\} \) is taken over an area that is large enough to allow averaging out of the shadowing as well as the small-scale fading \( E\{\} = E_{lsf}\{E_{ssf}\} \). Due to the frequency dependence of propagation effects in a UWB channel, the wideband pathloss is a function of frequency as well as of distance. It thus makes sense to define a frequency-dependent pathloss (related to wideband pathloss suggested in Refs. [1], [2])

\[ PL(f, d) = E\left\{ \int_{f-\Delta f/2}^{f+\Delta f/2} |H(f', d)|^2 df' \right\} \]  

where \( \Delta f \) is chosen small enough so that diffraction coefficients, dielectric constants, etc., can be considered constant within that bandwidth; the total pathloss is obtained by integrating over the whole bandwidth of interest. Integration over the frequency and expectation \( E_{ssf}\{\} \) thus essentially have the same effect, namely averaging out the small-scale fading. $$ $$

Discussion on a possible scaling of measurement results with frequency to account for the antennas $$ $$

To simplify computations, we assume that the pathloss as a function of the distance and frequency can be written as a product of the terms

\[ PL(f, d) = PL(f) PL(d). \]  

The frequency dependence of the pathloss is given as [3], [4]

\[ \sqrt{PL(f)} \propto f^{-\kappa} \]  

The distance dependence of the pathloss in dB is described by

\[ PL(d) = PL_0 + 10n \log_{10} \left( \frac{d}{d_0} \right) \]  

where the reference distance \( d_0 \) is set to 1 m, and \( PL_0 \) is the pathloss at the reference distance. The pathloss at the reference distance is computed according to the free-space pathloss law. \( n \) is the pathloss exponent. The pathloss exponent also depends on the environment, and on whether a line-of-sight (LOS) connection exists between the transmitter and receiver or not. Some papers even further differentiate between LOS, "soft" NLOS (non-LOS), also known as "obstructed LOS" (OLOS), and "hard NLOS". LOS pathloss exponents in indoor environments range from 1.0 in a corridor [3] to about 2 in an office environment. NLOS exponents typically range from 3 to 4 for soft NLOS, and 4 – 7 for hard NLOS. Note that this model is no different from the most common narrowband channel models. The many results available in the literature for this case can thus be re-used.

Remark 1: the above model for the distance dependence of the pathloss is known as "power law". Another model, which has been widely used, is the "breakpoint model", where different attenuation exponents are valid in different distance ranges. Due to the limited availability of measurement data, and concerns for keeping the simulation procedure simple, we decided not to use this breakpoint model for our purposes.

Remark 2: Refs. [4], [5], [6], [7], had suggested to model the pathloss exponent as a random variable that changes from building to building specified as a Gaussian distribution. The of the pathloss will be truncated to make sure that only physically reasonable exponents are chosen. This approach shows good agreement with measured data; however, it leads to a significant complication of the simulation procedure prescribed within 802.15.4a, and thus was not adopted for our model.
B. Shadowing

Shadowing, or large-scale fading, is defined as the variation of the local mean around the pathloss. Also this process is fairly similar to the narrowband fading. The pathloss (averaged over the small-scale fading) in dB can be written as

$$PL(d) = PL_0 + 10n \log_{10} \left( \frac{d}{d_0} \right) + S$$

where $S$ is a Gaussian-distributed random variable with zero mean and standard deviation $\sigma_S$.

Remark 3: While the shadowing shows a finite coherence time (distance), this is not considered in the model. The simulation procedure in 802.154a prescribes that each data packet is transmitted in a different channel realization, so that correlations of the shadowing from one packet to the next are not required/allowed in the simulations.

C. Power delay profile

The impulse response of the SV model is given in general as

$$h_{\text{discr}}(t) = \sum_{l=0}^{L} \sum_{k=0}^{K} a_{k,l} \delta(t - T_l - \tau_{k,l})$$

where $a_{k,l}$ is the tap weight of the $k^{th}$ component in the $l^{th}$ cluster, $T_l$ is the delay of the $l-th$ cluster, $\tau_{k,l}$ is the delay of the $k-th$ MPC relative to the $l-th$ cluster arrival time $T_l$. Following [?], the number of clusters $L$ is an important parameter of the model. It is assumed to be Poisson-distributed

$$pdf_L(L) = \frac{(L)^L \exp(-L)}{L!}$$

so that the mean $L$ completely characterizes the distribution.

By definition, we have $\tau_{0,l} = 0$. The distributions of the cluster arrival times and the ray arrival times are given by a Poisson processes

$$p(T_l|T_{l-1}) = \Lambda_l \exp \left[ -\Lambda_l(T_l - T_{l-1}) \right], \ l > 0$$

$$p(\tau_{k,l}|\tau_{(k-1),l}) = \lambda \exp \left[ -\lambda(\tau_{k,l} - \tau_{(k-1),l}) \right], \ k > 0$$

where $\Lambda_l$ is the cluster arrival rate, and $\lambda_l$ is the ray arrival rate.

Remark 4: while a delay dependence of these parameters has been conjectured, no measurements results have been found up to now to support this.

The next step is the determination of the cluster powers and cluster shapes.

$$E\{|a_{k,l}|^2\} = \begin{cases} \Omega_l \frac{K_{i,l}}{1 + K_{r,l}} \gamma_l \exp(\tau_{k,l}/\gamma_l) & \text{for } k = 0 \\ \Omega_l \frac{1}{1 + K_{r,l}} \gamma_l \exp(-\tau_{k,l}/\gamma_l) & \text{otherwise} \end{cases}$$

where $\Omega_l$ is the integrated energy of the $l-th$ cluster, $K_{r,l}$ is the Rice factor of the $l-th$ cluster, and $\gamma_l$ is the intra-cluster decay time constant. The cluster decay rates are found to depend linearly on the arrival time of the cluster,

$$\gamma_l \propto k_\gamma T_l + \gamma_0$$

The mean (over the cluster shadowing) mean (over the small-scale fading) energy (normalized to $\gamma_l$), of the $l-th$ cluster follows in general an exponential decay $\exp(-T_l/\Gamma)$, with lognormal variations $\sigma_{\text{cluster}}$ around it.

$$f[\tau, a, b, c] = -\gamma_1 \tau + b + c \exp[-dx]$$
The above parameters give a complete description of the power delay profile. Auxiliary parameters that are helpful in many contexts are the mean excess delay, rms delay spread, and number of multipath components that are within 10 dB of the peak amplitude. Those parameters are used only for informational purposes.

Remark 5: The rms delay spread is a quantity that has been used extensively in the past for the characterization of delay dispersion. It is defined as the second central moment of the PDP:

\[ S_T = \sqrt{\frac{\int_{-\infty}^{\infty} P(\tau)\tau^2 d\tau}{\int_{-\infty}^{\infty} P(\tau) d\tau} - \left(\frac{\int_{-\infty}^{\infty} P(\tau)\tau d\tau}{\int_{-\infty}^{\infty} P(\tau) d\tau}\right)^2}. \]  

and can thus be immediately related to the PDP as defined from the SV model. However, it is not possible to make the reverse transition, i.e., conclude about the parameters of the SV model from the rms delay spread. This quantity is therefore not considered henceforth as a basic quantity, but only as auxiliary parameter that allows better comparison with existing measurements.

It is also noticeable that the delay spread depends on the distance, as many measurement campaigns have shown. However, this effect is neglected in our channel model. The main reason for that is that it makes the simulations (e.g., coverage area) significantly simpler. As different values of the delay spread are implicit in the different environments, it is anticipated that this simplification does not have an impact on the selection, which is based on the relative performance of different systems anyway.

Remark 6: Another auxiliary parameter is the number of multipath components that is within \(x\) dB of the peak amplitude, or the number of MPCs that carries at least \(y\)% of the total energy. Those can be determined from the power delay profile in conjunction with the amplitude fading statistics (see below) and therefore are not a primary parameter.

D. Small-scale fading

The distribution of the small-scale fading is Nakagami

\[ pdf(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp\left(-\frac{m}{\Omega} x^2\right), \]  

where \(m \geq 1/2\) is the Nakagami m-factor, \(\Gamma(m)\) is the gamma function, and \(\Omega\) is the mean-square value of the amplitude. A conversion to a Rice distribution is possible with the conversion equations

\[ m = \frac{(K_r + 1)^2}{(2K_r + 1)} \]  

and

\[ K_r = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}. \]

where \(K\) and \(m\) are the Rice factor and Nakagami-m factor respectively.

The parameter \(\Omega\) corresponds to the mean power, and its delay dependence is thus given by the power delay profile above. The \(m\)-parameter is modeled as a lognormally distributed random variable, whose logarithm has a mean \(\mu_m\) and standard deviation \(\sigma_m\). Both of these can show a delay dependence can have a delay dependence

\[ \mu_m(\tau) = \mu_0 - k_m \tau \]  

\[ \sigma_m(\tau) = \sigma_0 - \tilde{k}_m \tau \]

For the first component of each cluster, the Nakagami factor is modeled differently. It is assumed to be deterministic, with a delay dependence that is different from those of the other components

\[ \mu_m(\tau) = \tilde{\mu}_0 - \tilde{k}_m \tau \]

The polarity (phase) is uniformly distributed, i.e., for a bandpass system, the phase is taken as a uniformly distributed random variable from the range \([0, 2\pi]\) in the case of a bandpass system, while for a baseband system, the polarity is +/- 1 with equal probability for all components with the exception of the LOS, which always has a polarity +1.
E. Antenna efficiency

In the link budget for the AWGN case, it is assumed that the antennas have isotropic radiation patterns and 100% efficiency. This is typically not fulfilled in realistic situations. Especially, the presence of a person (user) close to the antenna will lead to an attenuation. Measurements have shown this process to be stochastic, with attenuations varying between 1dB and more than 10dB, depending on the user [?]. However, we have decided - for the sake of simplicity - to model this process by a "antenna attenuation factor" that is fixed, and includes finite efficiency, absorption by bodies, and all other effects. The only exception is the 1MHz model - antenna efficiencies in that frequency range are far lower, and have to be modeled separately.

Directional information and polarization are not included in the model, due to lack of available measurements.

F. Complete list of parameters

The considered parameters are thus \( d_0, n, \sigma_S, A_{\text{ant}}, \kappa, \bar{\Lambda}, \Lambda, \lambda, \Gamma, K_{\ell, l}, k_\gamma, \gamma_0, \sigma_{\text{cluster}}, m_0, k_m, \bar{m}_0, \bar{k}_m, \bar{m}_1, \bar{k}_m, \bar{m}_0, \bar{k}_m \).

G. Flow graph for the generation of impulse responses

The above specifications are a complete description of the model. In order to help a practical implementation, the following procedure suggests a "cooking recipe" for the implementation of the model:

- select

III. UWB MODEL PARAMETERIZATION FOR 2-10 GHz

A. Residential environments [8]

The model was extracted based on measurements that cover a range from ??? to ???

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Residential</th>
<th>NLOS</th>
<th>comments</th>
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<td>( A_{\text{ant}} )</td>
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<td>3dB</td>
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<td>( \kappa )</td>
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<td>( m_0 )</td>
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</tr>
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<td>0.73±0.25</td>
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B. Indoor office environment

The model was extracted based on measurements that cover a range from $$$ ??? \text{ to } ??? $$$

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<th>NLOS</th>
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<td>$A_{\text{ant}}$</td>
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</tr>
<tr>
<td>$\kappa$</td>
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Power delay profile

$\Lambda$ [1/ns] 0.016 0.19
$\lambda$ [1/ns] 0.22 0.71
$\Gamma$ [ns] 14.6 19.8
$k_{\gamma}$ 0 0
$\gamma_0$ [ns] 6.4 11.2
$\sigma_{\text{cluster}}$ [dB]

Small-scale fading

$m_0$ 0.42dB 0.50dB
$k_m$ 0 0
$\tilde{m}_0$ 0.31 0.25
$\tilde{k}_m$ 0 0

What is $r$?
missing parameters: $K_{l,j}, \tilde{m}_0, \tilde{k}_m, \Lambda$

C. Outdoor environment

The model was extracted based on measurements that cover a range from $$$ ??? \text{ to } ??? $$$
### Residential LOS NLOS comments

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**Pathloss**

**Power delay profile**

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<tr>
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**Small-scale fading**

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### Industrial LOS NLOS comments

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**Pathloss**

**Power delay profile**

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**Small-scale fading**

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### D. Industrial environments

The model was extracted based on measurements that cover a range from ??? to ???

### Industrial LOS NLOS comments

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<td>-</td>
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<tr>
<td>$\sigma_S$</td>
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</tr>
<tr>
<td>$PL_0$</td>
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<tr>
<td>$A_{ant}$</td>
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<tr>
<td>$\kappa$</td>
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**Pathloss**

**Power delay profile**

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</thead>
<tbody>
<tr>
<td>$\Lambda$ [1/ns]</td>
<td>0.0709</td>
<td>0.0890</td>
</tr>
<tr>
<td>$\lambda$ [1/ns]</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Gamma$ [1/ns]</td>
<td>3.10</td>
<td>5.83</td>
</tr>
<tr>
<td>$k_\gamma$</td>
<td>0.21</td>
<td>0.44</td>
</tr>
<tr>
<td>$\gamma_0$ [1/ns]</td>
<td>0.15</td>
<td>0.30</td>
</tr>
</tbody>
</table>

**Small-scale fading**

<table>
<thead>
<tr>
<th>$m_0$</th>
<th>0.36 dB</th>
<th>0.30 dB</th>
</tr>
</thead>
</table>

**missing parameters:** $K_{t,l}, \tilde{m}_0, \tilde{k}_m, \mathcal{L}$
IV. UWB MODEL PARAMETERIZATION FOR 100-900 MHz

V. BODY AREA NETWORK

VI. CHANNEL MODEL FOR 1MHZ CARRIER FREQUENCY

VII. SUMMARY AND CONCLUSIONS

REFERENCES


VIII. APPENDIX: SUMMARY OF ALL CONTRIBUTIONS

A. Abstract for the small scale fading document

This document was intended to describe small scale fading phenomena and the statistical methods to model them. A short overview of linear time varying (LTV) and linear time invariant (LTI) stochastic systems is given, followed by a tutorial on hypothesis testing with common goodness-of-fit tests. Since most measurements for UWB channels are carried out in the frequency domain, another section deals with the way a VNA measures the channel, and the relation between the measured data points and the channel transfer function. This document was never finished since my research lead me to conclude that hypothesis testing is not the right way to establish a model of the small scale tap statistics, hence there are large gaps in the exposition, which most certainly will never be filled.
B. Abstract for the measurement document (802.15-04-0447-00-004a):

This presentation describes UWB channel measurements from 2 GHz to 8 GHz in the frequency domain, conducted in two office buildings at ETH Zurich, Switzerland. Measurements were taken for LOS, OLOS and NLOS settings in a corridor and a large lobby, with transmitter-receiver separations ranging from 8 m to 28m. The focus of the measurements was to establish a model suitable for theoretical analysis, but we also used the measured data to extract the IEEE 802.15.4a standard model parameters as presented in this document. We use a model selection approach to conclude that tap amplitude statistics are adequately described by the Rayleigh distribution in most cases, while the lognormal model as used by the IEEE 802.15.3a working group shows a consistently bad fit.

IX. APPENDIX: MATLAB PROGRAM FOR GENERATION OF IMPULSE RESPONSES

X. APPENDIX: RECOMMENDATIONS FOR MEASUREMENT PROCEDURE AND DATA EXTRACTION

This appendix gives recommendations for the performance of measurements, and the extraction of data. Most of these procedures have been followed in the course of the establishment of the model, though some of them were established only after many of the measurements have been performed. In any case, this part of the documentation is only informative.

A. Measurement procedure

Multipath profiles are to be measured at various locations, so that the statistics can be determined. We have to distinguish four different scales:

1) Small-scale fading:
   In order to determine this, a sufficient number of measurement points has to be taken in an area where large-scale parameters like shadowing are identical. Experience shows that some 50 measurement points per area are a minimum. The measurement point must be spaced lambda/2 or more apart, to allow the measurement points to experience independent fading (though for a small angular spread, this is not guaranteed with this spacing). The different realizations of the channel can be achieved by moving either the TX and/or the RX. Note that if the measurements are done in the 100-900 MHz range, it might be difficult to fit 50 measurement points into a small-scale area when only one of the link ends is moved. It is also important that the statistics within the measurement area are stationary. For example, the situation should not occur where one measurement point has a LOS, while another is shadowed behind an obstacle.

2) Large-scale fading:
   Different areas within one building should be measured, that are far enough apart that large-scale propagation processes (including shadowing) are different from area to area. However, the absolute distance between TX and RX should be the same for the different areas.

3) Large-scale areas with different distances between TX and RX should be measured.

4) Variations from building to building should be measured.

The statistics for all of the different scales should be extracted. When measurements of the angular spectra are also desired, this complicates the situation. The reason for this is the different requirement for the spacing of the (small-scale) measurement points. For the extraction of the small-scale statistics, we want the measurement points as far apart as possible - at a minimum, lambda/2 for the LOWEST involved frequency. For the determination of the angular spectra, we need the measurement points no farther apart than lambda/2 for the HIGHEST involved frequency. The main emphasis of the measurements for the 4a channel model will lie on the small-scale statistics, not on angular spectra.

***********************************************************

Parameters that must be determined

- Frequency range
- Number of frequency points
- Number of array elements
- Array element spacing
- Transmit power
- Number of measurement points
- IF bandwidth
- Estimated runtime for one measurement

Equipment to bring
- Network analyser
- Spectrum analyser (to check interference level)
- (At least) 2 HF antenna cables of desired length – with calibration (attenuation) curves
- 2 antennas - with calibration curves
- 2 virtual arrays - with stepper motors
- 2 tripods of the same (achievable) height

Information of the measurement site
- Maps - with lengths, scale and material information
- Pre-determined measurement positions, marked on the map

B. Extraction of large-scale parameters

This section describes the more general case where the pathloss exponent and the shadowing variance are treated as random variables. The case where they are considered as deterministic variables follows as a special case.

Unlike the narrow band case, it has been observed that the path loss in an UWB system depends on both distance \( d \) and frequency \( f \). This frequency dependency will complicate the large-scale parameter extraction procedure. However, it has been reported in [1,2] that if we consider a spatially averaged data instead of a single snap shot, this frequency dependency can be removed. Thus in this document, we will use the spatially averaged data to extract the large-scale parameters.

In [1], it was observed that the path loss exponent, \( \gamma \) and the standard deviation of the shadowing component, \( \sigma \) varied from one building to another and therefore, they were modeled by random variables as shown below:

\[
\gamma = \mu + \sigma n_1
\]
\[
\sigma = \mu + \sigma n_3
\]

where \( n_1 \) and \( n_3 \) are zero mean, unit variance Gaussian random variables. Now the path loss can be written as

\[
PL(d) = PL0 + 10\mu \lambda \log 10(d) + 10\sigma n_1 \log 10(d) + n2\mu \sigma + n2n3\sigma
\]
\[
PL(d) - PL0 = 10\mu \lambda \log 10(d) + 10\sigma n_1 \log 10(d) + n2\mu \sigma + n2n3\sigma
\]

(22)

where \( n3 \) is zero mean, unit variance Gaussian random variable and \( PL0 \) denotes the path loss at a reference distance.

As one can see from (22), we have 4 large-scale parameters to be extracted. At a fixed distance, \( d \) the first term in (22) is a constant (median path loss) and the last three terms together have a random variation about the median path loss. As explained in [1,2], these last three terms can be approximated as zero mean Gaussian variate with standard deviation of \( \sigma_{var} \), where

\[
\sigma_{var} = \sqrt{100\sigma^2_n (\log_{10} d)^2 + \mu^2 + \sigma^2_\sigma}
\]

(23)

In this case, we need to extract only two parameters: \( \mu \lambda \) and \( \sigma_{var} \). From the above arguments, we can see that at fixed distance \( d \), \( (PL(d) - PL0) \) is a Gaussian random variate with mean, \( 10\mu \lambda \log 10(d) \) and standard deviation \( \sigma_{var} \).
How to get these parameters?
- From the frequency domain responses at different homes/rooms, get the spatially averaged path loss, $PL(d)$ at various distances.
- Plot the $10 \log_{10}(PL(d) - PL_0)$ vs $10\log_{10}(d)$ and apply the linear regression fit.
- From the gradient, calculate $\mu/\lambda$. Square root of the second central moment will give the $\sigma_{var}$.
- Verify the validity of Gaussian variate assumption by comparing the empirical c.d.f. and the theoretical Gaussian c.d.f.

What about the other parameters $\sigma/\lambda$, $\mu/\sigma$ and $\sigma/\sigma$?
As we have explained earlier, $\gamma$ varies with from building to building and $\sigma$ varies from location to location. One way to extract these parameters is to obtain the $\gamma$ and $\sigma$ over various buildings and locations and then calculate the respective means ($\mu/\sigma, \mu/\lambda$) and the variances ($\sigma/\sigma, \sigma/\lambda$).

C. Extraction of SV parameters

We consider the SV model as defined in Chapter 2 of the main text.

Fig. 1 and Fig. 2 illustrate the S-V model channel impulse responses (CIRs) and double exponential decay model, respectively.

![SV2.eps](width=10.00in,height=5.69in)

1) Data Post-Processing: Since the measurement system measured the “radio channel” (i.e. including the effect of amplifiers, cables and antennas), in order to remove these hardware effects, all raw data are normalized with the calibration data so that only the “propagation channel” data will be used for further analysis. For measurements conducted using VNA, the CTFs are transformed into the CIRs through inverse Fourier transform (IFT). Frequency domain windowing is applied prior to the transformation to reduce the leakage problem. Then, the CIRs are analyzed by divided the temporal axis into small intervals (or delay bins), $\Delta \tau$. This delay bin is corresponding to the width of a path and is determined by the reciprocal of the bandwidth swept (i.e. time resolution of the measurement system). The CIRs are then normalized such that the total power in each power delay profile (PDP), $P(\tau)$, is equal to one. A cutoff threshold of 20 dB below the strongest path was applied to the PDP so that any paths arrived below his threshold is set to zero. This is to ensure that only the effective paths are used for the channel modeling. The initial delay for each of the transmission links was extracted from the PDP. This value was removed from the results so that all PDPs can be aligned with first path arrives at 0 ns.

2) Cluster Identification: The first task is to identify clusters. Different researchers have different definitions of a cluster. The position and the size of the clusters will be heavily dependent on the superstructure and physical layout of the considered environments. However, clustering identification employing statistical techniques such as clustering algorithms are inappropriate for this application as it is very difficult to develop a robust algorithm for the automatic identification of cluster regions. Thus, cluster regions were selected manually by visual inspection.

3) Arrival Statistics: In order to analyze the statistics of the clustering effects, the clusters in each data set must be identified. With the times and amplitudes of all major arrivals identified, as well as their clustering patterns, the data could be used to analyze the statistics and arrive at a model. As shown in Section II, there are 5 key parameters that define the S-V model:
- $\Lambda$ is the cluster arrival rate
- $\lambda$ is the ray arrival rate, i.e. the arrival rate of path within each cluster
- $\Gamma$ is the cluster exponential decay factor
- $\gamma$ is the ray exponential decay factor
- $\sigma$ is the standard deviation of the lognormal fading term (dB)
Following [13], the above parameters can be found using “brute force search” by trying to fit the measurement data to match different important characteristics of the channel. The main characteristics of the channel that are used to derive the above model parameters are the following:

- Mean excess delay, \( \tau_m \)
- rms delay spread, \( \tau_{rms} \)
- Number of MPCs within 20 dB threshold, \( N P_{20dB} \).

Following the methodology in [16], firstly, the cluster and ray decay time constants, \( \Gamma \) and \( \gamma \), were estimated by superimposing clusters with normalized amplitudes and time delays and selecting a mean decay rate. For example, in order to estimate \( \Gamma \), the first cluster arrival in each set was normalized to an amplitude of one and a time delay of zero. All cluster arrivals were superimposed and plotted on a semi-logarithmic plot. The estimate for \( \Gamma \) was found by curve fitting the line (representing an exponential curve) such that the mean squared error was minimized. Similarly, in order to estimate \( \gamma \), the first arrival in each cluster was set to a time of zero and amplitude of one, and all other ray arrivals were then adjusted accordingly and superimposed. Following this model, the best fit exponential distributions were determined from the cluster and ray arrival times, respectively. In order to estimate the Poisson cluster arrival rate, \( \Lambda \) the first arrival in each cluster was considered to be the beginning of the cluster, regardless of whether or not it had the largest amplitude. The arrival time of each cluster was subtracted from its successor, so that the conditional probability distribution given in (21) could be estimated. The Poisson ray arrival rate, \( \lambda \) was guessed based on the average separation time between arrivals. Estimates for \( \Lambda \) and \( \lambda \) were both done by fitting the sample pdf to the corresponding probability for each bin. The fitting was done using a least mean square criterion.

For the case of overlapping clusters, procedure as proposed in [21] is adopted. By assuming that each cluster has an exponential shape, a straight-line extrapolation function (in dB) is deployed on the first cluster and then subtract the PDP of the first cluster from the total PDP. Then, the next non-overlapping region is used to extract the decay factor for the next cluster. This process is repeated for all clusters in the total PDP until the last cluster is reached. Note that the powers of overlapping rays are calculated so that the total sum of the powers of overlapping rays corresponding to different clusters equals to the powers of the original total PDP. More details of this procedure is reported in [21].

D. Extraction of amplitude statistics

1) Linear time Varying Systems: Modeling Radio channels is a complicated task. The complexity of the solution to Maxwell’s equations needs to be reduced to a couple of parameters and some mathematically amenable formulas. The two most important steps towards this goal are the assumption of a linear channel and the description by stochastic methods. Linearity follows from Maxwell’s theory as long as the materials are linear. This is a good assumption in general. A stochastic description helps to overcome the complexity of the real propagation environment. The tradeoff here is between the optimal utilization of site-specific propagation features and system robustness. A system designed with full knowledge of the propagation conditions at a certain site would be able to exploit these conditions, resulting in superior performance, whereas a system design based on a stochastic channel model will only achieve average performance — but it will achieve this performance at a wide variety of sites whereas the former will not.

a) The System Functions: The most general description within the framework outlined is thus a stochastic linear time-varying (LTV) system. In a classical paper, Bello [9] derived the canonical representation in terms of system functions. The input-output relation is described by the two-dimensional linear operator with kernel \( h_0(t, t') \) as\(^1\) \( y(t) = (x)(t) = \int h_0(t, t')x(t')dt' \). The kernel represents the response of the system at time \( t \) to a unit impulse launched at time \( t' \). A more convenient representation for the following derivations can be obtained by changing the time origin\(^2\): \( h(t, \tau) = h_0(t, t - \tau) \), representing the response of the system at time \( t \) to a unit impulse launched \( \tau \) seconds earlier. This representation is commonly referred to as the time-varying impulse response. The input-output relation now reads \( y(t) = \int h(t, \tau)x(t - \tau)d\tau \). Equivalent representations can be obtained by Fourier transforms of the time-varying impulse response. \( (t, f) \)

\(^1\)Unless otherwise indicated, integrals are from \(-\infty \) to \(+\infty \).

\(^2\)The following choice of the time origin is just one possibility. For an in depth discussion see the report by Artés et al. [10]
b) Stochastic Characterization: For a stochastic description, the system functions are modeled as random processes. A complete characterization via associated joint distributions is far too complicated to be of practical interest, hence the description is normally confined to first and second order statistics. If the processes are Gaussian and the channel hence Rayleigh fading, a second order description is indeed a complete statistical characterization. According to the four equivalent system functions, there are four equivalent correlation functions $R_{lí}(t, t, τ, τt)
$

The WSS assumption is generally accepted, at least locally over a reasonable time frame. If shadowing effects come into play, the overall channel is of course no longer WSS. The US assumption however needs to be questioned for UWB channels since it is obvious that channel correlation properties change with frequency. One solution to this problem is to separate the nonstationary behavior from the small scale fading, as for example proposed by Kunisch and Pamp [11]; another possibility is the use of local scattering functions as proposed by Matz [12].

c) UWB Channel Models: The system functions do not depend on the bandwidth and are thus readily applicable to UWB channels. The correlation functions however only contain all statistical information if the channel process is assumed Gaussian. The notion of an infinite continuum of scatterers is approximately satisfied for narrowband channels since many reflections are not resolvable and hence the superposition of many arrivals justifies the invocation of the central limit theorem. In real world UWB channels, the number of scatterers does not necessarily scale linearly with the bandwidth and the Gaussian assumption becomes questionable due to insufficient averaging.

In narrowband channels, a model often used is a tapped delay line expression, where the channel impulse response is described as [13] $h(t, τt) = \sum_{i=1}^{N(τt)} c_{í}(t) \delta(t - τ_{í}(t)) e^{jθ_{í}(t)}. N(τ)$ is the number of multipath components, $c_{í}(t)$ the time-varying amplitude, $τ_{í}(t)$ the time-varying path delay and $θ_{í}(t)$ the time-varying phase. The underlying assumption here is that each arrival can be associated with a single propagation path, like in a ray-tracing model. This is no longer true for UWB channels since diffraction and dispersion leads to a frequency dependent distortion of every echo. One way to get around this problem is to include linear filters in every path, as in the paper by Qiu [14]. The other possibility is to continue using a tapped delay-line model but dispose of the physical intuition relating distinct paths to channel taps and consider the tapped delay line model just as the standard discretization of a bandlimited random process without ascribe any physical meaning to the individual terms.

2) VNA Channel Measurements: Because of the wide bandwidth, UWB channel measurements have been performed predominantly in the frequency domain using a vector network analyzer (VNA) [15], [11], [16], [17], [18], [19]. Because the sweep time is quite long, the channel has to remain stationary throughout the whole measurement, practically precluding the sounding of time-variant channels. It is thus sufficient to consider a time invariant channel model with impulse response $h(τ)$ and frequency response $H(f)$. The VNA samples the channel at different frequencies. However, the measurement points returned are not true samples of the channel transfer function.

a) VNA Measurement System response: An idealized VNA transmits a sinusoid for a fixed amount of time according to $x(t) = 2g_T(t) \cos 2πkFt$ where $g_T$ is a time-windowing function modeling the limited sample time, $F$ is the frequency step size and $k$ indicates the current measurement point. In the frequency domain, the transmitted signal is thus $X(f) = G_T(f - kF) + G_T(f + kF)$. The channel output as measured at the receiving end is given by $V(f) = \left( G_T(f - kF) + G_T(f + kF) \right)H(f)$. The VNA filters the signal with an RF prefilter of bandwidth $(-B_{RF}/2, B_{RF}/2)$ and baseband equivalent transfer function $G_{RF}(f)$, to obtain $Y_{RF}(f)$

b) VNA Measurement Noise: Using the standard assumption of white Gaussian noise introduced at the receiver front end with power spectral density (PSD) $2G_{RF}(f - kF)^2$, and after demodulation and baseband filtering $2G_{BB}(f)^2=2G_{BB}(f)^2$, again assuming that the baseband filters contain $n(t)g_R(t)dt$. Let the correlation function of $n(t)$ be denoted $R_n(τ)$, given by the inverse Fourier transform of $(X-D.2.b). Then the sampled noise is characterized by the correlation $N[k]N[k'] = δ_{k,k'} \int n(t)n(t')g_{RF}(t)g_{RF}(t')dt\,dt' = δ_{k,k'} \int R_n(t - t')g_{RF}(t)g_{RF}(t')dt\,dt'$. Thus, the noise samples are uncorrelated between the different frequency points since
3) Channel Tap Distribution: As already mentioned in Section X-D.1.c, the central limit theorem does not necessarily hold for UWB channels since there might not be enough unresolvable arrivals. It is thus important to characterize the distribution of the channel taps. The samples measured by the VNA are not ideal, hence any statement about densities and distributions of these VNA samples does not necessarily carry over to the original physical channel. If the channel process can be modeled as Gaussian, then the VNA samples from equation (??) will also be Gaussian. However, since the VNA samples are a smoothed version of the channel frequency response, they might still appear Gaussian due to the inherent averaging, even if the channel frequency response can no longer be described by a Gaussian process. In addition to the averaging effect, the receiver noise is always present, adding another Gaussian component. Hence to get close to the original channel, the baseband bandwidth $B_{BB}$ should be chosen as small as practically possible, and high SNR conditions should always be ensured.

a) Time Domain Channel Tap Distribution: The frequency domain representation and the time domain representations are linked via the discrete Fourier transform (DFT). Because only a finite bandwidth $B = KF$ is measured, the frequency samples are implicitly windowed. To reduce sidelobes due to the rectangular window, further windowing is often performed in practice. Let $W[k]$ denote the window function, then the IDFT is given by $y[n] = \frac{1}{K} \sum_{k=0}^{K-1} W[k] e^{j2\pi\frac{nk}{K}}$. With the definition $G(f) = \mathcal{F}(y[n])$, the DFT gives $y[n] = \frac{1}{K} \sum_{k=0}^{K-1} W[k] e^{j2\pi\frac{nk}{K}} \times \frac{1}{K} \int \mathcal{H}(\phi) e^{j2\pi f \phi} G(\phi) e^{j2\pi f \phi} d\phi$.

b) Testing Distributions: The tap gain distribution commonly refers to the distribution of the tap magnitude. Because the phase undergoes rapid changes whenever the path distance changes by more than a fraction of a wavelength, the standard assumption is a uniform phase distribution. For UWB signals with lower frequency bound over 1 GHz, this assumption still seems to be valid, hence in the following I will focus on the distribution of the $|y[n]|$ only.

The empirical probability density function (PDF) and cumulative distribution function (CDF) of a measured channel tap $y[n]$ can be obtained from the histogram, provided that a sufficient number of independent samples is available. Estimating the true distribution however is more a philosophical problem as to be of practical interest, since the concept of a true distribution drawn from which samples are observed, requires a probability model within which to operate. Hence the notion of a single true distribution is not relevant — the goal is to find a model that is supported by the measured data and at the same time amenable for analytical and simulation use. The goal is then to test a certain number of predefined mathematical models against the data. The choice of candidate PDFs in this case is based on experience and mathematical convenience. The more degrees of freedom a PDF has the better the fit in general, but the higher the complexity. Thus the right way to proceed is not to find the model with the best fit but the model attaining a prescribed goodness of fit with the least complexity. Typical candidate PDFs for mobile radio channels are Rayleigh, Rice, Nakagami, Gamma, Lognormal and to a lesser extend Weibull. This is a model selection problem problem, and several methods were developed by statisticians starting with the work of Akaike in the early 1970s [2]. Among the established criteria for model selection are the Akaike information Criterion (AIC), the Bayesian information Criterion (BIC) and the principle of minimum description length (MDL).

Surprisingly, these model selection techniques are hardly ever used in the field of channel modeling. Instead most researchers rely on hypothesis testing to find the best fitting distribution. All the candidate models have one or more free parameters, so the hypothesis is the statement that the channel tap random variable is drawn from a distribution belonging to the Rayleigh, the Rice, the Nakagami etc. family. This is a different question than the one posed before, asking for the best approximating model from a family of a priori models. there are several problems associated with the hypothesis testing approach, especially that there is no universally adopted criterion to decide in favor of one out of many candidate models, since confidence levels, discretization and parameter estimation are always left unspecified. However, because the hypothesis testing approach is prevalent in fading channel modeling, we will focus on it. A short summary of hypothesis testing and a discussion of goodness-of-fit tests is contained in Appendix X-D.6.

Some researchers propose to first estimate the parameters of all candidate PDFs and then perform the
simple hypothesis test only for these parametrized PDFs. Yet though intuitively appealing, this method is
not well justified for some tests.

The Kolmogorov-Smirnov Test
A common hypothesis test for distributions is the Kolmogorov-Smirnov test for continuous CDFs. It is
based on the fact that the test statistic \( \sqrt{n}D_n := \sqrt{n} \sup_x F_n(x) - F(x) \) has a limiting CDF for \( n \to \infty \)
which does not depend on the test CDF \( F \) and the empirical CDF \( F_n \), derived from \( n \) samples of the process. Now, if a CDF with estimated parameters is used instead of the fixed CDF, this theorem no longer holds and the test result is meaningless [?].

\( \chi^2 \) Test
The \( \chi^2 \) test was originally developed to test a sample against a discrete distribution. The procedure
can be extended to continuous distributions and it even works to some extend for distributions where the
parameters need to be estimated. Some theory and explanations are summarized in Appendix X-D.6. The
general procedure is as follows:

- Partition the range of the random variable in intervals \( C_j \). There is no rule how to choose these intervals,
  but a equidistant partition seems to make sense. Even for distributions with unlimited range, only
  a limited number \( r \) of intervals are needed, since only intervals containing measured data points are
  necessary.
- Count the number \( N_j \) of measurements that lie in each interval \( j \).
- Either estimate the parameters of the distribution under test from the unpartitioned or the partitioned
  data. See Appendix X-D.6 for elaboration.
- Compute the test statistic according to (X-D.6.b) or (X-D.6.b), depending on the type of parameter
  estimate.
- compare the statistic to the integral over the right tail of the \( \chi^2_{r-1-k} \) PDF at confidence level \( \alpha \) and with
  \( r - 1 - k \) degrees of freedom, or equivalently the value of the CDF \( Q(1 - \alpha) \). Here \( k \) is the number
  of parameters estimated from the data. Hence the number of degrees of freedom of the distribution is
  reduced if parameters need to be estimated first. If the test statistic is larger than the probability obtained
  by evaluating the integral, the hypotheses must be rejected.

4) Parameter Estimation:
   a) Nakagami \( m \) Parameter: Maximum likelihood estimation of the Nakagami parameters \( \Omega \) and \( m \) is
       not possible in closed form. Several approximations to the true ML solution exist, like the estimators recently
       proposed by Cheng [?] and Ko [?], and the classical approximation by Greenwood and Durand [?], recently
       reintroduced by Zhang [?]. The latter ML approximation is in effect for the gamma distribution, but since
       the square of a Nakagami distributed random variable is gamma distributed, the estimates are equivalent. It
       is difficult to compare the performance of the various estimators
   b) Delay Spread: Mean delay and delay spread are tied to the uncorrelated scattering assumption. In this case, the correlation matrix of the channel is diagonal, and can be obtained by averaging several measured impulse responses, also referred to as the power delay profile. The mean delay is now the mean of the PDP, the delay spread the standard deviation. Thus both parameters together are a measure about the number of independently fading taps and thus about the diversity order, although not in a precise way as analyzed in Section ?? . For these numbers to be meaningful, the PDP needs to be compactly supported and also otherwise mathematically well behaved, an assumption which can almost always be made for the channels under consideration.

In the following, we estimate the mean delay \( \overline{\tau} \) and the delay spread \( s \) of every recorded impulse response
in the different environments and for different separations between receiver and transmitter. All impulse
responses are normalized to have unit energy. Then the mean delay is given as \( \overline{\tau} = \sum_{l=1}^{L} \tilde{h}[l]l \) and the
delay spread as \( s = \sqrt{\sum_{l=1}^{L} (l - \overline{\tau})^2 \tilde{h}[l]} \) where \( \tilde{h} \) is the normalized channel impulse response. The average
empirical mean delay is recorded as \( \mu \overline{\tau} \), the standard deviation as \( \sigma_s \); the same quantities are also computed
for the empirical delay spread \( s \).
5) Common Densities: The following is a short summary of probability density functions and associated parameters for the common channel tap models and the distributions used in some of the tests. The formulas are compiled from the books by Papoulis [20], Proakis [21] and Weisstein [22].

a) Rayleigh:

- PDF $f_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$
- CDF $F_X(x) = 1 - e^{\frac{x^2}{2\sigma^2}}$
- domain $[0, \infty)$
- mean $X = \sqrt{\frac{\pi \sigma^2}{2}}$
- variance $X = \left(2 - \frac{3}{2}\right)\sigma^2$
- higher moments $X^k = (2\sigma^2)^\frac{k}{2} \Gamma \left(1 + \frac{1}{2}k\right)$

b) Rice:

- PDF $f_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2 + s^2}{2\sigma^2}} I_0 \left(\frac{sx}{\sigma^2}\right)$ with the modified Bessel function of the first kind $I_0$
- CDF $F_X(x) = 1 - Q_1 \left(\frac{s}{\sigma}, \frac{x}{\sigma}\right)$ with the Marcum $Q$ function $Q_1(a, b) = e^{-\frac{a^2 + b^2}{2}} \sum_{k=0}^\infty (\frac{a}{b})^k I_k(ab)$, $b > a > 0$
- domain $[0, \infty)$
- mean $X = \sigma \sqrt{\frac{\pi}{2}} \left(1 + \frac{s^2}{2\sigma^2}\right) I_0 \left(\frac{s}{2}\right) + s I_1 \left(\frac{s}{2}\right) e^{-\frac{s^2}{4}}$

c) Nakagami:

- PDF $f_X(x) = \frac{2}{\Omega^m} \left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-\frac{x^2}{\Omega}}$
- mean $X = \Gamma(m + 1/2) \sqrt{\frac{\Gamma(m)}{m}}$
- variance $X = \Omega \left[1 - \frac{1}{m} \left(\frac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2\right]$
- higher moments $X^n = \frac{\Gamma \left(m + \frac{1}{2}n\right)}{\Gamma(m)} \left(\frac{\Omega}{\mu}\right)^n$
- domain $(0, \infty)$
- remarks: the parameters are defined as follows
  - $\Omega = X^2$
  - $m = \frac{\Omega^2}{(X^2 - \Omega)^2}$, $m \geq \frac{1}{2}$ miscalled the fading figure
  for $m = 1$, (X-D.5.c) reduces to the Rayleigh PDF (X-D.5.a).

d) Lognormal:

- PDF $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln(x) - \mu)^2}{2\sigma^2}}$
- CDF $F_X(x) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\ln(x) - \mu}{\sqrt{2\sigma^2}}\right)\right]$
- domain $(0, \infty)$
- mean $X = e^{\mu + \frac{\sigma^2}{2}}$
- variance $X = e^{\sigma^2 + 2\mu} \left(e^{\sigma^2} - 1\right)$

e) Central $\chi^2$ with $n$ Degrees of Freedom: The central $\chi^2$ distribution arises as the distribution of the sum of $n$ independent zero mean Gaussian random variables. If the variance is normalized to unity, the following expressions are obtained.

- PDF $f_X(x) = \frac{1}{2^n \Gamma \left(\frac{n}{2}\right)} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$
- CDF $F_X(x) = 1 - \frac{\Gamma \left(\frac{1}{2}, \frac{x}{2}\right)}{\Gamma \left(\frac{n}{2}\right)}$
- domain $[0, \infty)$
mean $X = n$

• variance $X = 2n$

A slightly different result is obtained for the sum of Gaussian random variables with variance $\sigma^2$.

• $f_X(x) = \frac{x^{2n-1}}{\sigma^{2n}2^{n}1(\frac{x}{\sigma})} e^{-\frac{x^2}{2\sigma^2}}$

• mean $X = n\sigma^2$

• variance $X = 2n\sigma^4$

f) Non-central $\chi^2$: The non-central $\chi^2$ distribution arises as the distribution of the sum of $n$ independent Gaussian

6) Hypothesis Testing: The following is a short summary about hypothesis testing, extracted from the books by Papoulis [20], Bartoszyński [?] and Dixon [?]. Hypothesis testing is part of decision theory. The simplest case is the binary hypothesis testing problem, where some assumption, called the null hypothesis $H_0$ is tested against the alternate hypothesis $H_1$. The null hypothesis might be for example the assumption that the distribution of a random variable $X$ has parameter $\theta = \theta_0$. The alternate hypothesis would then be $\theta \neq \theta_0$. Hypothesis testing is not about determining whether $H_0$ or $H_1$ is true. It is to establish if the evidence in form of available data supports the hypothesis or not. Therefore the sample space is partitioned into the critical region $c$ and the region of acceptance $c^c$. Depending on the location of the data points within the sample space, the hypothesis of the test is rejected or not. Some basic terminology in hypothesis testing is summarized as follows.

• If $H_0$ is true and $\bar{c}$, $H_0$ is rejected, called a Type I error. The probability $\alpha = (\bar{c}, H_0)$ is called the significance level.

• If $H_0$ is false and $\bar{c}$, $H_0$ is accepted, called a Type II error. The probability of error is a function $\beta(\theta)$, called the operating characteristic (OC) of the test.

• The difference $P(\theta) := 1 - \beta(\theta)$ is the probability of rejecting $H_0$ when false, called the power of the test.

For so called “goodness-of-fit” tests, $H_0$ does not involve parameters. The hypothesis is, that a given function $F_0(x)$ equals the distribution $F(x)$ of a random variable $X$, $H_0 : F(x) \equiv F_0(x)$ against $H_1 : F(x) \neq F_0(x)$. These types of tests normally rely on some limiting behavior of a function of the data and the distribution under test, called the test statistic. The distribution of this statistic converges to some other distribution if the data is indeed drawn according to the distribution under test. If not, then the test statistic will yield a value that would occur only with low probability according to the limiting distribution. This probability is set by the confidence level $\alpha$ of the test, and hence hypothesis where the test statistic exceeds the value of the CDF $Q(\alpha)$ need to be rejected.

a) The Kolmogorov-Smirnov Test: Let $F_i(x)$ be the empirical estimate of the CDF of the random variable $X$ from the sample $i$ and let $F_n(x)$ be the empirical CDF obtained from $n$ independent samples. Then the distance $D_n := \sup_x F_n(x) - F(x)$ converges to zero a.s. for $n \to \infty$. Hence for large $n$, $D_n$ is close to zero if $H_0$ is true and close to $\sup_x F_n(x) - F(x)$ if $H_1$ is true. The distribution of $\sqrt{n}D_n$ can be shown to converge to the Kolmogorov distribution $\lim_{n \to \infty}(\sqrt{n}D_n \leq z) = 1 - 2\sum_{k=1}^{\infty}(-1)^{k-1}e^{-2k^2z^2} = Q(z)$. The test should reject $H_0$ if the observed value of the statistic $\sqrt{n}D_n$ exceeds the critical value determined from the right tail of the distribution according to the significance level. $Q(z)$ is tabulated in any standard textbook, eg. [?, Table A7]. The test has power 1 against any alternative in the limit $n \to \infty$.

The test only applies if the distribution $F(x)$ of the null hypothesis is fixed. If the parameters need to be estimated from the samples, the corresponding distribution $F^*(x)$ is now random, depending on the same samples as the ones used to determine the empirical distribution $F_n$, and the limiting distribution of $\sqrt{n}\sup_x F_n(x) - F^*(x)$ is not given by $Q(z)$.

b) The $\chi^2$ Test: Discrete Distribution

Let $X$ be a discrete random variable defined on some finite alphabet with associated probabilities $p_i = (X = x_i)$. In a random sample of size $N$, each letter appears with frequency $N_i$, such that $\sum N_i = N$. The vector $[N_1, \ldots, N_r]$ is called the count vector. The hypothesis to test is $H_0 : p_i = p_i^0$, $i = 1, \ldots, r$, against the general alternative $H_1 : H_0$ is false. Here $0 = [p_1^0, \ldots, p_r^0]$ is some fixed distribution. The
test statistic \( Q^2 := \sum_{j=1}^{r} \frac{(N_j-\hat{np}_j)^2}{np_j} \), i.e. a central \( \chi^2 \) distribution with \( r-1 \) degrees of freedom, if the distribution of \( X \) equals the distribution of the null hypothesis. To obtain a good approximation, the counts should exceed 10. When there are many letters in the alphabet, the approximation is good enough even if few expected frequencies are as small as 1. The critical region of the test is the right tail of the \( \chi^2 \) distribution with confidence level \( \alpha \), denoted \( \chi^2_{\alpha,r-1} \) and tabulated in any standard statistics textbook [?, Table A4]. If now the test statistic exceeds this value, then the distribution of the sample can be only be drawn according to the distribution under test with low probability (with probability less than \( \alpha \) to be precise). Hence this hypothesis has to be rejected.

### Continuous Distribution

The above outlined test can be adapted to continuous distributions by partitioning the range of the random variable \( X \), i.e. by creating \( r \) sets \( C_1, \ldots, C_r \) that are disjoint and cover the whole range\(^3\). If \( f(x) \) is the density of \( X \) specified by the null hypothesis, then \( p_j^0 = \int_{C_j} f(x)dx, \quad j = 1, \ldots, r \). The test now depends also on the choice of the sets.

#### Discrete Parametric Distribution

Often the Distribution of the null hypothesis is not completely specified, such that just the family (e.g. Bernoulli, Poisson etc.) is known and the parameters are not. Denote the \( k \)-dimensional parameter vector by \( \hat{\theta} \). Then the distribution of the discrete random variable \( X \) is given by \( p(.) = [p_1(.), \ldots, p_r(.)] \) with \( p_j(.) > 0 \). Let the maximum likelihood estimate (MLE) of \( \hat{\theta} \) be denoted by \( \hat{\theta}^* \). Then the statistic \( Q^2 := \sum_{j=1}^{r} \frac{(N_j-\hat{np}_j)^2}{np_j} \) has the limiting \( \chi^2 \) distribution with \( r-1-k \) degrees of freedom as \( n \to \infty \). Thus the test proceeds as before, but to compute the critical region, the distribution with the reduced number of degrees of freedom needs to be used.

#### Continuous Parametric Distribution

If the parametric distribution is continuous, the test methodology remains the same, i.e. the range of \( X \) needs to be partitioned and the respective probabilities are computed via the integral over the density function. However, the MLE of the parameter vector is now in general very hard to obtain. The key point is that \( \hat{\theta} \) is no longer the same for the continuous distribution and the discrete distribution obtained through partitioning. However, it is the latter MLE that is required to form the statistic (X-D.6.b). In most cases, \( \hat{\theta} \) can only be obtained numerically. An example borrowed from Bartoszyński [?] illustrates this problem. Assume \( X \sim (\mu, \sigma^2) \) and \( N_j \) is the count of observations in the interval \( [t_{j-1}, t_j) \). The MLE is the solution to the system of equations \( \hat{\mu} = \frac{\int_{t_{j-1}}^{t_j} (x-\mu)^2 dx}{\int_{t_{j-1}}^{t_j} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx} \).

If now the MLE from the complete data instead of the grouped data is used, the limiting distribution is unknown. However, there exists a bound. Let \( \hat{\theta}^* \) be the MLE of the parameter vector based on the complete observation. Then the statistic \( Q^{\hat{\theta}^*} := \sum_{j=1}^{r} \frac{(N_j-\hat{np}_j)^2}{np_j} \) satisfies, \( \text{asn} \to \infty (\frac{Q^{\hat{\theta}^*}}{\chi^2_{r-1-k}}) \lim_{n \to \infty} (Q^{\hat{\theta}^*}) \) for \( 0 \). This implies that if the hypothesis can be rejected on the basis of the partitioned distribution with the unpartitioned parameter estimates, it will also be rejected if the partitioned parameter estimates are used.

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\(^3\)The sets need not necessarily be intervals.