Space-Time Codes and Signal Processing for Slow Fading Channels

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Base Document: The presentation provides an overview of recent advances in space-time codes and signal processing. Since space-time techniques have potentially large benefits for MMDS systems, it is recommended that 802.16.3 investigate their applications in the new standard under development.

Purpose: Discussion

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Space-Time Codes and Signal Processing for Slow Fading Channels

A. Roger Hammons Jr. and Hesham El Gamal

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Germantown, Maryland USA
Space-Time Modems Exploit “Hidden” Capacity of the Multipath Channel

- **Space-time codes**: $L_t > L_r$. Seek diversity through code design.
- **BLAST**: $L_r = L_t$. Seek high throughput through signal processing.
- **Hybrid schemes**: Also possible.
Space-Time Technology: Code Design and Signal Processing

AT&T Research has popularized “Space-Time Channel Codes”
(Tarokh, Seshadri, Calderbank)
• **Primary objective**: Increased diversity
• **Method**: Channel coding performed across antennas as well as time

Lucent has popularized “Layered Space-Time Architecture” or “BLAST” (Foschini, Gans)
• **Primary objective**: Increased throughput
• **Method**: Independent spatial channels via interference avoidance and cancellation

We investigated synergistic approaches that advance the state of the art in both space-time codes and space-time modems.
Design Criteria for Space-Time Codes

Pairwise error probability for Rayleigh fading channel:

\[ P(c \rightarrow e) \leq \left( \frac{\eta E_s}{4N_0} \right)^{-r} \]

where \( r = \text{rank}(f(c) - f(e)) \) ← rank of baseband difference

\[ \eta = (\lambda_1 \lambda_2 \cdots \lambda_r)^{1/r} \] ← geometric mean of eigenvalues

Design Criteria [Fitz (Ohio State Univ.), Tarokh (AT&T)]

- Rank Criterion: Maximize diversity advantage \( r \) over all distinct code word pairs \( c \) and \( e \).
- Product Distance Criterion: Maximize coding advantage \( \eta \) over all distinct code word pairs \( c \) and \( e \).
Space-Time Modulation Format Gives 3GPP “Open Loop” Transmit Diversity

Simplified Block Diagrams of 3GPP Transmitter with “Space-Time Transmit Diversity” (STTD)

Channel Encoder → Rate Matching → Channel Interleaver → QPSK Space-Time Formatter → Diversity Antenna Pilot → MUX → Tx. Ant #1, Tx. Ant #2

QPSK Space-Time Formatter

\[
\begin{array}{c|c}
S_1 & S_2 \\
S_2^* & S_1^* \\
\end{array}
\]

Ant #1, Ant #2

Alamouti space-time “block code”

No Diversity Data Pattern

N_pilot | N_data | N_pilot | N_data

Slot 1

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Handcrafted Trellis Codes Achieving Full Spatial Diversity are Known

Tarokh-Seshadri-Calderbank (TSC) 4-State Trellis Code for QPSK Modulation

Achieves maximum 2-level spatial diversity.

Binary Formulation

\[ x_t^{(1)} = 2b_{t-1} + a_{t-1} \]
\[ x_t^{(2)} = 2b_t + a_t \]

\[ Z_4 \text{ Formulation} \]
Handcrafted Trellis Codes Achieving Full Spatial Diversity are Known

Tarokh-Seshadri-Calderbank (TSC) 8-State Trellis Code for QPSK Modulation

Binary Formulation

\[ x_t^{(1)} = 2a_{t-2} + 2b_{t-1} + a_{t-1} \]
\[ x_t^{(2)} = 2a_{t-2} + 2b_t + a_t \]

\[ Z_4 \text{ Formulation} \]

Achieves maximum 2-level spatial diversity.
Binary Criteria Identify Full-Diversity Space-Time Codes

• **BPSK Binary Rank Criterion**: Let $C$ be a linear $L \times n$ space-time code with $n \geq L$. Suppose that every non-zero binary code word $c \in C$ is a matrix of full rank over the binary field $F$. Then, for BPSK transmission, the space-time code $C$ satisfies the space-time rank criterion and achieves full spatial diversity $L$.

• **QPSK Binary Rank Criterion**: Let $C$ be a linear $L \times n$ space-time code over $\mathbb{Z}_4$ with $n \geq L$. Suppose that, for every non-zero binary code word $c \in C$, the row-based indicant $\Xi(c)$ or the column-based indicant $\Psi(c)$ has full rank $L$ over $F$. Then, for QPSK transmission, the space-time code $C$ satisfies the space-time rank criterion and achieves full spatial diversity $L$.

• **Extensions to Higher-Order Modulation**: Use multi-level construction and apply binary rank criteria to the design of the constituent codes at each level.
“Stacking” Constructions Yield New Full-Diversity Space-Time Codes

**Stacking Construction**

\[
\begin{bmatrix}
T_1(\bar{x}) \\
T_2(\bar{x}) \\
M \\
T_L(\bar{x})
\end{bmatrix}
= \begin{bmatrix}
\triangle\triangle\triangle\triangle\triangle\triangle \\
\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit \\
M M M M O M \\
\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit
\end{bmatrix} \in C
\]

\( C \) satisfies BPSK binary rank criterion iff \( T = a_1T_1 + a_2T_2 + \Lambda + a_LT_L \) is non-singular unless \( a_1 = a_2 = \Lambda = a_L = 0 \).

**Multi-Stacking Construction**

\[
\begin{bmatrix}
\triangle\triangle\triangle\triangle\triangle\triangle \\
\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit \\
M M M M O M \\
\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit
\end{bmatrix} \in C
\]

Concatenations of space-time codes satisfying binary rank criterion form full-diversity space-time codes \( C \).

**Transformation Theorem**

Full-Diversity Space-Time Code

\[
\begin{bmatrix}
\bar{c}_1 \\
\bar{c}_2 \\
M \\
\bar{c}_L
\end{bmatrix} = \begin{bmatrix}
\triangle\triangle\triangle\triangle\triangle\triangle \\
\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit \\
M M M M O M \\
\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit
\end{bmatrix} \in C
\]

Linear transformation of full-rank

\[
\begin{bmatrix}
T(\bar{c}_1) \\
T(\bar{c}_2) \\
M \\
T(\bar{c}_L)
\end{bmatrix} = \begin{bmatrix}
\text{full-rank} \\
\text{full-rank} \\
\text{full-rank} \\
\text{full-rank}
\end{bmatrix} \in T(C)
\]

Full-Diversity Space-Time Code
Convolutional Codes with Optimal $d_{\text{free}}$ Yield Full-Diversity Space-Time Codes

"Natural" space-time code associated with rate $1/L$ convolutional code:

*Multiplex $L$ output coded bits in space (among antennas) rather than time.*

Practical examples of our general space-time "Stacking Constructions."

<table>
<thead>
<tr>
<th>$L$</th>
<th>$v$</th>
<th>Connection Polynomials</th>
<th>$d_{\text{free}}$</th>
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<td>536, 466, 646, 562, 736</td>
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</table>
Performance of BPSK Space-Time Codes with 4 Transmit Antennas

- Optimal $d_{\text{free}}$ code is 3 dB better than the delay diversity scheme.
- Optimal $d_{\text{free}}$ code is 1 dB better than Fitz-Grimm zeroes symmetry code.

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Performance of BPSK Space-Time Codes with 5 Transmit Antennas

- Optimal $d_{\text{free}}$ code is 3 dB better than the delay diversity scheme.
- Optimal $d_{\text{free}}$ code is 2 dB better than Fitz-Grimm zeroes symmetry code.
Performance of QPSK Space-Time Codes in Quasi-Static Fading Channels

![Graph showing the performance of QPSK Space-Time Codes in Quasi-Static Fading Channels. The graph plots FER against SNR (dB) with two codes compared: New Code (16 state) and Tarokh's Code (16 state).]
Performance of 8-PSK Space-Time Codes in Quasi-Static Fading Channels

![Graph showing the performance of 8-PSK codes with 2 transmit and 2 receive antennas. The graph plots FER (Frame Error Rate) against SNR (Signal-to-Noise Ratio) in dB. The graph compares a new code (8-state) and Tarokh's code (8-state).]
Stacking Construction Yields New Codes for Space-Time Appliques

1. Decoded Bit Error Rate
   - L=2, Alamouti, QPSK Orthogonal Design
   - L=3, Tarokh, 16-QAM Orthogonal Design
   - L=3, Hammons-EI Gamal, QPSK 3x3 Stacking Construction

2. SNR dB

3. 2 bps/Hz
Foschini showed that outage capacity increases linearly when $L_r$ equals $L_t$.

Bit Rate = Bandwidth · Number of antennas · Bit rate/antenna · Coding rate

$30 \text{ Mbps} = 3 \cdot 5 \cdot 4 \cdot 1/2$
Layered Space-Time Architectures

**Definition:** A layer is an assignment of space-time transmission resources to a component channel encoder in which at most one antenna is available each transmitted symbol interval.

**Properties:**
- No spatial interference within a layer.
- Decoding is performed layer by layer.
- Conventional channel codes can be used.
Lucent’s BLAST Technology

Two steps to decode a given layer:

1. Subtract interference from previously decoded layers

2. Project away from interference from undecoded layers

Potential Limitations of BLAST Signal Processing

- Requires equal number of transmit and receive antennas
- Spatial diversity varies within a code word
- Errors can propagate both spatially and temporally
- Limited ability to interleave for temporal diversity
- Loss in throughput due to diagonal layering

One code word per diagonal layer
Hughes Offers Threaded Space-Time Architecture

Characteristics of Threaded Space-Time Architecture

- Generalized layering exploits spatial and temporal diversity
- Threaded space-time channel codes ensure full-diversity
- Receiver uses new, efficient, multi-user detection techniques
  - Soft-decision feedback reduces spatial error propagation.
  - Iterative MMSE processing results in a symmetrical performance.
• Iterative multi-user detection (MUD) is one key to threaded space-time architecture.
• Threaded channel codes, based on space-time principles, are optimized for MUD.
Threaded Space-Time Code Design
for Quasi-static Fading Channels

Theorem (Threaded Stacking Construction):
Let $L$ be a layer of spatial span $n$. Given binary matrices $M_1, M_2, \ldots, M_n$ of dimension $k \times \lambda$, let $C$ be the binary code of dimension $k$ consisting of all code words $g(x) = \bar{x}M_1 | \bar{x}M_2 | \Lambda | \bar{x}M_n$, where $\bar{x}$ denotes an arbitrary $k$-tuple of information bits. Let $f_L$ denote the spatial modulator having the property that the modulated symbols $\mu(\bar{x}M_j)$ are transmitted in the symbol intervals of $L$ that are assigned to antenna $j$.

Then, as the space-time code in a communication system with $n$ transmit antennas and $m$ receive antennas, the space-time code $C$ consisting of $C$ and $f_L$ achieves spatial diversity $dm$ in a quasi-static fading channel if and only if $d$ is the largest integer such that $M_1, M_2, \ldots, M_n$ have the property that

$$\forall a_1, a_2, \Lambda, a_n \in F, \quad a_1 + a_2 + \Lambda + a_n = n - d + 1:$$

$$M = [a_1M_1, a_2M_2, \Lambda, a_nM_n]$$

is of rank $k$ over $F$. 

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Threaded-STC Yields Bigger Bang than the BLAST Technology

At 1% FER, advantage is more than 3 dB.
Threaded Space-Time Outperforms AT&T’s Group Suppression Approach

At 1% FER, advantage is more than 4 dB.
Conclusions

• MMDS must contend with slow fading channels
• Space-time technology offers potentially large gains in this environment
• 802.16.3 should be aggressive in study and adoption of best space-time solutions
References

Journal Papers


• El Gamal and Hammons, “A new approach to layered space-time signal processing and code design,” accepted for publication (pending revision) in *IEEE Transactions on Information Theory*.

Conference Papers


