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## Title: Wideband Extension of the ITU profiles with desired spaced-frequency correlation.

## Source:

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#### Abstract

: The spaced-frequency correlation function (FCF) of any of the ITU channel models is oscillatory and hence does not correspond to the field measurements. To alleviate this problem, we first fit an exponential-decay power-delay profile to each of the Pedestrian A, Pedestrian B, and Vehicular A profiles giving the desired FCFs. Then for a given ITU channel model, a path is replaced by a cluster (symmetrically placed or otherwise) of paths (say 2-4) such that its FCF is 'close' to the desired one. The path delays are chosen to be a multiple of 10 ns . The power-delay profiles of the new channel models are given and their spaced-frequency correlation functions are plotted from $0-20 \mathrm{MHz}$. This approach modifies the existing ITU models making them suitable for multi-carrier DO evolution.


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## 1 Spaced-Frequency Correlation Function (FCF)

Let us consider a $L$-path wide-band channel model given by

$$
\begin{equation*}
c(\tau ; t)=\sum_{l=1}^{L} g_{l}(t) \delta\left(\tau-\tau_{l}\right), \tag{1}
\end{equation*}
$$

where $g_{l}(t)$ is the complex base-band time-varying channel gain of the $l$ th path, $E\left\{\left|g_{l}(t)\right|^{2}\right\}=\sigma_{l}^{2}$, with different paths having un-correlated channel gains, and $\delta(\cdot)$ is the Dirac-delta function.

The spaced-frequency spaced-time correlation function is defined in [1] as

$$
\begin{equation*}
\phi_{C}(\Delta f ; \Delta t)=E\left\{C^{*}(f ; t) C(f+\Delta f ; t+\Delta t)\right\}, \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
C(f ; t)=\int_{-\infty}^{\infty} c(\tau ; t) \exp (-j 2 \pi f \tau) d \tau \tag{3}
\end{equation*}
$$

The spaced-frequency correlation function of the channel is given by $\phi_{C}(\Delta f ; 0)$.
It can be shown that the spaced-frequency correlation function for the channel model in Eq. 1 is given by

$$
\begin{equation*}
\phi_{C}(\Delta f ; 0)=\sum_{l=1}^{L} \sigma_{l}^{2} \exp \left(-j 2 \pi \Delta f \tau_{l}\right) . \tag{4}
\end{equation*}
$$

## 2 FCF for the ITU channel models

The power-delay profiles for the ITU channel models are given in Table 1 .

Table 1: Power-delay profiles of the ITU channel models.

| Ped. A |  | Ped. B |  | Veh. A |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Power | Delay (ns) | Power | Delay (ns) | Power | Delay (ns) |
| 0.8893 | 0 | 0.4057 | 0 | 0.4850 | 0 |
| 0.0953 | 110 | 0.3298 | 200 | 0.3853 | 310 |
| 0.0107 | 190 | 0.1313 | 800 | 0.0611 | 710 |
| 0.0047 | 410 | 0.0643 | 1200 | 0.0485 | 1090 |
| - | - | 0.0673 | 2300 | 0.0153 | 1730 |
| - | - | 0.0017 | 3700 | 0.0049 | 2510 |

FCF for the ITU channel models are plotted in Figs. 1. 2, and 3. As one can see from the figures, the FCF is oscillatory in nature. It is easy to show that if $\tau_{l}$ 's are rational i.e. $\tau_{l}=p_{l} / q_{l}$, where $p_{l}, q_{l}$ are integers for all $l$, then using Eq. 4

$$
\begin{equation*}
\phi_{C}\left(\Delta f+\operatorname{lcm}\left(q_{1}, q_{2}, \cdots\right) ; 0\right)=\phi_{C}(\Delta f ; 0), \tag{5}
\end{equation*}
$$



Figure 1: Absolute value of the spaced-frequency correlation function for the Pedestrian A channel model from Eq. 4 .
where $\operatorname{lcm}\left(q_{1}, q_{2}, \cdots\right)$ denotes the least common multiple of $q_{1}, q_{2}, \cdots$. So FCF is a periodic function if $\tau_{l}$ 's are rational as is the case for the ITU channel models.

This oscillatory behavior is however, contrary to the field data where FCF decays with the frequency difference [2, 3] for several cases of interest. The genesis of the oscillatory behavior is essentially due to the discrete nature of the channel with finite paths in Eq. 1. It is well-understood that there are a continuum of paths in reality and the approximation of a continuum of paths to one path with total average power is done for the ease of simulation. But this has an un-intended consequence that the FCF becomes oscillatory. This was not a big issue till now since for single carrier DO systems, FCF is reasonably well-behaved for all the channel models up to 1.25 MHz . But it is an important issue for the multi-carrier DO and for any OFDM based solution.

## 3 Exponential power-delay profile fit to the ITU channel models

In order to find a solution to this issue, we first wish to find out what should be the desired FCF of the new channel model. The Cost 207 channel model in [1] gives an exponentially decaying power-delay profile. Let us assume that the ITU channel models are taken as a discrete approximation to the Cost 207 model. Hence the power of the $l$ th path is given by

$$
\begin{equation*}
\sigma_{l}^{2}=\alpha e^{-\beta \tau_{l}}, \tag{6}
\end{equation*}
$$



Figure 2: Absolute value of the spaced-frequency correlation function for the Pedestrian B channel model from Eq. 4 .
where we will assume that the first path comes at zero delay. We now attempt to fit this model to each of the ITU channel models and determine $\alpha$ and $\beta$ for each one of them. We note that for the Spatial Channel Model (SCM) discussed jointly in 3GPP/3GPP2, the power of each multi-path has a negative exponential dependence on the time delay with an additional random component. Because of the exponentially decaying feature of the power-delay profile (barring the randomness), the method described below should also be applicable for the SCM.

We define a quantity $J$ as

$$
\begin{equation*}
J(\alpha, \beta)=\sum_{l=1}^{L}\left(\sigma_{l}^{2}-\alpha e^{-\beta \tau_{l}}\right)^{2}, \tag{7}
\end{equation*}
$$

and for each of the ITU channel models, we will determine

$$
\begin{equation*}
\left\{\alpha^{*}, \beta^{*}\right\}=\arg \min _{\alpha, \beta} J(\alpha, \beta) . \tag{8}
\end{equation*}
$$

For a given $\beta$, one can choose $\alpha^{*}$ as follows. Differentiating $J(\alpha, \beta)$ w.r.t. $\alpha$, we get

$$
\begin{equation*}
\frac{\partial J}{\partial \alpha}=-2 \sum_{l=1}^{L}\left(\sigma_{l}^{2}-\alpha e^{-\beta \tau_{l}}\right) e^{-\beta \tau_{l}} . \tag{9}
\end{equation*}
$$



Figure 3: Absolute value of the spaced-frequency correlation function for the Vehicular A channel model from Eq. 4 .

Note that since

$$
\begin{equation*}
\frac{\partial^{2} J}{\partial \alpha^{2}}=2 \sum_{l=1}^{L} e^{-2 \beta \tau_{l}}>0, \tag{10}
\end{equation*}
$$

hence the solution of $\partial J / \partial \alpha=0$ gives the minima and is given by

$$
\begin{equation*}
\alpha^{*}(\beta)=\frac{\sum_{l=1}^{L} \sigma_{l}^{2} e^{-\beta \tau_{l}}}{\sum_{l=1}^{L} e^{-2 \beta \tau_{l}}} . \tag{11}
\end{equation*}
$$

Determining $\beta^{*}$ analytically is more difficult and we find it out by numerically computing

$$
\begin{equation*}
\beta^{*}=\arg \min _{\beta} J\left(\alpha^{*}(\beta), \beta\right) . \tag{12}
\end{equation*}
$$

It can be shown that the FCF for the power-delay profile of Eq. 6 with a continuum of paths (i.e. infinite paths) is given by

$$
\begin{equation*}
\phi_{C}(\Delta f ; 0)=\frac{1}{1+j 2 \pi \Delta f / \beta} . \tag{13}
\end{equation*}
$$

Hence the frequency spacing beyond which the FCF remains below 0.5 in magnitude is given by

$$
\begin{equation*}
\mathrm{BW}_{0.5}=\frac{\sqrt{3}}{2 \pi} \beta . \tag{14}
\end{equation*}
$$



Figure 4: The fitted exponential-decay power-delay profile with the original power-delay profile for Pedestrian A.

Table 2: $\alpha^{*}, \beta^{*}$, and $\mathrm{BW}_{0.5}=\sqrt{3} \beta /(2 \pi)$ for the ITU channel models.

|  | $\alpha^{*}$ | $\beta^{*}$ | $\mathrm{BW}_{0.5}(\mathrm{MHz})$ |
| :---: | :---: | :---: | :---: |
| Ped. A | 0.889132 | $20.30368 \times 10^{6}$ | 5.597003 |
| Ped. B | 0.418353 | $1.447822 \times 10^{6}$ | 0.399341 |
| Veh. A | 0.526032 | $2.045900 \times 10^{6}$ | 0.563654 |

The values of $\alpha^{*}, \beta^{*}$, and $\mathrm{BW}_{0.5}$ are given in Table 2 ,
The fitted power-delay profiles are plotted with the original ITU power-delay profiles in Figs. 4, 5, and 6.

## 4 Modified ITU models

We now replace each original path by a cluster of $N$ number of paths such that the time delays are within $\epsilon$ of the time-delay of the original path and the total average power of the cluster is the same as that of the original path. So the new channel model becomes

$$
\begin{equation*}
c(\tau ; t)=\sum_{l=1}^{L} \sum_{n=1}^{N} g_{l, n}(t) \delta\left(\tau-\tau_{l}-\delta_{l, n}\right), \tag{15}
\end{equation*}
$$



Figure 5: The fitted exponential-decay power-delay profile with the original power-delay profile for Pedestrian B.
where $-\epsilon \leq \delta_{l, n} \leq \epsilon, \forall \quad l, n, \sigma_{l, n}^{2}=E\left\{\left|g_{l, n}(t)\right|^{2}\right\}$, and

$$
\begin{equation*}
\sigma_{l}^{2}=\sum_{n=1}^{N} \sigma_{l, n}^{2} \quad \forall l . \tag{16}
\end{equation*}
$$

The problem to solve can now be formulated as follows.
Problem 1: Given a system with power-delay profile specified by $\left\{\sigma_{l}^{2}\right\},\left\{\tau_{l}\right\}, l=1, \cdots, L$. Let the desired FCF be given by $\phi_{C}^{D}(\Delta f ; 0)$. Find a system with power-delay profile specified by $\left\{\sigma_{l, n}^{2}\right\},\left\{\tau_{l}-\delta_{l, n}\right\}(l=1, \cdots, L ; n=1, \cdots, N)$ with FCF $\phi_{C}(\Delta f ; 0)$ such that

- $\left|\delta_{l, n}\right| \leq \epsilon \forall \quad l, n$.
- $\sigma_{l}^{2}=\sum_{n=1}^{N} \sigma_{l, n}^{2} \forall l$.
- $\phi_{C}(\Delta f ; 0)$ is a good approximation of $\phi_{C}^{D}(\Delta f ; 0)$. This could be accomplished by minimizing

$$
\begin{equation*}
T=\int_{0}^{\Delta f_{m}}\left|\phi_{C}^{D}(\Delta f ; 0)-\phi_{C}(\Delta f ; 0)\right|^{2} d \Delta f \tag{17}
\end{equation*}
$$

where $\Delta f_{m}$ could be 20 MHz .


Figure 6: The fitted exponential-decay power-delay profile with the original power-delay profile for Vehicular A.

We can also impose a symmetry constraint as follows.
Problem 2: Solve Problem 1 with an additional constraint that for all $l$, the set $\left\{\delta_{l, n}\right\}$ should be symmetric around 0 , and any pair of paths in a cluster that are symmetrically placed around the path in the original profile should have the same average powers $\sigma_{l, n}^{2}$.

Symmetry constraint may be useful to ensure that one doesn't change the mean delays of the paths.
We find solutions to both these problems numerically by generating randomly many power-delay profiles with the constraints in Problems 1 and 2, and choosing those that makes the FCF close to the desired function (as in Eq. 17 for example). The cluster size is chosen as $N=2,3,4$, and $\epsilon=100 \mathrm{~ns}$.

Tables 3, 4, 5, 6, 7, and 8 give the power-delay profiles for the modified channel models. Note that in some cases, the delays of more than one path is the same. One could make a single path out of it by adding their powers (since the paths are un-correlated, the powers add up). Also note that when the cluster around the first path of the ITU channel models (with zero delay) is within $\pm \epsilon$. Since the path-delays can't be negative, hence we shift all the delays so that all of them are positive.

As an example, we consider the case of $N=2$ for Pedestrian A channel with the symmetry constraint. The delays which we get by the computer search are given by $\{-20,20,110,110,110,270,360,460\}$. The first cluster of two paths is placed at $-20,20 \mathrm{~ns}$. The second cluster is placed at $110,110 \mathrm{~ns}$ i.e. both the paths are at the same place. The third cluster is placed at 110,270 . This choice places three paths at 110 ns . To make the delays non-negative, we add the above set with 20 to get
$\{0,40,130,130,130,290,380,480\}$.
Figs. 7, 8, 9, 10, 11, and 12 plot the FCF for various values of $N$ for various channel models with or without the symmetry constraint. As the figures show, the match becomes better as the cluster size increases. Also note that the symmetry constraint degrades the match though it doesn't have a debilitating effect.

## 5 Conclusions

We propose modifications in the existing ITU channel models such that their FCFs decay with increasing frequency difference. The approach is also applicable for the SCM.

## References

[1] J. G. Proakis, Digital Communications, Second Edition, McGraw-Hill, 1989.
[2] A. M. D. Turkmani, D. A. Demery, and J. D. Parsons, Measurement and modelling of wideband mobile radio channels at 900 MHz , IEE Proc., vol. 138, pp. 447-457, Oct. 1991.
[3] Robert J. C. Bultitude, Estimating Frequency Correlation Functions From Propagation Measurements on Fading Radio Channels: A Critical Review, IEEE J. Sel. Areas Commun., vol. 20, pp. 1133-1143, Aug. 2002.
[4] Spatial Channel Model (Version 7.0), 3GPP-3GPP2 Adhoc, Aug 2003, ftp://ftp.3gpp2.org/TSGC/Working/2003/3GPP_3GPP2_SCM_(Spatial_Modeling)/ConfCall-16-20030417/ SCM-135 Text v7.0.zip.

Table 3: Modified ITU Pedestrian A channel model with $L=4$ with the symmetry constraint.

| $N=2$ | $\begin{aligned} & \hline \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative (dB) } \end{aligned}$ | 0, 40, 130, 130, 130, 290, 380, 480 $0.44465,0.44465,0.04765,0.04765,0.00535,0.00535,0.00235,0.00235$, 0.0000, 0.0000, -9.6996, -9.6996, -19.1966, -19.1966, -22.7695, -22.7695, |
| :---: | :---: | :---: |
| $N=3$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative (dB) } \end{aligned}$ | $0,30,60,140,140,140,220,220,220,370,440,510$ $0.28209,0.32511,0.28209,0.04623,0.00285,0.04623,0.00351,0.00368$, $0.00351,0.00067,0.00336,0.00067$ $-0.6164,0.0000,-0.6164,-8.4714,-20.5766,-8.4714,-19.6682,-19.4602$, $-19.6682,-26.8632,-19.8556,-26.86315$ |
| $N=4$ | $\begin{aligned} & \hline \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative (dB) } \end{aligned}$ | $0,30,50,80,120,120,180,180,190,220,240,270,390,410,490,510$ $0.13758,0.30707,0.30707,0.13758,0.02562,0.02203,0.02203,0.02562$, $0.00285,0.00250,0.00250,0.00285,0.00109,0.00126,0.00126,0.00109$ $-3.4867,0.0000,0.0000,-3.4867,-10.7863,-11.4425,-11.4425,-10.7863$, $-20.3230,-20.8940,-20.8940,-20.3230,-24.5006,-23.8664,-23.8664,-24.5006$ |

Table 4: Modified ITU Pedestrian A channel model with $L=4$ with no symmetry constraint.

| $N=2$ | $\begin{aligned} & \hline \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative (dB) } \\ & \hline \end{aligned}$ | 0, 40, 70, 120, 150, 170, 320, 420 0.04971, 0.41094, 0.47836, 0.04559, 0.00596, 0.00474, 0.00029, 0.00441 $-9.8334,-0.6598,0.0000,-10.2085,-19.0448,-20.0402,-32.1428,-20.3552$ |
| :---: | :---: | :---: |
| $N=3$ | $\begin{aligned} & \hline \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative (dB) } \end{aligned}$ | $0,40,70,130,150,160,250,250,300,470,480,520$ $0.19731,0.38581,0.30618,0.04261,0.02166,0.03103,0.00130,0.00479$, $0.00461,0.00122,0.00218,0.00131$ $-2.9121,0.0000,-1.0039,-9.5684,-12.5067,-10.9464,-24.7298,-19.0618$, $-19.2236,-25.0091,-22.4869,-24.70283$ |
| $N=4$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,70,90,120,120,150,160,180,220,220,280,330,420,480,540,560$ $0.09413,0.23742,0.30251,0.00116,0.25524,0.04660,0.02443,0.02312$, $0.00284,0.00136,0.00380,0.00270,0.00098,0.00134,0.00061,0.00176$ $-5.0701,-1.0521,0.0000,-24.1696,-0.7378,-8.1239,-10.9288,-11.1676$, $-20.2707,-23.4597,-19.0116,-20.5007,-24.8916,-23.5266,-26.9366,-22.3430$ |

Table 5: Modified ITU Pedestrian B channel model with $L=6$ with the symmetry constraint.

| $N=2$ | $\begin{aligned} & \hline \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,40,130,310,800,840,1200,1240,2270,2370,3700,3740$ $0.20285,0.20285,0.16490,0.16490,0.06565,0.06565,0.03215,0.03215$, $0.03365,0.03365,0.00085,0.00085$ $0.0000,0.0000,-0.8995,-0.8995,-4.8994,-4.8994,-7.9999,-7.9999$, $-7.8019,-7.8019,-23.7776,-23.77756$ |
| :---: | :---: | :---: |
| $N=3$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,40,80,200,240,280,800,840,880,1190,1240,1290,2310,2340$, $2370,3730,3740,3750$ $0.12913,0.14745,0.12913,0.10975,0.11031,0.10975,0.04330,0.04470$, $0.04330,0.02681,0.01068,0.02681,0.01758,0.03214,0.01758,0.00052$, $0.00067,0.00052$ $-0.5762,0.0000,-0.5762,-1.2824,-1.2603,-1.2824,-5.3216,-5.1830$, $-5.3216,-7.4036,-11.3994,-7.4036,-9.2358,-6.6163,-9.2358,-24.5540$, $-23.4477,-24.55395$ |
| $N=$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,40,80,120,170,210,310,350,770,800,920,950,1230,1250$, $1270,1290,2300,2330,2390,2420,3690,3740,3780,3830$ $0.09985,0.10300,0.10300,0.09985,0.07172,0.09318,0.09318,0.07172$, $0.03463,0.03102,0.03102,0.03463,0.01311,0.01904,0.01904,0.01311$, $0.01941,0.01424,0.01424,0.01941,0.00064,0.00021,0.00021,0.00064$ $-0.1346,0.0000,0.0000,-0.1346,-1.5717,-0.4351,-0.4351,-1.5717$, $-4.7335,-5.2121,-5.2121,-4.7335,-8.9523,-7.3315,-7.3315,-8.9523$, $-7.2482,-8.5928,-8.5928,-7.2482,-22.0988,-26.8089,-26.8089,-22.0988$ |

Table 6: Modified ITU Pedestrian B channel model with $L=6$ with no symmetry constraint.

| $N=2$ | $\tau_{l}(\mathrm{~ns})$ $\sigma_{l}^{2}$ Relative (dB) | $0,40,80,120,760,840,1100,1160,2250,2370,3650,3760$ $0.20404,0.20166,0.18905,0.14075,0.03758,0.09372,0.04509,0.01921$, $0.04200,0.02530,0.00115,0.00055$ $0.0000,-0.0511,-0.3314,-1.6127,-7.3476,-3.3789,-6.5566,-10.2614$, $-6.8652,-9.0652,-22.5066,-25.65963$ 0 |
| :---: | :---: | :---: |
| $N=3$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,50,90,130,170,280,770,790,840,1170,1230,1290,2280,2290$, $2360,3630,3630,3630$ $0.13258,0.12033,0.15280,0.14560,0.09301,0.09119,0.00562,0.07694$, $0.04874,0.02727,0.01972,0.01731,0.01854,0.03151,0.01725,0.00060$, $0.00087,0.00023$ $-0.6164,-1.0374,0.0000,-0.2096,-2.1558,-2.2415,-14.3457,-2.9795$, $-4.9622,-7.4842,-8.8924,-9.4578,-9.1590,-6.8573,-9.4730,-24.0919$, $-22.4255,-28.21729$ |
| $N=4$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | 0, 40, 70, 120, 210, 250, 290, 350, 780, 830, 880, 920, 1200, 1250, 1310, 1350, 2290, 2350, 2380, 2400, 3700, 3730, 3730, 3870 $0.08419,0.11035,0.10604,0.10511,0.10379,0.10073,0.04081,0.08447$, 0.01001, 0.02957, 0.04952, 0.04220, 0.01076, 0.03007, 0.01083, 0.01264, $0.00445,0.01331,0.03056,0.01898,0.00002,0.00065,0.00027,0.00076$ $-1.1750,0.0000,-0.1729,-0.2113,-0.2661,-0.3963,-4.3200,-1.1608$, $-10.4232,-5.7198,-3.4798,-4.1745,-10.1101,-5.6460,-10.0817,-9.4109$, <br> $-13.9434,-9.1845,-5.5766,-7.6455,-38.1923,-22.3097,-26.0472,-21.6155$ |

Table 7: Modified ITU Vehicular A channel model with $L=6$ with the symmetry constraint.

| $N=2$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $\begin{aligned} & 0,40,290,370,680,780,1070,1150,1680,1820,2510,2550 \\ & 0.24250,0.24250,0.19265,0.19265,0.03055,0.03055,0.02425,0.02425, \\ & 0.00765,0.00765,0.00245,0.00245 \\ & 0.0000,0.0000,-0.9994,-0.9994,-8.9970,-8.9970,-10.0000,-10.0000, \\ & -15.0105,-15.0105,-19.9555,-19.95546 \end{aligned}$ |
| :---: | :---: | :---: |
| $N=3$ | $\begin{aligned} & \hline \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $\begin{aligned} & \hline 0,40,80,310,350,390,730,750,770,1050,1130,1210,1730,1770, \\ & 1810,2480,2550,2620 \\ & 0.15777,0.16947,0.15777,0.12521,0.13489,0.12521,0.02332,0.01446, \\ & 0.02332,0.00994,0.02862,0.00994,0.00561,0.00407,0.00561,0.00157, \\ & 0.00177,0.00157 \\ & -0.3107,0.0000,-0.3107,-1.3146,-0.9911,-1.3146,-8.6134,-10.6898, \\ & -8.6134,-12.3166,-7.7244,-12.3166,-14.7987,-16.1911,-14.7987,-20.3437, \\ & -19.8147,-20.34367 \end{aligned}$ |
| $N=4$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,40,80,120,280,320,420,460,750,750,790,790,1060,1150$, $1150,1240,1700,1710,1870,1880,2510,2530,2610,2630$ $0.10022,0.14228,0.14228,0.10022,0.10295,0.08970,0.08970,0.10295$, $0.00804,0.02251,0.02251,0.00804,0.01168,0.01257,0.01257,0.01168$, $0.00259,0.00506,0.00506,0.00259,0.00130,0.00115,0.00115,0.00130$ $-1.5218,0.0000,0.0000,-1.5218,-1.4051,-2.0035,-2.0035,-1.4051$, $-12.4802,-8.0072,-8.0072,-12.4802,-10.8563,-10.5387,-10.5387,-10.8563$, $-17.4022,-14.4880,-14.4880,-17.4022,-20.4059,-20.9087,-20.9087,-20.4059$ |

Table 8: Modified ITU Vehicular A channel model with $L=6$ with no symmetry constraint.

| $N=2$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,40,180,220,600,730,1000,1060,1610,1690,2470,2510$ $0.24343,0.24157,0.17677,0.20853,0.05368,0.00742,0.02632,0.02218$, $0.00792,0.00738,0.00295,0.00195$ $0.0000,-0.0332,-1.3897,-0.6719,-6.5658,-15.1578,-9.6614,-10.4034$, $-14.8759,-15.1837,-19.1714,-20.95431$ |
| :---: | :---: | :---: |
| $N=3$ | $\begin{aligned} & \hline \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | $0,40,80,330,370,410,770,790,820,1040,1130,1210,1790,1790$, $1870,2600,2600,2630$ $0.11267,0.17034,0.20200,0.12620,0.13494,0.12416,0.05366,0.00471$, $0.00273,0.01628,0.01618,0.01604,0.00046,0.00773,0.00712,0.00098$, $0.00200,0.00191$ $-2.5355,-0.7404,0.0000,-2.0428,-1.7522,-2.1135,-5.7572,-16.3188$, $-18.6941,-10.9359,-10.9646,-11.0014,-26.4533,-14.1739,-14.5305,-23.1200$, $-20.0383,-20.23640$ |
| $N=4$ | $\begin{aligned} & \tau_{l}(\mathrm{~ns}) \\ & \sigma_{l}^{2} \\ & \text { Relative }(\mathrm{dB}) \end{aligned}$ | 0, 50, 90, 130, 270, 300, 390, 420, 670, 750, 770, 800, 1040, 1060, 1070, 1190, 1670, 1710, 1820, 1840, 2480, 2500, 2540, 2620 <br> $0.07439,0.13808,0.15198,0.12055,0.10989,0.10900,0.10650,0.05990$, <br> 0.00330, 0.00552, 0.04889, 0.00339, 0.01818, 0.00980, 0.01472, 0.00580, <br> $0.00304,0.00719,0.00493,0.00014,0.00170,0.00190,0.00002,0.00128$ <br> $-3.1031,-0.4166,0.0000,-1.0065,-1.4083,-1.4436,-1.5443,-4.0437$, <br> $-16.6369,-14.3955,-4.9259,-16.5160,-9.2222,-11.9058,-10.1378,-14.1861$, <br> $-16.9901,-13.2515,-14.8881,-30.3480,-19.5257,-19.0286,-38.1504,-20.7436$ |



Figure 7: FCF of a modified Pedestrian A channel model with the symmetry constraint.


Figure 8: FCF of a modified Pedestrian A channel model with no symmetry constraint.

Ped. B: With symmetry constraint


Figure 9: FCF of a modified Pedestrian B channel model with the symmetry constraint.


Figure 10: FCF of a modified Pedestrian B channel model with no symmetry constraint.


Figure 11: FCF of a modified Vehicular A channel model with the symmetry constraint.


Figure 12: FCF of a modified Vehicular A channel model with no symmetry constraint.

