Proposal of an approach for statistical modeling of OM1 multimode fiber within the IEEE 802.3aq channel modeling ad-hoc committee

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A Purpose of this document

This document proposes an approach for statistical modeling of OM1 multimode fiber (MMF) for the consideration of the IEEE 802.3aq channel modeling ad-hoc committee, with the eventual aim being to generate outputs of value to those working towards extended-reach MMF solutions within the IEEE 802.3aq Task Force.

The approach proposed here is based upon the work described in detail in M. Webster *et al.*, "A statistical analysis of conditioned launch for Gigabit Ethernet links using multimode fiber", *Journal of Lightwave Technology*, vol. 17, no. 9, pp. 1532-1541, September 1999. A recent supplementary document distributed to the 10GMMF email reflector by J. D. Ingham *et al.*: "Statistical modeling of multimode-fiber links: a supplement to the information provided in the document of 20 February 2004 in relation to Release 1.0 from the University of Cambridge," provides clarification of some points and further detailed information.

This document describes the proposal in terms of the three topics of: (i) the inputs to the proposed model of OM1 MMF; (ii) the proposed model of OM1 MMF; (iii) the outputs from the proposed model of OM1 MMF.

B Inputs to the proposed model of OM1 MMF

Refractive-index profiles form the input to the proposed model of OM1 MMF. These refractive-index profiles are intended to incorporate perturbations from the ideal refractive-index profile which are representative of the imperfections found in the installed base of OM1 MMF. This proposal is concerned exclusively with MMF which has a core diameter of 62.5 μm and which is operated at a wavelength of 1300 nm. Therefore, a near-ideal refractive-index profile is defined here as a profile with: (i) a power-law parameter $\alpha = 1.97$; (ii) a core radius $r_{core} = 31.25$ μm; (iii) an axial refractive index $n_{core} = 1.5$; (iv) a cladding refractive index $n_{clad} = 1.474$, which is equivalent to a numerical aperture $\mathrm{NA} = \sqrt{(n_{core}^2 - n_{clad}^2)} = 0.28$.

Four types of perturbation of this near-ideal power-law refractive-index profile are proposed. Each of the four types of perturbation, which are described in detail below, is associated with three possible values, which leads to $3^4 = 81$ distinct refractive-index profiles as inputs to the proposed model of OM1 MMF.

(1) Deviation of the power-law parameter α from near-ideal within the *inner* region of the fiber core
In addition to its near-ideal value of 1.97, the power-law parameter α for the inner region of the fiber core ($0 \leq r \leq r_{core}/2$) may assume values of 1.89 and 2.05. The figures of 1.89 and 2.05 are chosen since for a pure power-law fiber they result in an OFLBWL close to the ISO/IEC 11801 specification of 500 MHz km at 1300 nm.

(2) Deviation of the power-law parameter α from near-ideal within the *outer* region of the fiber core
In addition to its near-ideal value of 1.97, the power-law parameter α for the outer region of the fiber core ($r_{core}/2 < r \leq r_{core}$) may assume values of 1.89 and 2.05, as for perturbation (1).

For perturbations (1) and (2), the continuity of the refractive-index profile is maintained by adjusting the outer refractive-index profile ($r_{core}/2 < r \leq r_{core}$) by a correction factor n_1/n_2, where n_1 is the refractive index at $r = r_{core}/2$ calculated using the inner power-law parameter, and n_2 is the refractive index at $r = r_{core}/2$ calculated using the outer power-law parameter. The refractive index of the cladding ($r > r_{core}$) is unaffected by this adjustment.

(3) Dip or peak located on the axis of the fiber
A dip or peak located on the axis of the fiber ($r = 0$) is modeled by a gaussian function which is added to the refractive-index profile. The dip and peak both have a full width at half maximum FWHM = 3 μm and are modeled by the addition of: $n(r) = A\exp(-r^2/\delta^2)$, where $\delta = \mathrm{FWHM}/[2\sqrt{(\log_e 2)}]$ and the amplitude of the perturbation $A = -0.004$ (dip), 0 (no perturbation) or 0.002 (peak).

(4) Imperfect transition from the fiber core to the fiber cladding
Two possible types of transition at the core-cladding interface are considered in addition to the ideal: (i) a step transition at $r = 28$ μm to the cladding refractive index n_{clad}; (ii) an exponential decay, such that for $r > r_0 = 28$ μm: $n(r) = (n_0 - n_{clad})\exp[-\beta(r - r_0)] + n_{clad}$, where n_0 is the refractive index at $r = r_0$ before the perturbation is applied. The decay constant $\beta = 3.0457 \times 10^5\ \mathrm{m}^{-1}$.

C The proposed model of OM1 MMF

The model proceeds by: (i) for each perturbed input fiber, calculating the electric-field distributions and propagation delays of the guided modes supported by the fiber; (ii) for each perturbed input fiber, calculating the differential modal delay (DMD) and overfilled-launch (OFL) bandwidth-length product (BWL); (iii) for each perturbed input fiber, comparing the calculated DMD with a worst-case DMD target, *which is chosen to be 2 ns/km in order to model the worst 5% of the installed base of OM1 MMF*; (iv) for each perturbed input fiber, scaling the perturbations to create a scaled perturbed refractive-index profile with a DMD close to the worst-case target.

The proposed 81 perturbed refractive-index profiles described in part *B* are input separately to a scalar-wave-equation mode solver. This mode solver determines which linearly-polarized (LP) guided modes are supported by a given fiber at a desired wavelength, which is 1300 nm throughout this work. Each LP guided mode is associated with: (i) a cylindrical order $\nu \geq 0$; (ii) a radial order $\mu \geq 1$; (iii) a mode-group order $g = 2\mu + \nu + 1 \geq 3$. Moreover, the mode solver provides: (i) the electric-field distribution $E_{\nu\mu}(r, \phi)$ and (ii) the propagation delay $\tau_{\nu\mu}$ (ns/km) of each LP guided mode supported by the fiber. All modes for which the mode-group order $g > 20$ are ignored from this point onwards. This is equivalent to a mode-dependent loss (MDL) of 0 dB for mode groups with $g \leq 20$ and a MDL of ∞ dB for all other mode groups. No further modeling of MDL is proposed.

At this point in the proposed model, each LP guided mode is associated with a unique propagation delay $\tau_{\nu\mu}$. In general, this propagation delay will vary between mode groups and also within mode groups. The propagation delay $\tau_{\nu\mu}$ of each guided mode is replaced by the arithmetic mean τ_g of the propagation delays for the mode group g to which the guided mode belongs.

DMD

The first step in the calculation of the DMD of a fiber is to determine the modal excitation in response to a radially-scanned single-mode fiber (SMF). The SMF output beam is modeled by a gaussian beam for which the electric-field distribution $E_I(r, \phi)$ has a FHWM = 7 µm. For radial offsets of this SMF beam from 0 µm to 30 µm, in increments of 1 µm, $E_I(r, \phi)$ is overlapped with $E_{\nu\mu}(r, \phi)$, according to the overlap integrals of equation (3) in Webster *et al*. This generates a power-coupling coefficient $P_{\nu\mu}$ for each guided mode. The power-coupling coefficients $P_{\nu\mu}$ are then averaged within each mode group, such that the power-coupling coefficient $P_{\nu\mu}$ for each guided mode is replaced by the arithmetic mean P_g of the power-coupling coefficients for the mode group g to which the guided mode belongs. This is performed in order to model the effect of complete mode-mixing *within* mode groups. Note that mode mixing *between* mode groups is not proposed in this work. Once the power-coupling coefficients P_g have been obtained for each radial offset of the SMF beam, an "intermediate" DMD value may be calculated for each of the 31 offsets using the P_g values in combination with the propagation delays τ_g. This "intermediate" DMD is defined as: $\Sigma N_g P_g \tau_g / \Sigma N_g P_g$ *. Note that although the propagation delays and power-coupling coefficients have been averaged within each mode group g, the multiplicity of modes within each mode group is retained, i.e. the contribution of each mode group g is weighted by the number of modes N_g within the mode group g. Once the "intermediate" DMD has been calculated for each radial offset of the SMF beam, the final DMD is obtained by: (i) subtracting the minimum "intermediate" DMD from the set of 31 values; (ii) extracting the maximum value from the resulting set of normalized "intermediate" DMDs. Note that this definition of DMD is often referred to as *mean* DMD, which distinguishes it from other definitions of DMD, e.g. the definition of DMD described in TIA/EIA-455-220.

OFLBWL

To calculate the OFLBWL, the initial step is to calculate the impulse response $h(t)$ for the case of OFL. Since OFL indicates equal excitation of all guided modes, no power-coupling calculation is required. Therefore, $h(t) = \Sigma N_g \delta(t - \tau_g)$ *, where δ is the Dirac delta function. The OFLBWL is then obtained by extracting the –3-dB bandwidth from the corresponding frequency response $H(f) = \Sigma N_g \exp(-j\, 2\pi f \tau_g)$ *, where the delays are set for 1 km of fiber in order for the extracted –3-dB bandwidth to be the OFLBWL.

* The summations run over all mode-group orders g such that: $3 \leq g \leq 20$.

DMD scaling

The DMD and OFLBWL form the input to the DMD scaling process in which each fiber is considered separately. The purpose of this process is to generate a DMD scaling factor S, which indicates how close the fiber is to the worst-case DMD target. This DMD scaling factor S is then used to adjust each of the perturbations of the fiber such that its DMD is close to the worst-case DMD target.

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_C_ The proposed model of OM1 MMF _continued_

Importantly, it is necessary to monitor the effect of the scaling on the OFLBWL. Special care must be taken for those cases where the resulting OFLBWL is below the specification of 500 MHz km. The four cases that may be encountered during DMD scaling of each fiber are now considered. Note that the bandwidth scaling factor is the reciprocal of the DMD scaling factor *S*.

(1) **DMD ≥ 2 ns/km OFLBWL × DMD / (2 ns/km) ≥ 500 MHz km**
The DMD is excessive in relation to the target or equal to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is greater than or equal to unity. The scaled OFLBWL meets the 500 MHz km specification and therefore the fiber is retained with a scaled DMD of 2 ns/km and a bandwidth scale factor of DMD / (2 ns/km).

(2) **DMD ≥ 2 ns/km OFLBWL × DMD / (2 ns/km) < 500 MHz km**
The DMD is excessive in relation to the target or equal to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is greater than or equal to unity. The scaled OFLBWL does not meet the 500 MHz km specification. Therefore, the fiber is rejected.

(3) **DMD < 2 ns/km OFLBWL × DMD / (2 ns/km) ≥ 500 MHz km**
The DMD is insufficient in relation to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is less than unity. The scaled OFLBWL meets the 500 MHz km specification and therefore the fiber is retained with a scaled DMD of 2 ns/km and a bandwidth scale factor of DMD / (2 ns/km).

(4) **DMD < 2 ns/km OFLBWL × DMD / (2 ns/km) < 500 MHz km**
The DMD is insufficient in relation to the target. Therefore, the bandwidth scale factor DMD / (2 ns/km) is less than unity. The scaled OFLBWL does not meet the 500 MHz km specification. The immediate reaction would be to reject the fiber. However, an alternative is to define a bandwidth scale factor as 500 MHz km / OFLBWL, which results in the fiber being scaled to the OFLBWL specification of 500 MHz km.

When the DMD scaling process is complete, there are two important outcomes: (i) those fibers associated with case (2) are rejected; (ii) the remaining fibers are each equipped with a unique DMD scaling factor *S*. *This DMD scaling factor S is then applied to the original perturbations in order to generate a new refractive-index profile with a DMD close to the worst-case target.* This approach will use the equations for the scaling of the refractive index per the Task Force contribution entitled "More information on statistical modeling of MMF optical links" from the May 2004 meeting (cam_1_0504.pdf).

Once the scaled refractive-index profiles have been created, power-coupling coefficients and modal delays may be generated as required by repeating the earlier procedure, but with the scaled refractive-index profiles as the inputs to the modesolver.

_D_ Outputs from the proposed model of OM1 MMF

The outputs from the proposed model of OM1 MMF should meet the requirements of *all* of those working towards extended-reach MMF solutions within the IEEE 802.3aq Task Force. Different contributors to the Task Force have different needs. The needs of two communities are of particular interest: (i) those working towards a solution which is (mainly) based upon electronic dispersion compensation (EDC); (ii) those working towards an alternative solution, perhaps based in the optical domain and using specific restricted transmit and receive optics.

For those working towards an EDC-based solution, suitable outputs from the model possibly include power-coupling coefficients for each of the perturbed fibers for a radially-scanned beam from an input SMF, together with the corresponding modal delays. The proposed model allows these outputs to be readily generated and distributed. Indeed, Release 1.0 from the University of Cambridge is an example of how such information can be provided to the Task Force. *A distinct advantage of the proposal is that it is concerned with modeling a worst-case population of MMF, rather than the entire population of MMF. Therefore, the outputs from the model are for the limiting cases which are of greatest interest to those developing an EDC-based solution.* Corresponding frequency responses and –3 dB bandwidths may also be easily generated and distributed.

For those working towards an alternative solution, based in the optical domain, *the proposal is capable of providing the refractive-index profile data that is of considerable value to this community*. The refractive-index profile data from all stages of the simulation process may be distributed easily, i.e. both the perturbed refractive-index profiles which form the input to the proposed model, and the refractive-index profiles with perturbations scaled to meet the worst-case DMD target. Of course, this community will also have access to the power-coupling coefficients and modal delay data used by the EDC community (and vice versa), which will allow useful assessments and benchmarking of an optical technique.

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D* Outputs from the proposed model of OM1 MMF *continued

Note on Optical Launches
If the optical launch is defined in terms of an implementation-independent method, encircled flux for example, then the proposed model can be used as follows:

(1) To determine the statistical performance of transmitters which meet the encircled flux template by applying compliant user-defined or agreed example mode power distributions (MPDs) in the computation of impulse responses and associated performance metrics. Examples of user-defined MPDs would be those due to vortex, donut or offset launches or other launches.

(2) To specify practical implementation conformance tests by simulation of realistic TP2 MPDs and associated performance metrics.

Note on Connectors
Connectors will be part of the channel model and are not part of this proposal for OM1 MMF. However, from the extensive literature on the subject, it is clear that connectors can be dealt with using standard overlap integrals and an agreed distribution of connector offset. Normally, the use of the modes of a nominal fiber for the overlap integrals is adequate. However, since this model provides as an output the scaled refractive-index profiles, then those contributors especially interested in connections between the various combinations of "worst-case" 81 fibers can use wavesolvers to enable the required overlap calculations.