Some Useful Formulas for Analysis of Interferometric Noise

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1.0 Introduction

It is well known that phase noise fluctuations at the output of a semiconductor laser can produce intensity noise fluctuations upon transmission through a fiber-optic link due to interferometric phase-to-intensity conversion [1-4]. In a single-mode fiber link, the interferometric conversion occurs when multiple reflections occur between a pair of fiber interfaces (Figure 1). Even in the absence of such fiber discontinuities, Rayleigh scattering in a sufficiently long piece of fiber can cause similar effects [5]. In an optical network using passive couplers, interferometric noise might occur if the difference between the two signal paths at the receiving node is small enough. The nature of the interferometric noise has been studied in [1-3]. It was shown that this excess noise can cause bit-error-rate floors [6], and the system performance has been evaluated as a function of the number and magnitude of the reflections [7].

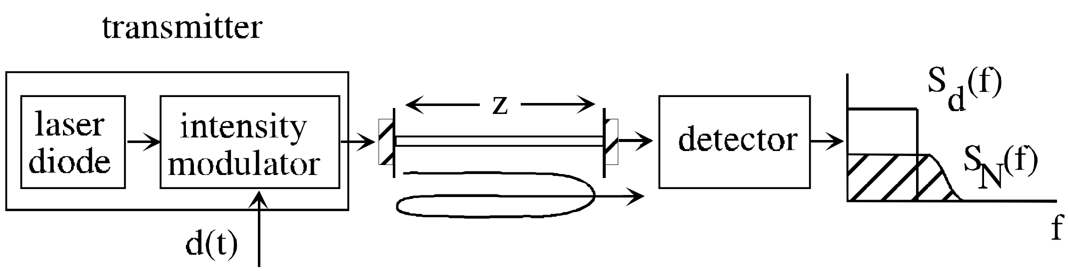

FIGURE 1. Fiber-optic link experiencing multiple reflections from fiber interfaces (connectors, fiber ends etc.). The graph right of the detector shows schematically the data spectrum $S_d(f)$ accompanied with the interferometric noise, whose power spectral density is $S_N(f)$.

2.0 Introduction to interferometric noise

As was mentioned before, the results derived in this section are well known [1-3], but are repeated here for the sake of defining the symbols used here.

Consider the intensity noise generated in a single mode (SM) fiber optic link through interferometric FM-AM conversion due to, for example, double reflection between two pairs of connectors (Figure 1). The laser is assumed to be single-mode, and it is also assumed that the data is intensity modulated.

The electric field at the input of the fiber is given by:

$$E(t) = \sqrt{P(t)}exp[j\Omega_0 t + \varphi(t)] \qquad \text{Eq.2.1}$$

where P(t) is the output laser power, Ω_0 is the optical carrier frequency and $\varphi(t)$ is the laser phase noise. The field at the output of the fiber is represented by the superposition of the original input field with a delayed version of itself:

$$E_{out}(t) = \psi_1 E(t-t_1) + \psi_2 E(t-t_2) \qquad \text{Eq.2.2}$$

where ψ_1 and ψ_2 are the relative field intensities, and t_1 and t_2 are the delays. ψ_1 and ψ_2 take into account the attenuation in the fiber link and the reflection coefficients at the fiber interfaces. We neglect any phase changes in the electric field due to the reflections. Without loss of generality, we can assume that $\psi_1 = 1$, $\psi_2 = \psi$, t_1=0 and t_2=τ.

The optical power of the laser can be written as:

$$P(t) = P_0 d(t) \qquad \text{Eq.2.3}$$

where P_0 is the average output laser power (in the absence of modulation) and d(t) is data which is band limited and contains the modulation format. For example, for direct intensity modulation, $d(t)= 1+ m\, d_I(t)$, where *m* is the modulation index and $d_I(t)$ is the data. The intrinsic laser intensity noise is neglected in the following analysis, since it is much smaller than the other noises considered.

The electric field at the fiber output will be:

$$\begin{aligned} E_{out}(t) = {} & \sqrt{P_0 d(t)} exp[j\Omega_0 t + \varphi(t)] + \\ & \psi\sqrt{P_0 d(t)} exp[j\Omega_0 (t-\tau) + \varphi(t-\tau)] \end{aligned} \qquad \text{Eq.2.4}$$

Then, the intensity at the output of the fiber is:

$$i(z,t) = |E(t)|^2 + \psi^2 |E(t-\tau)|^2 + 2\psi Re\{E(t)E^*(t-\tau)\} \qquad \text{Eq.2.5}$$

We can identify the signal i_S(z,t) and the noise part i_N(z,t) of i(z,t) as:

$$i_S(z,t) = P_0[d(t) + \psi d(t-\tau)] \approx P_0 d(t) \qquad \text{where we have assumed } \psi \ll 1 \text{ and}$$

$$i_N(z,t) = 2\psi P_0 \sqrt{d(t)d(t-\tau)} \cos[\Omega_0 \tau + \varphi(t) - \varphi(t-\tau)] \qquad \text{Eq.2.6}$$

Because of the random processes involved, the impact of the noise has to be treated through the standard communications theory, i.e. autocorrelation function of the noise term. The worst case approach to calculating the impact of the interferometric noise gives overly pessimistic results, and better upper bound on the probability of error can be found. Because of the fast changes cosine in the interferometric noise, can't sustain its value for more than one instance, not to talk about the entire interval.

The laser phase noise φ(t) is modeled to follow Gaussian probability density function and φ(t) and φ(t-τ) are correlated in such a way that [1]:

$$\langle(\varphi(t) - \varphi(t-\tau))^2\rangle = \frac{|\tau|}{\tau_c} \qquad \text{Eq.2.7}$$

τ_c is the laser coherence time, and $\langle x \rangle$ denotes statistical averaging. In this case, Eq. 2.7 corresponds to a Lorentzian lineshape of the spectral distribution. The expression in Eq. 2.6 is a general result, which will be used to derive the noise spectrum in several cases.

Finding the signal and the noise spectrum is a straightforward matter. The corresponding autocorrelation functions need to be found. They are:

$$R_S(\delta\tau) = E\{i_S(z,t)i_S(z,t+\delta\tau)\} = (1+\psi^2)P_0^2 R_d(\delta\tau) \qquad \text{Eq.2.8}$$

$$
\begin{aligned}
R_N(\delta\tau) &= E\{i_N(z,t)i_N(z,t+\delta\tau)\} \\
&= 2\psi^2 P_0^2 R_{dd}(\delta\tau)[R_-(\delta\tau) + R_+(\delta\tau)\cos(\Omega_0\tau)]
\end{aligned}
\qquad \text{Eq.2.9}
$$

where E{} denotes statistical averaging and $R_d(\delta\tau)$ is the autocorrelation function of d(t), and

$$
R_{dd}(\delta\tau) = E\{\sqrt{d(t)d(t-\tau)d(t+\delta\tau)d(t-\tau+\delta\tau)}\} \qquad \text{Eq.2.10}
$$

The corresponding power spectral densities are denoted by $S_d(f)$ and $S_{dd}(f)$. To compute $R_{dd}(\delta\tau)$ one needs to specify the data statistics. The expression in [] in Eq. 2.9 is recognized as that due to the interferometrically converted laser phase noise of a cw laser in the absence of data modulation. The expressions R_+ and R_- are given by:

$$
R_-(\delta\tau) = \langle\cos[\varphi(t) - \varphi(t-\tau) - \varphi(t+\delta\tau) + \varphi(t+\delta\tau-\tau)]\rangle \qquad \text{Eq.2.11}
$$

$$
R_+(\delta\tau) = \langle\cos[\varphi(t) - \varphi(t-\tau) + \varphi(t+\delta\tau) - \varphi(t+\delta\tau-\tau)]\rangle \qquad \text{Eq.2.12}
$$

and have been previously calculated [1,2]. The variations of the term $R_+\cos[\Omega_0\tau]$ are of the order of the laser wavelength. We are interested in the macroscopic variations, which are on a much bigger scale than those due to the term involving $R_+(\delta\tau)$. For this reason the term including $R_+(\delta\tau)$ will be neglected.

$R_-(\delta\tau)$ is given by [1]:

$$
R_-(\delta\tau) = exp\left[-\frac{1}{2\tau_c}(2|\tau| - |\tau-\delta\tau| - |\tau+\delta\tau| + 2|\delta\tau|)\right] \qquad \text{Eq.2.13}
$$

For sufficiently long τ, the corresponding power spectral density of $R_-(\delta\tau)$ (denoted by $S_-(f)$ and given by the Fourier transform of $R_-(\delta\tau)$, assumes the Lorentzian lineshape of the lasing field down-converted to baseband, with a typical width of 10MHz to 100MHz.

Then, the noise autocorrelation function becomes:

$$
R_N(\delta\tau) = 2\psi^2 P_0^2 R_{dd}(\delta\tau) R_-(\delta\tau) \qquad \text{Eq.2.14}
$$

Its power spectral density is given by:

$$
S_N(f) = 2\psi^2 P_0^2 S_{dd}(f) \otimes S_-(f) \qquad \text{Eq.2.15}
$$

where $\otimes$ denotes convolution.

The power spectral densities are schematically illustrated in Figure 3. The signal to noise ratio S/N can be easily calculated as:

$$
\frac{S}{N} = \frac{P_0^2\int S_d(f)df}{2\psi^2 P_0^2\int S_{dd}(f) \otimes S_-(f)df} = \frac{1}{2\psi^2 R_-(0)} \qquad \text{Eq.2.16}
$$

where for small a modulation index m it was assumed that $S_d(f)=S_{dd}(f)$. Eq. 2.16 illustrates clearly the deleterious effect of interferometric FM - IM noise on the maximum achievable S/N ratio of the transmitted data. In the event of large signal modulation, Eq. 2.16 has to be calculated without the above asumption, and most likely need to be evaluated numerically.

At this point, we need to point out that although the S/N expression given above is only an approximation, and can not be used to calculate the BER using the standard formulas assuming Gaussian noise. That is clear once we examine equation Eq. 2.6, which shows that the noise is bounded.

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