Achievable Bit Rates and Choice of Modulation Rate for 10GBASE-T

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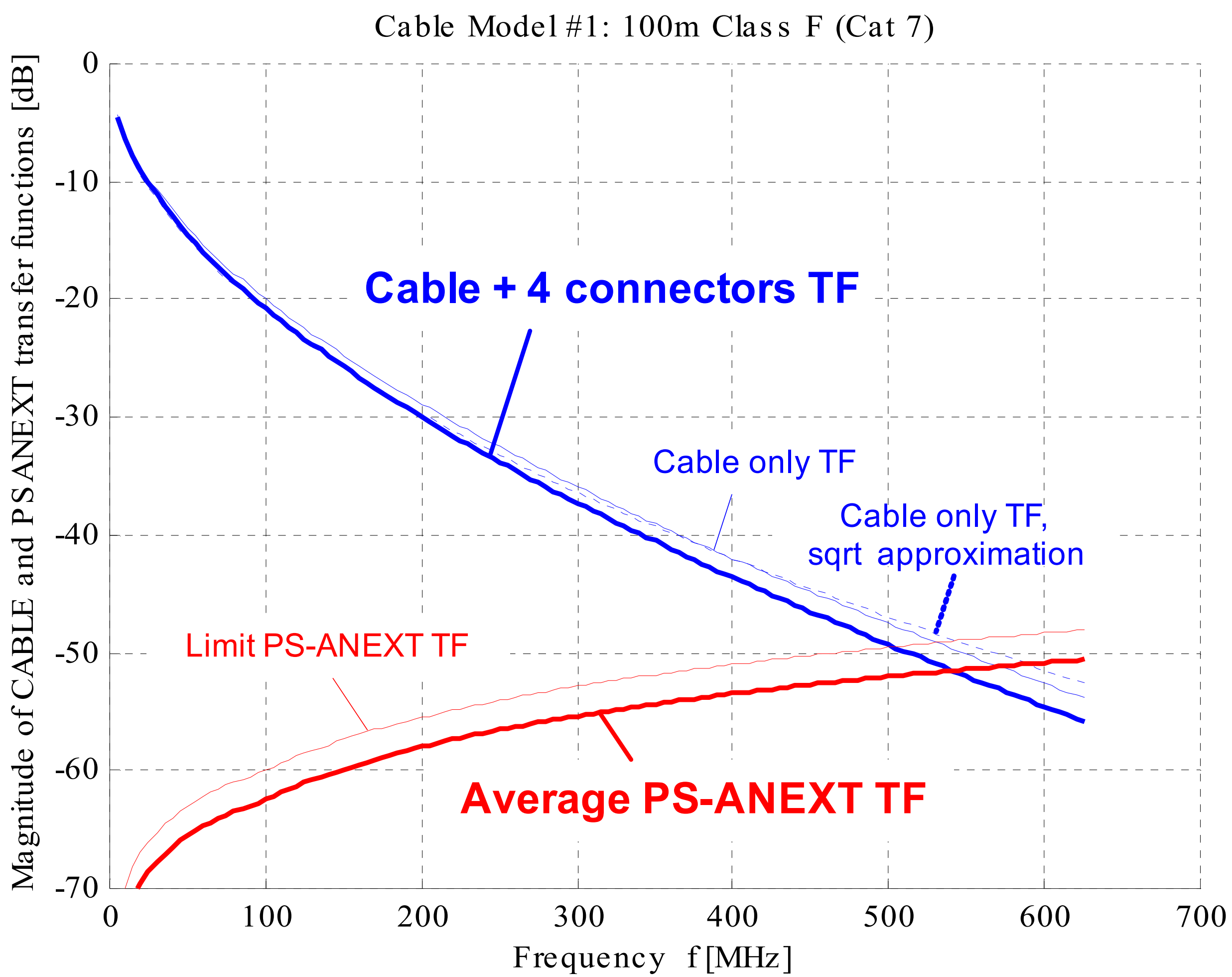
Agenda

- Transfer functions of cable models #1, #2, #3: cable and ANEXT; fitting by physical cable model
- Achievable rates with gap to capacity
- Optimum TX PSD for alien NEXT from same-type transmission and AWGN
- Practical choice of TX PSD
- Achievable rate as function of modulation rate
- Conclusions

Cable and PS-ANEXT transfer functions: model #1



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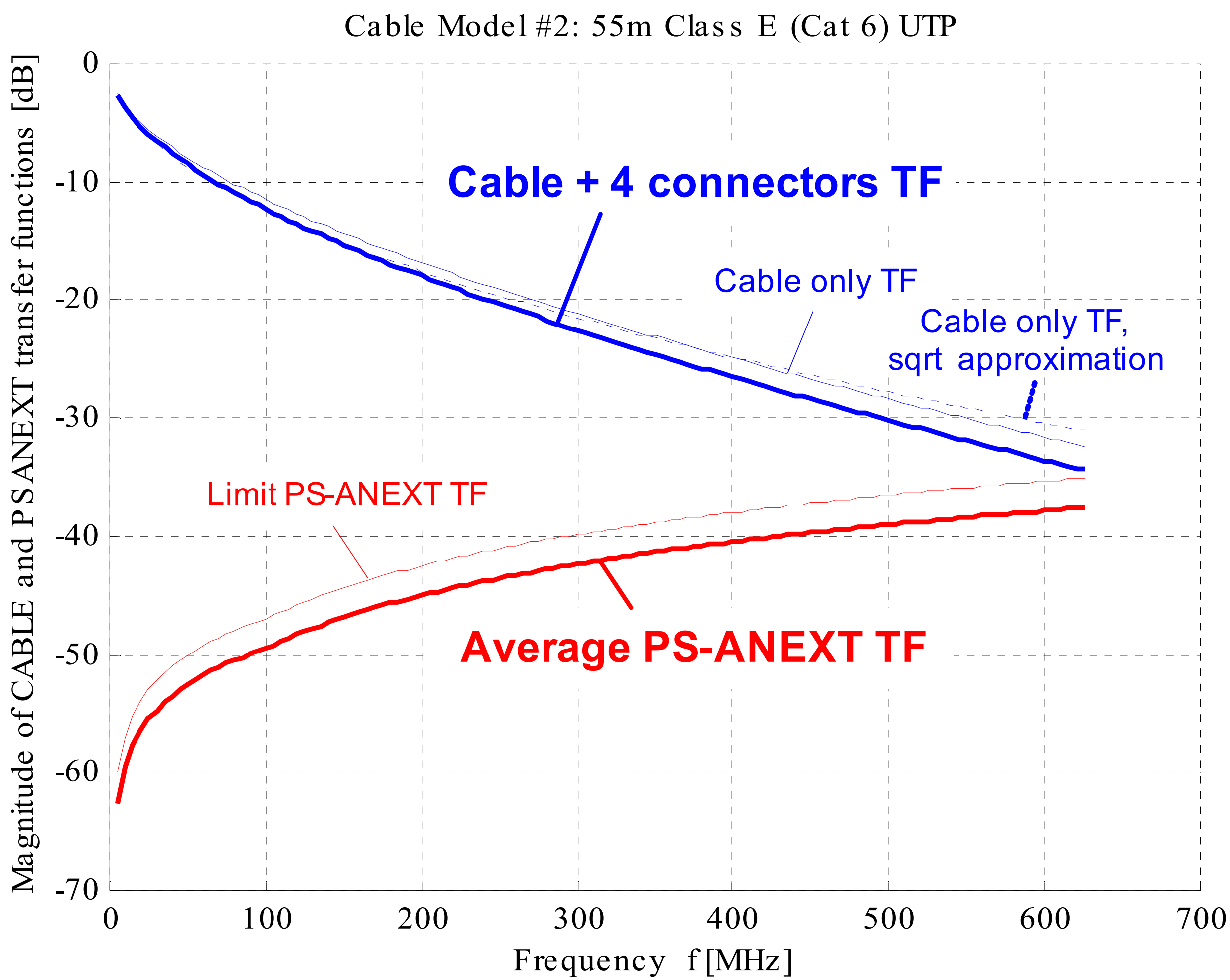
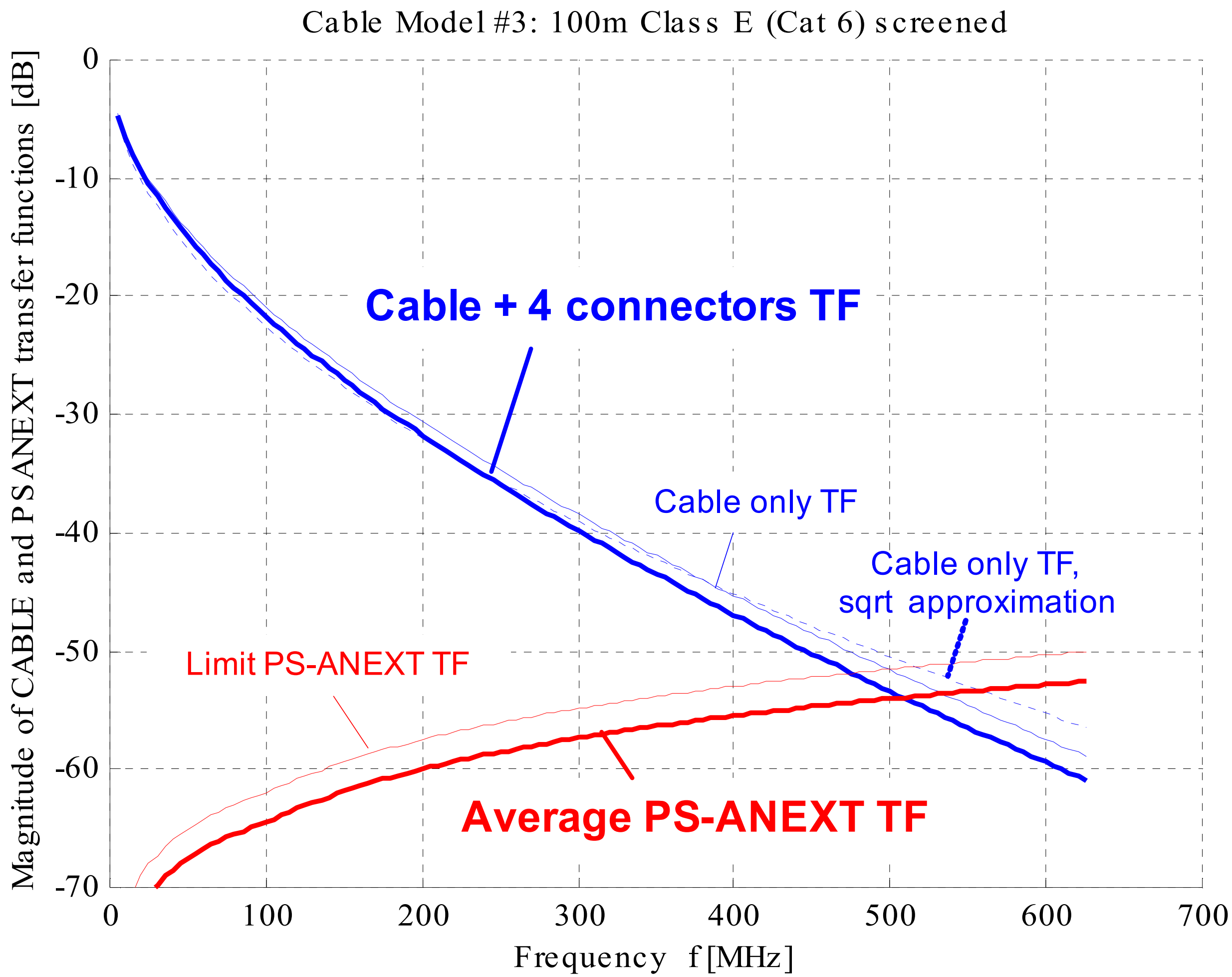
Cable and PS-ANEXT transfer functions: model #2



Cable and PS-ANEXT transfer functions: model #3



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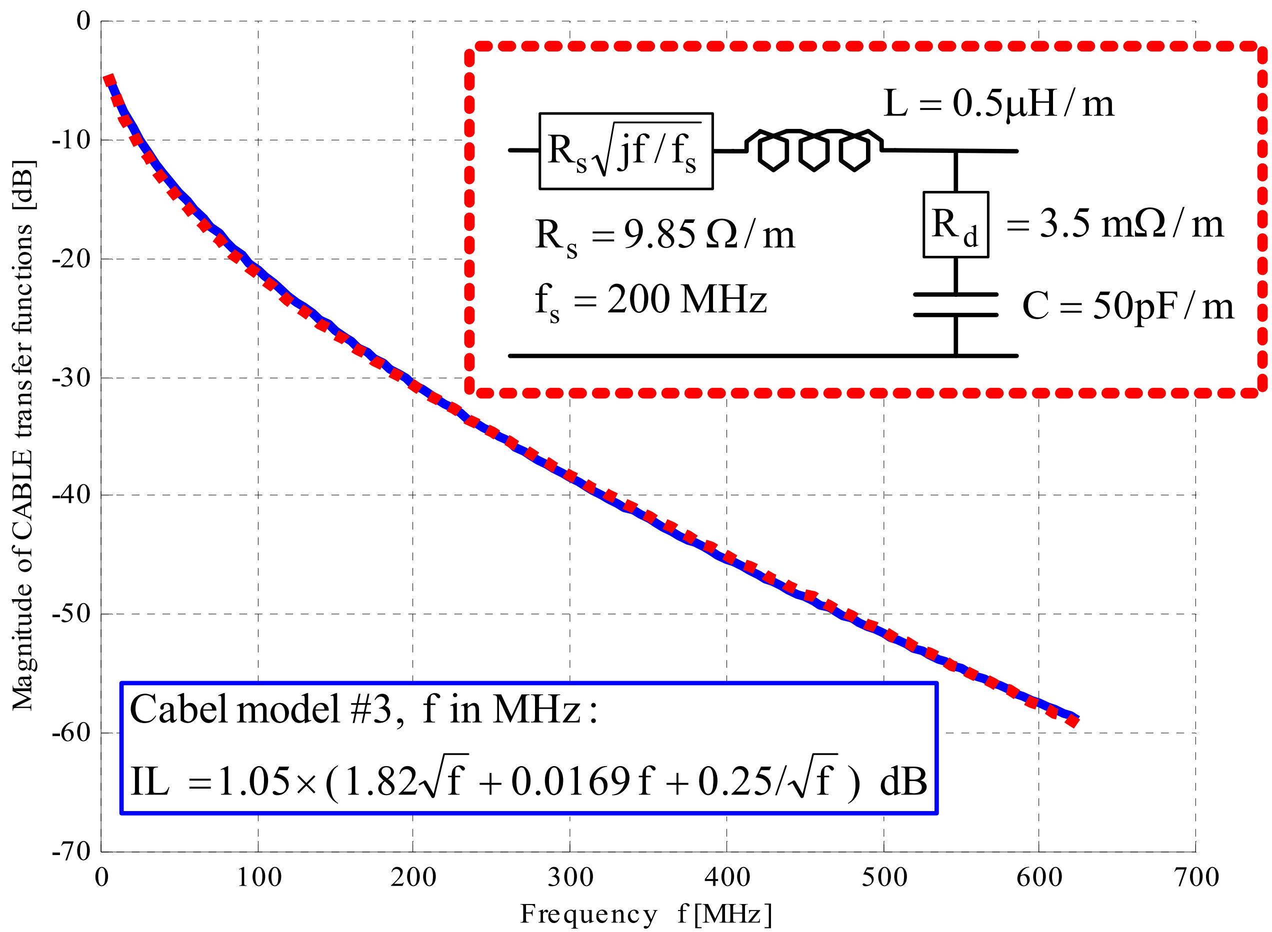
Fitting cable model #3 by physical cable model



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Achievable rate

Transmit - signal PSD $S_T(f)$. Average transmit power $P_T = \int_{f \geq 0} S_T(f)\, df$

Received - signal spectral SNR function $\mathrm{SNR}(f) = \frac{S_T(f)\left|G_C(f)\right|^2}{N(f)}$

Maximally achievable rate (capacity) for given SNR(f)

$$C_{[\mathrm{bit/s}]} = \int_{f \geq 0} \log_2\left(1 + \mathrm{SNR}(f)\right) df$$

Achievable rate with gap $\Gamma \geq 1$ to capacity

$$R_{[\mathrm{bit/s}]} = \int_{f \geq 0} \log_2\left(1 + \frac{\mathrm{SNR}(f)}{\Gamma}\right) df \quad \leq \quad C_{[\mathrm{bit/s}]}$$

No assumption on type of modulation
(multi carrier , single carrier, linear/nonlinear …)

Optimizing $S_T(f)$ for ANEXT from same-type transmission and AWGN

$$\text{Maximize} \quad R_{[\text{bit/s}]} = \int_{f\ge 0} \log_2\left(1+\frac{1}{\Gamma}\frac{S_T(f)\left|G_C(f)\right|^2}{N(f)+S_T(f)\left|G_A(f)\right|^2}\right) df$$

$$\text{w.r.t. } S_T(f) \text{ with constraint } P_T = \int_{f\ge 0} S_T(f)\, df$$

Exact solution

$$\text{Let} \quad s(f) = s_{o(pt)}(f) + \varepsilon g(f) \ge 0, \quad q(f) \ge 0, \quad a(f) \ge 0$$

$$\text{Lagrange}: \quad J = \int_f \ln\left(1+\frac{s(f)}{q(f)+s(f)a(f)}\right) dx - \frac{1}{\lambda}\left(\int_f s(f)\, df - P\right) \rightarrow \max$$

$$\left.\frac{\partial J}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_f \underbrace{\left[\frac{q}{\left[q+s_o(f)(1+a)\right]\left[q+s_o(f)a\right]} - \frac{1}{\lambda}\right]}_{=0} g(f)\, df = 0 \Rightarrow s_o^2(f)(1+a)a + s_o(f)(1+2a)q - (\lambda - q)q = 0$$

$$a = 0: \quad s_o(f) = \max(\lambda - q, 0) \quad \text{... water pouring solution}$$

$$a > 0: \quad s_o(f) = \max\left(\frac{-(1+2a)q + \sqrt{q^2 + 4(1+a)a\lambda q}}{2(1+a)a}, 0\right)$$

Optimizing $S_T(f)$ for ANEXT from same-type transmission and AWGN

$$\text{Maximize} \quad R_{[\text{bit/s}]} = \int_{f \geq 0} \log_2\left(1 + \frac{1}{\Gamma}\frac{S_T(f)\left|G_C(f)\right|^2}{N(f) + S_T(f)\left|G_A(f)\right|^2}\right) df$$

$$\text{w.r.t. } S_T(f) \text{ with constraint } P_T = \int_{f \geq 0} S_T(f)\, df$$

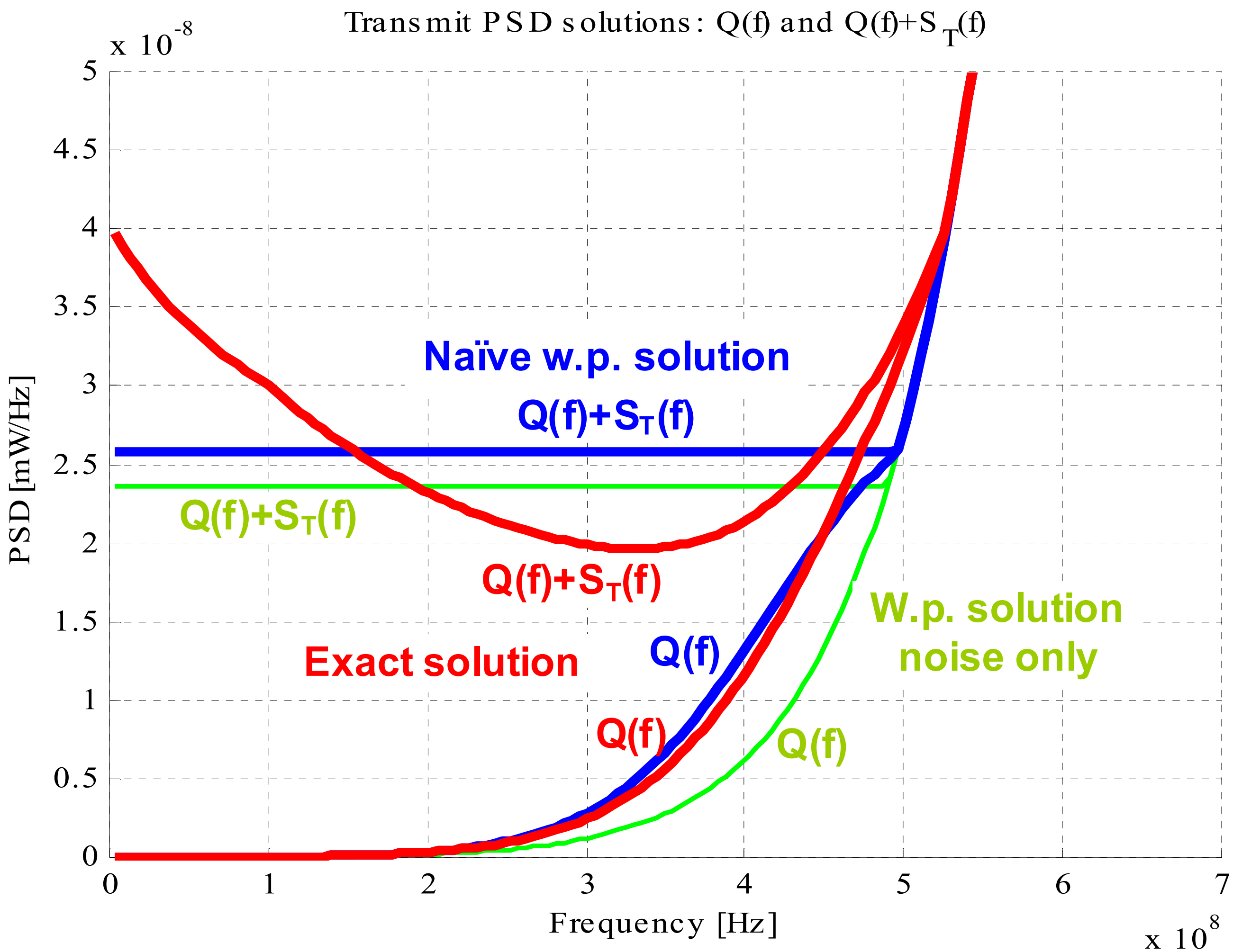
Naïve = water pouring solution

$$S_T(f) = \max\left(\lambda - Q(f), 0\right) \; ; \quad Q(f) = \frac{\Gamma\left(N(f) + S_T(f)\left|G_A(f)\right|^2\right)}{\left|G_C(f)\right|^2}$$

$$\left|G_C(f)\right|^2\left(\lambda - S_T(f)\right) = \Gamma\left(N(f) + S_T(f)\left|G_A(f)\right|^2\right)$$

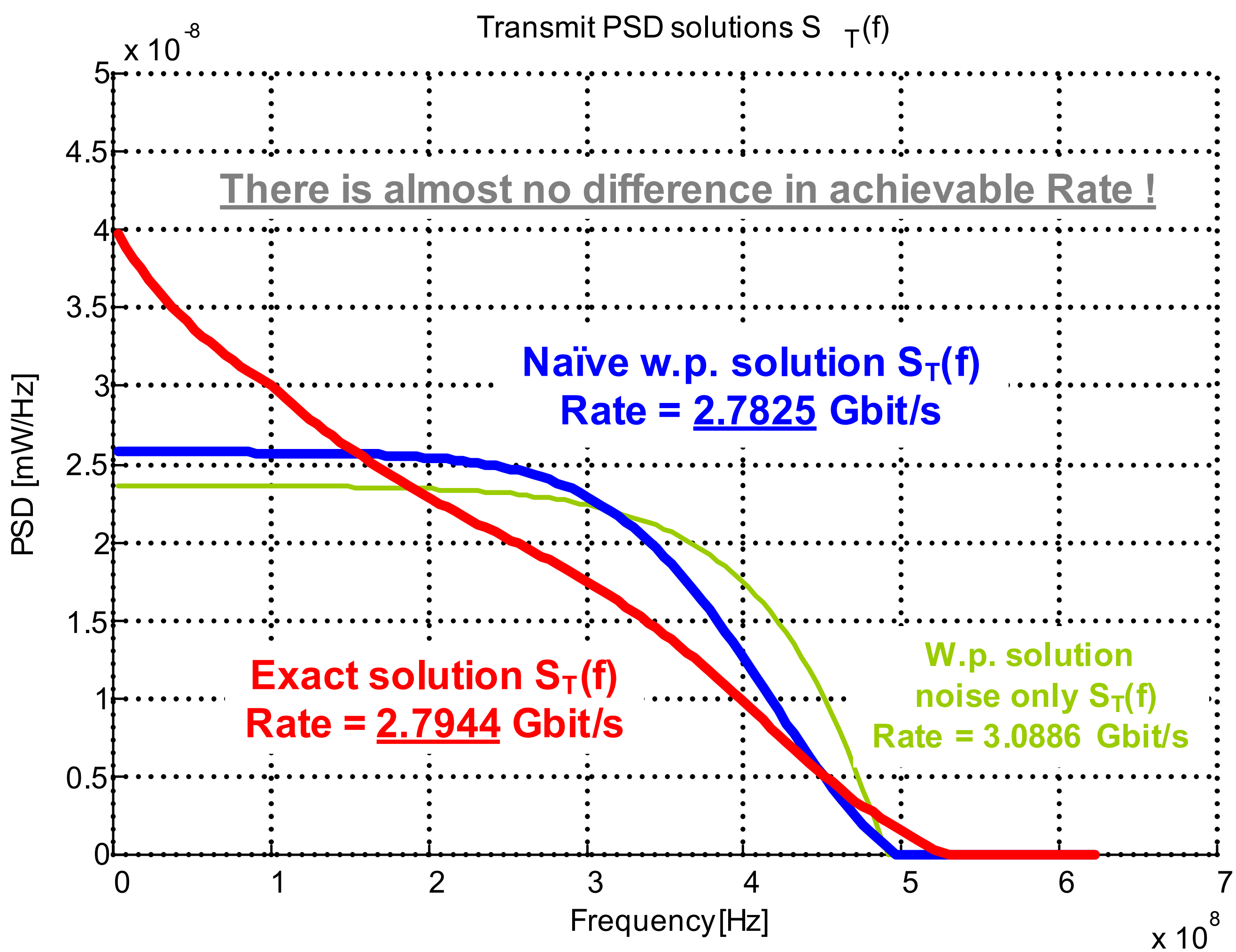
$$S_T(f) = \max\left(\frac{\lambda\left|G_C(f)\right|^2 - \Gamma\, N(f)}{\left|G_C(f)\right|^2 + \Gamma\left|G_A(f)\right|^2}\; , \; 0\right)$$

TX PSD solutions



Cable model #3: $P_T = 10\text{dBm}$, $N_0 = -135\ \text{dBm/Hz}$, $\Gamma = 6\ \text{dB}$

TX PSD solutions



Cable model #3: $P_T = 10\text{dBm}, N_0 = -135 \text{ dBm/Hz}, \Gamma = 6 \text{ dB}$

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Achievable rate: MC and SC modulation

$$\text{Multi-carrier modulation} \quad R^{MC}_{[bit/s]} = \sum_{i=0}^{N-1} \log_2\left(1 + \frac{SNR_i}{\Gamma}\right)\Delta f \overset{N\to\infty}{=} \int_{f>0} \log_2\left(1 + \frac{SNR(f)}{\Gamma}\right) df$$

$$\text{Single-carrier modulation} \quad R^{SC}_{[bit/s]} = \frac{1}{T}\log_2\left(1 + \frac{SNR_{mmse\text{-}wmf\text{-}u}}{\Gamma}\right)$$

$$SNR^*(f) = SNR(f) + SNR\left(\frac{1}{T} - f\right), \quad \underbrace{\log_2\left(1 + SNR_{mmse\text{-}wmf\text{-}u}\right)}_{SNR_{mmse-wmf}} = 2T \int_{f=0}^{1/2T} \log_2\left(1 + SNR^*(f)\right) df$$

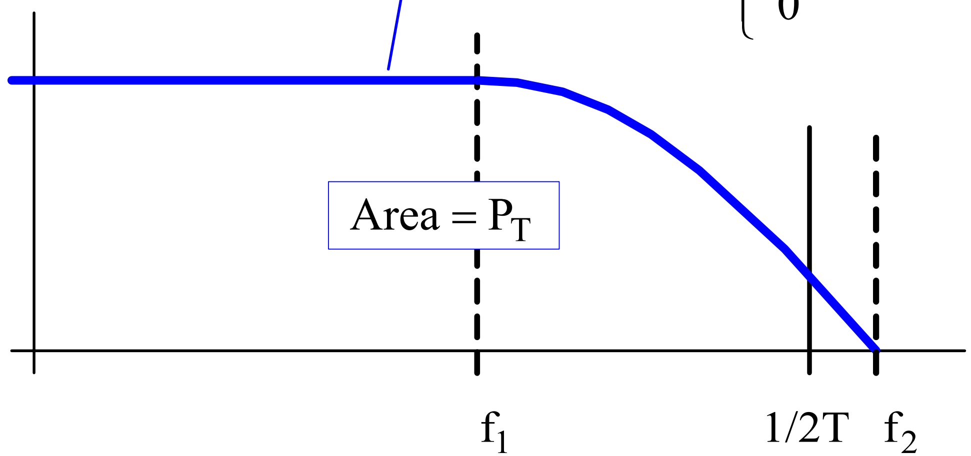
$\Gamma = 1: \quad R^{SC}_{[bit/s]} = R^{MC}_{[bit/s]}$ in absence of SC spectral folding (zero excess bandwidth)

$\Gamma > 1: \quad R^{SC}_{[bit/s]} < R^{MC}_{[bit/s]}$ if SNR(f) very low in parts of (0, 1/2T)

SC modulation: practical choice of TX PSD

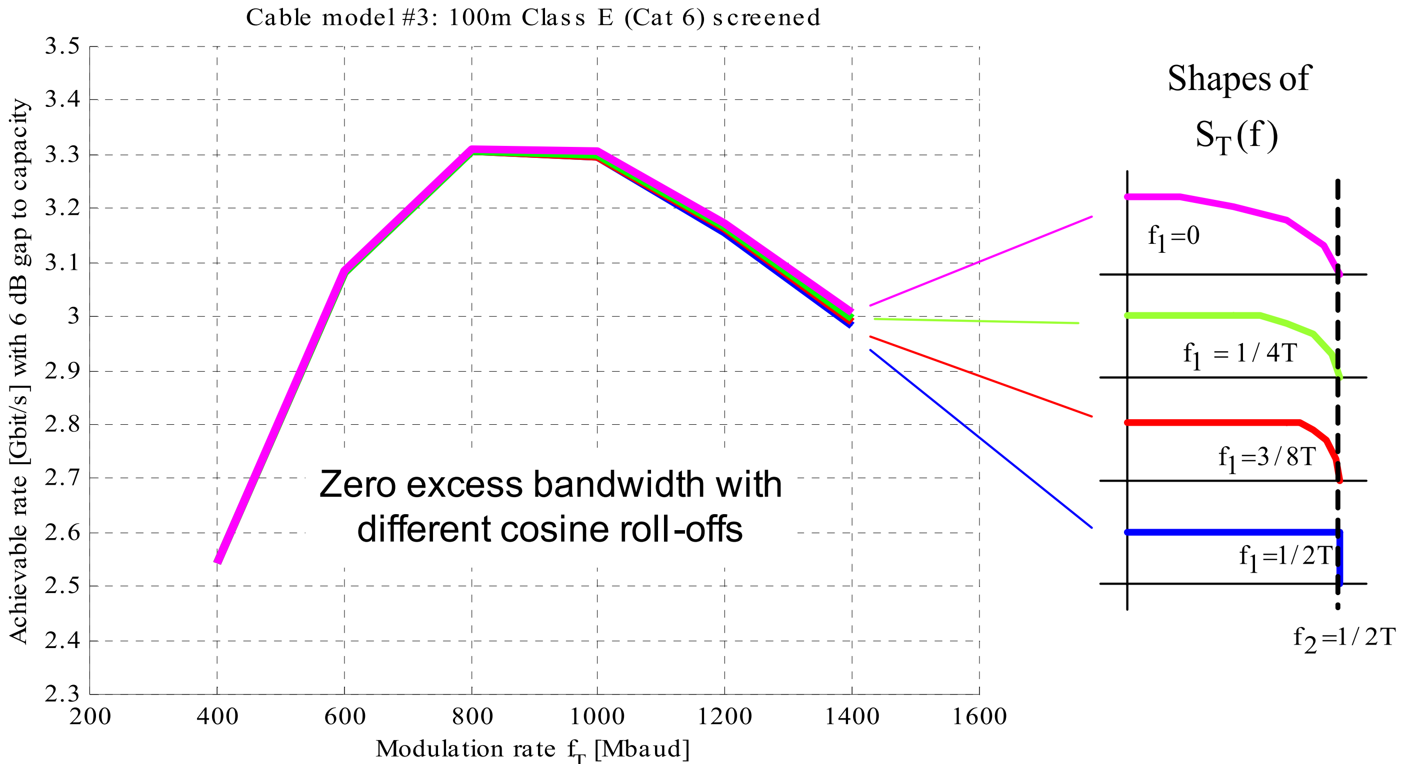
- **Achievable rate is insensitive to exact shape of TX PSD, as long as bandwidth (modulation rate) is appropriately chosen.**
- **TX PSD assumed for further investigations:**

$$\text{Spectrum with cosine roll-offs: } S_T(f) = S_0 \begin{cases} 1 & , \quad |f| \le f_1 \\ \cos\left(\dfrac{\pi}{2}\dfrac{(f-f_1)}{(f_2-f_1)}\right) & , \quad f_1 < |f| \le f_2 \\ 0 & , \quad f_2 < |f| \end{cases}$$



Achievable bit rate vs modulation rate

- **Cable Model #3: 100m Class E (Cat 6) screened**
- **TX PSD with cosine roll-offs f_1 = 0 to 1/2T, f_2 = 1/2T (zero excess bandwidth)**
- **Modulation rates f_T=1/T = 400 to 1400 Mbaud; P_T=10dBm, N_0=-150 dBm/Hz, Γ = 4 (6 dB).**



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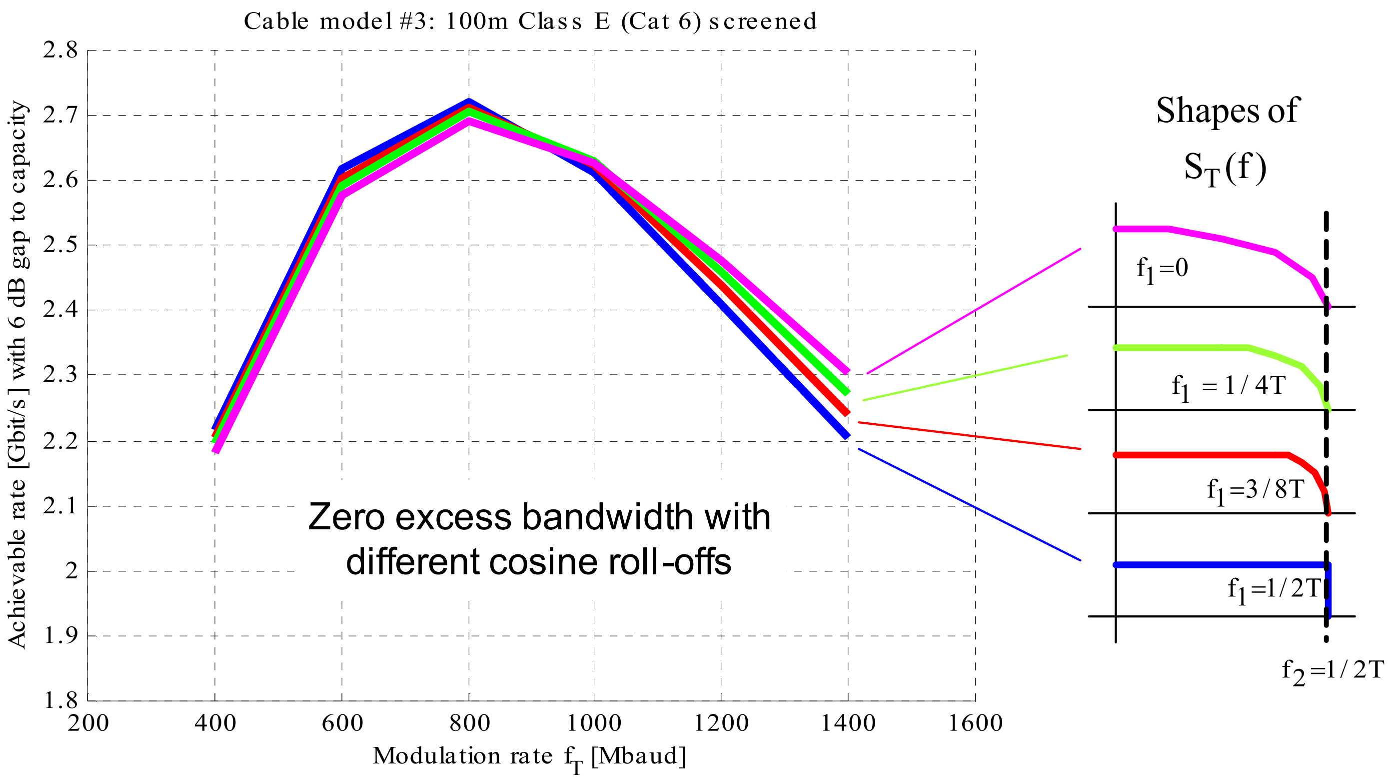
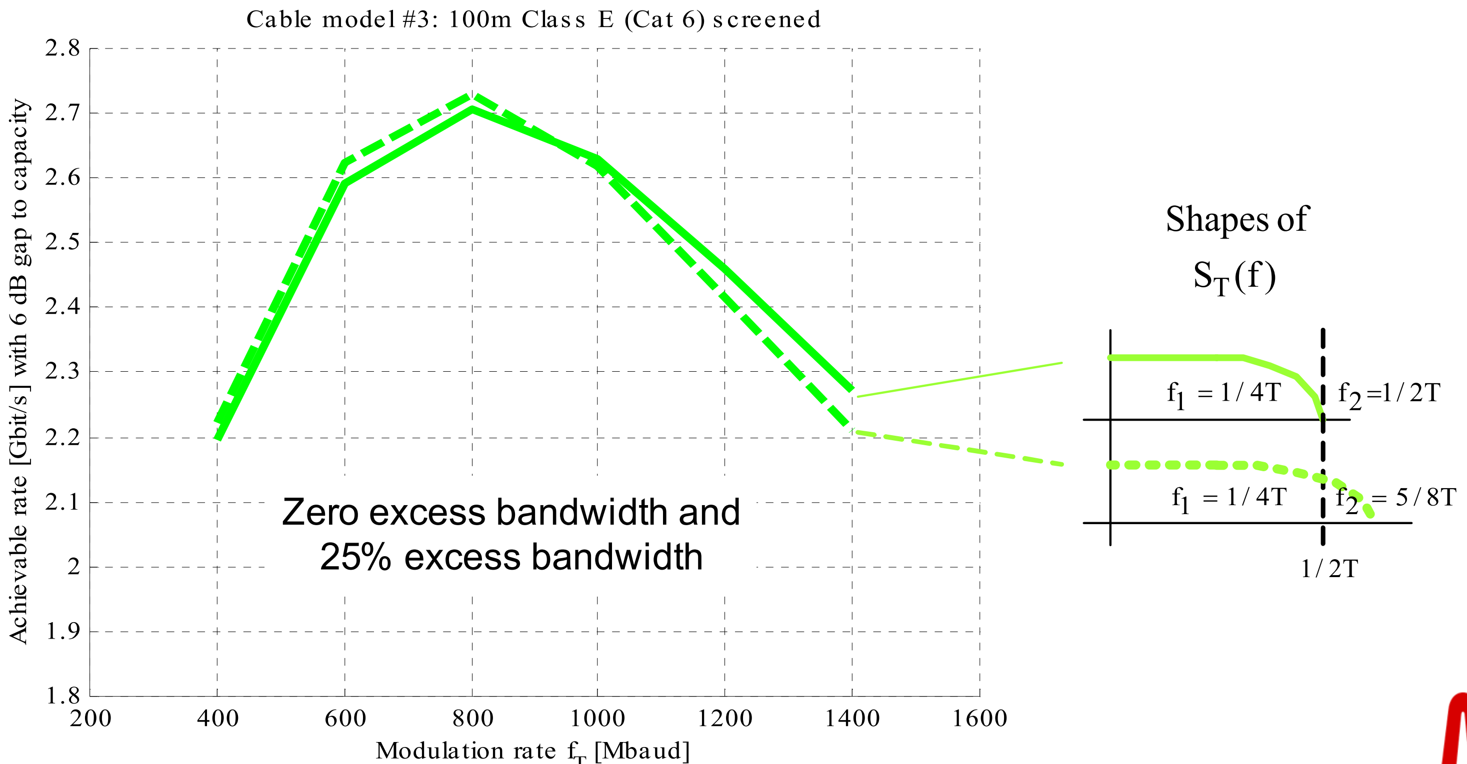


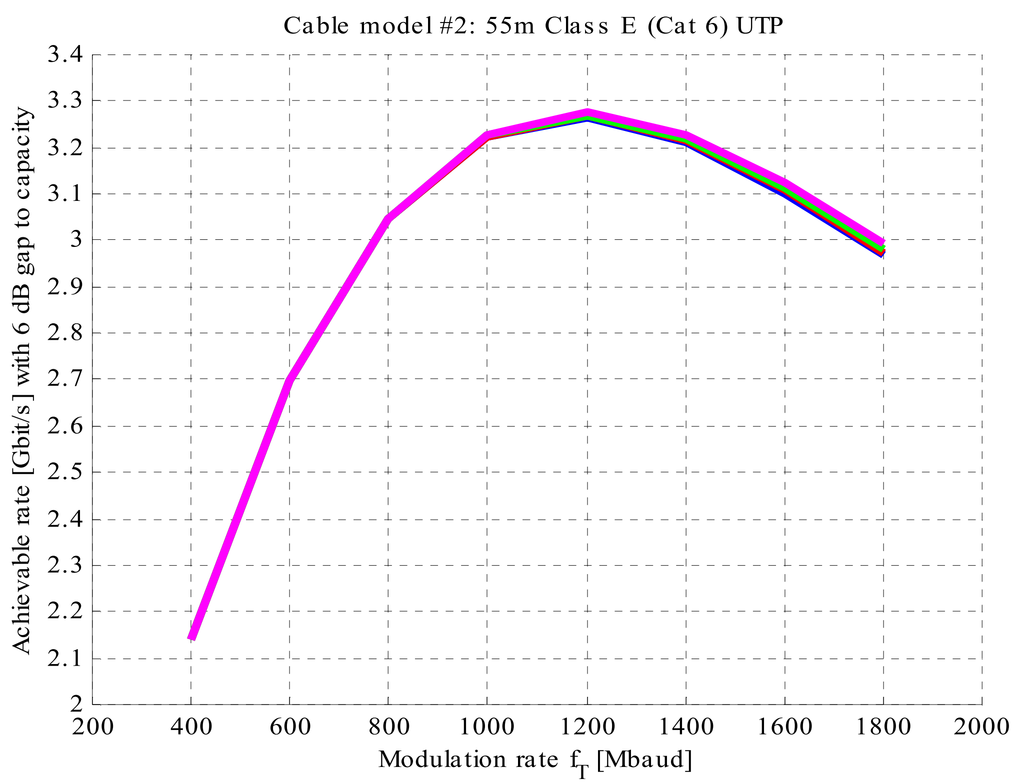
Achievable bit rate vs modulation rate

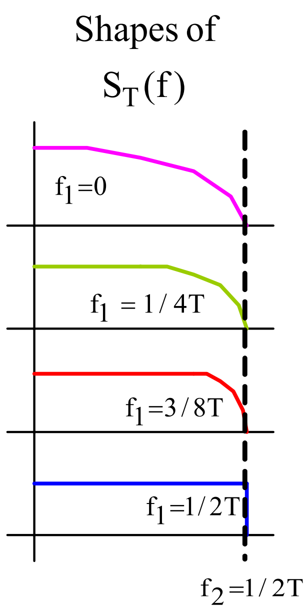
- **Cable Model #3: 100m Class E (Cat 6) screened**
- **TX PSD with cosine roll-offs: zero excess bandwidth and 25% excess bandwidth**
- **Modulation rates f_T=1/T = 400 to 1400 Mbaud; P_T=10dBm, N_0=-135 dBm/Hz, Γ = 4 (6 dB).**



Achievable bit rate vs modulation rate

- **Cable Model #2: 55m Class E (Cat 6) UTP**
- **TX PSD with cosine roll-offs f_1 = 0 to 1/2T, f_2 = 1/2T (zero excess bandwidth)**
- **Modulation rate f_T=1/T = 400 to 1800 Mbaud; P_T=10dBm, N_0=-135 dBm/Hz, Γ = 4 (6 dB).**





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Conclusions

- Achievable rate depends only moderately on spectral shape of TX PSD. Zero excess bandwidth has some advantages.
- Modulation rate is more important than spectral shape of TX PSD. Peak of achievable rate vs modulation rate still shallow.
- For cable model #3 (worst case?), a modulation rate of about 800 Mbaud (= 3.12 bit/PAM symbol) appears to be a good choice, from viewpoint of achievable rate.

  Drawback: signal-converter and signal-processing precision; sensitivity to disturbances not covered by present cable model, e.g., narrow-band interference.
- For cable model #2, a modulation rate of about 1000 Mbaud (= 2.5 bit/PAM symbol) represents a good choice.

  Drawback: higher signal processing rate, 10 Gbit/s not achieved for cable model #3 (if N_0 = -135 dBm/Hz).