10GBASE-T Coding and Modulation: 128-DSQ + LDPC

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Precoding system and definition of SNR

Signal-to-noise ratio
\[
\text{SNR} = \frac{E_x}{\sigma_w^2}
\]

where
- \(E_x\) is the signal energy
- \(\sigma_w^2\) is the noise variance

\(a_n \in M\text{-PAM} = \{\pm 1, \pm 3, \ldots, \pm(M-1)\}\)

\[x(D) = \frac{a(D) + 2Mk(D)}{h(D)}\]

\[y(D) = a(D) + 2Mk(D) + w(D)\]

\(2M \times k_n\)

\(h(D) - 1\)

DAC, TX filter, Channel, RX filter, ADC

FF equalizer

Soft demapping and decoding

Connecting everything

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2-D constellations for modulation rates in 800+ Mb range

**12-PAM² (with or w/o hole)**

- **12-PAM**
- **M - PAM = \{±1,±3,…±(M-1)\}

**128-DSQ (Double SQuare)**

- **16-PAM**
- **2M = 32**
- **Δ₀ = 2√2**

Signal energy per dimension at precoder output: \( E_x = \frac{(2M)^2}{12} \)

- \( E_x / Δ₀^2 = 48 / 4 = 12 \)
- \( E_x / Δ₀^2 = \left(\frac{256}{3}\right) / 8 = 10.666 \)

-0.5115 dB
128-DSQ: probability of intra-subset errors

\[ \text{Probability of intra-subset (ISS) error for 9 dB set partitioning (8 subsets)} \]

\[ \text{Probability of intra-subset (ISS) error for 12 dB set partitioning (16 subsets)} \]

BER limited by ISS distance

128-DSQ: \( M = 16; \ \sigma^2_w = (2M)^2 / 12; \Delta_0^2 = 8 \)

\[ \Delta_3^2 = 8\Delta_0^2: \Pr(e)_{\text{ISS-lev3}} = \frac{1}{2} \times 4 \times Q\left(\frac{\Delta_3}{2\sigma_w}\right); \Delta_4^2 = 16\Delta_0^2: \Pr(e)_{\text{ISS-lev4}} = \frac{1}{2} \times 4 \times Q\left(\frac{\Delta_4}{2\sigma_w}\right) \]

This confirms the need for 12 dB set partitioning
Coding, modulation, framing: two variants

Variant I: 128-DSQ + LDPC(1024,821) (M = 384, d_H \geq 14)

- LDPC coding weak w.r.t. to uncoded-bit-only error performance
- Code rate 3.1035 bit/dim
- Framing example: 1 frame = 8 code blocks → modulation rate 821.51 Mbaud, 0.29% overhead for synch and aux. channel.

Variant II: 128-DSQ + LDPC(1024,797) (M = 512, d_H \geq 18)

- Stronger LDPC coding better matched to uncoded-bit-only error performance
- Code rate 3.0566 bit/dim (-0.0469 bit/dim vs. 0.28 dB gain)
- Framing example: 1 frame = 1 code block → modulation rate 833.33 Mbaud (25 MHz x 100/3), 0.28% overhead for synch and aux. channel.
128-DSQ modulation with 12 dB set partitioning (16 2-D subsets) and (1024,821) LDPC coding

256 128-DSQ symbols = 512 redundant 16-PAM symbols

3*256 = 768 uncoded info bits
821 coded info bits
203 check bits

Code block = 1589 info bits encoded into 512 PAM symbols (3.1035 bit/dim)

Framing example

XGMII
2 x (32 TXD + 4 TXC) bits

TX_CLK:
10 GHz / 64 = 156.25 MHz

64+/65 transcoding

65-bit blocks

Framing + Coding + Modulation

10GBASE-T Frame
8 code blocks over four pairs ≈ 8 x 1589 bits
= 195 x 65-bit blocks + 37 overhead bits (0.29%)

Modulation rate = 10 GHz/64 x 1024/195 = 820.51 Mbaud

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Coding, modulation, framing: variant II

128-DSQ modulation with 12 dB set partitioning (16 2-D subsets) and (1024,797) LDPC coding

Code block = 1565 info bits encoded into 512 PAM symbols (3.0566 bit/dim)

Framing example

10GBASE-T Frame =
1 Code block over four pairs: 1565 bits = 24 x 65-bit blocks + 5 overhead bits (0.28%)
128-DSQ bit mapping: 3 uncoded bits, 4 coded bits

4 coded bits:
Gray ($d_H = 1$)

3 uncoded bits:
pseudo-Gray ($d_H = 1$ or 2)

Basic 128-DSQ with cyclic precoding extensions
128-DSQ bit mapping

**Step 1:** \( 0 \leq (x_i = 8x_i^3 + 4x_i^2 + 2x_i^1 + x_i^0) \leq 15, \quad i = 1, 2 \)

\[
\begin{align*}
  x_3 &= \bar{u}_1 \& u_3 \\
  x_2 &= (u_2 \& u_3) \lor (u_1 \& \bar{u}_2) \\
  x_1 &= u_1 \oplus u_3 \\
  x_0 &= u_1 \oplus c_2 \\
  c_1 &= 000 \\
  c_2 &= 001 \\
  c_3 &= 011 \\
  c_4 &= 101
\end{align*}
\]

**Step 2:**
\[
\begin{bmatrix}
  y_1 \\
  y_2
\end{bmatrix} = \begin{bmatrix}
  1 & 1 \\
  -1 & 1
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} \mod 16
\]

**Step 3:**
\[
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix} = 2 \begin{bmatrix}
  y_1 \\
  y_2
\end{bmatrix} - \begin{bmatrix}
  15 \\
  15
\end{bmatrix}
\]
128-DSQ bit mapping: implementation

7-bit label

\[ x_1^3 = \overline{u}_1 \& u_3 \]
\[ x_1^2 = u_1 \oplus u_3 \]
\[ x_1^1 = c_1 \]
\[ x_1^0 = c_1 \oplus c_2 \]

\[ x_2^3 = (u_2 \& u_3) \lor (u_1 \& \overline{u}_2) \]
\[ x_2^2 = u_2 \oplus u_3 \]
\[ x_2^1 = c_3 \]
\[ x_2^0 = c_3 \oplus c_4 \]

Two 16-PAM symbols

\[ a_1 \in \{\pm 1, \pm 3, \cdots, \pm 15\} \]
\[ a_2 \in \{\pm 1, \pm 3, \cdots, \pm 15\} \]
128-DSQ soft demapping: 4 coded bits

\[
R^{-1} = \begin{bmatrix}
0.5 & -0.5 \\
0.5 & 0.5 \\
\end{bmatrix}
\begin{bmatrix}
(s_1 + 15)/2 \\
(s_2 + 15)/2 \\
\end{bmatrix}
\]

Extended constellation points caused by precoding

\[
\text{llrb}(x) = \ln \left( \frac{1}{\sqrt{\pi \sigma^2}} \right) \left( -\frac{x^2}{2\sigma^2} \right) + \ln \left( \frac{1}{\sqrt{\pi \sigma^2}} \right) \left( -\frac{(x + 1)^2}{2\sigma^2} \right)
\]

\[
\sum_k \exp \left( -\frac{[x - (4k + 0)]^2}{2\sigma^2} \right) + \exp \left( -\frac{[x - (4k + 1)]^2}{2\sigma^2} \right)
\]

\[
\sum_k \exp \left( -\frac{[x - (4k + 2)]^2}{2\sigma^2} \right) + \exp \left( -\frac{[x - (4k + 3)]^2}{2\sigma^2} \right)
\]

\[
\log \frac{\Pr(c_1 = 0 / x_1)}{\Pr(c_1 = 1 / x_1)} = \text{llrb} \left( x_1 \mod 4 \right)
\]

\[
\log \frac{\Pr(c_2 = 0 / x_1)}{\Pr(c_2 = 1 / x_1)} = \text{llrb} \left( x_1 + 1 \mod 4 \right)
\]

\[
\log \frac{\Pr(c_3 = 0 / x_2)}{\Pr(c_3 = 1 / x_2)} = \text{llrb} \left( x_2 \mod 4 \right)
\]

\[
\log \frac{\Pr(c_4 = 0 / x_2)}{\Pr(c_4 = 1 / x_2)} = \text{llrb} \left( x_2 + 1 \mod 4 \right)
\]
The function \( \text{llrb}(x) \)

\[
\text{llrb}(x) = \ln \left( \sum_{k} \exp\left(-\frac{(x - (4k + 0))^2}{2\sigma^2}\right) + \exp\left(-\frac{(x - (4k + 1))^2}{2\sigma^2}\right) \right)
\]

\[
\sum_{k} \exp\left(-\frac{(x - (4k + 2))^2}{2\sigma^2}\right) + \exp\left(-\frac{(x - (4k + 3))^2}{2\sigma^2}\right)
\]

\[
\approx \frac{1}{\sigma^2} \left\{ \begin{array}{ll}
    x + 0.5 & : 0 \leq x \leq 0.5 \\
    1.5 - x & : 0.5 < x \leq 2.5 \\
    x - 3.5 & : 2.5 < x \leq 4
\end{array} \right.
\]

\[
\text{SNR} = 23 \text{ dB}
\]

\[
\text{SNR} = 13 \text{ dB}
\]

\[
\text{SNR} = 33 \text{ dB}
\]
128-DSQ + LDPC performance

Simulation: precoding extensions included, messages (LLRs): sssssssxxxx.xxxxx

Problem in implementation of BP algorithm found. Correct result are better by about 0.3 dB.
12-PAM-T and 128-DSQ + LDPC performance

Long simulations by Bazhong Shen (Broadcom) without effect of precoding extensions

12-PAM-T + LDPC(1024,821) iter = 6

128-DSQ + LDPC(1024,821) iter = 6
Conclusions

- 128-DSQ constellation is the natural in-between 8-PAM and 16-PAM modulation

- Bit mapping, precoding, metric calculation, subset decoding: all based on simple logic and power-of-two based arithmetic

- Stronger LDPC(1024,797) code is better matched to uncoded-bit-only error performance

- and leads to simple low-overhead framing and easy clock generation.