Modeling MM Light Propagation using measured index error, DMD, and bandwidth

John Abbott
Corning Incorporated

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Discovering Beyond Imagination

Summary

1a. Predicting mode delays, DMD & BW from index error

1b. Predicting index error from DMD measurements.
  (i) matrix approach
  (ii) basis function approach

2. Common index errors from the 1GbE MBI study group reduced to mode delays; enriching the Cambridge 81 fiber mix with realistic worst-case perturbations.

Topic1: Index Profiles → mode delays →DMD

One convenient way to calculate light in multimode fibers given an actual measured index profile is to use perturbation methods.

All index perturbations are referenced to a base profile for which the scalar wave modal functions $\psi_i(r)$ and propagation constants β_i have been solved. Modes with the same β_i couple strongly and are considered to be in the same mode group.

If the index perturbation of the profile of interest is $\delta n(r)$, then [Snyder & Love section 18-5] the corresponding $\delta\beta$ is

$$\delta\beta = \frac{k\int_0^\infty \delta n \psi^2 \, r\, dr}{\int_0^\infty \psi^2 \, r\, dr}$$

Calculation of Mode Delays & DMD

For each individual mode the group delay is $\tau_i = \frac{1}{c}\frac{d\beta_i}{dk}$,
and the change in mode delay for the actual fiber relative to that of the base fiber is

$$\delta\tau_i = \frac{1}{c}\frac{d}{dk}\delta\beta_i = \frac{1}{c}\int \delta n(r)\frac{d}{dk}(\psi_i^2(r)k)\, r\, dr$$

Because the individual modes share energy, it is convenient to simplify the calculation to 20 mode groups rather than 200 individual modes using (weight = 1 radial, = 2 skew modes)

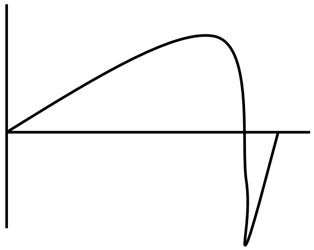
$$\psi_m^2(r) = \sum_{i\,\mathrm{in}\,m} \frac{w_i}{W_{tot}}\psi_i^2(r) \qquad \delta\tau_m = \sum_{i\,\mathrm{in}\,m} \frac{w_i}{W_{tot}}\delta\tau_i$$

Converting to Matrix format

Then $$\delta\tau_m = \frac{1}{c}\int \delta n \frac{d}{dk}(\psi_m^2(r)k)\, r\, dr$$

We note in passing that this is the modal form of an equation derived by Petermann for an ideal DMD,

$$c\tau = -\delta n + 2\frac{\int \delta n dA}{\int dA}$$



The first equation can be written in vector/matrix form as

$$\boxed{\boldsymbol{\tau}_{\mathbf{m}} = C_{mr}\mathbf{N}_{\mathbf{r}}}$$

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Calculating BW and DMD

The bandwidth etc. for a given modal power distribution can then be modeled in the usual way as the sum of delta functions to represent the pulse:

$$\mathbf{P(t)} = \sum_m P_m \delta(t - (\tau_m - \tau_{\mathbf{ave}})) \qquad \tau_{\mathbf{ave}} = \sum_m P_m \tau_m$$

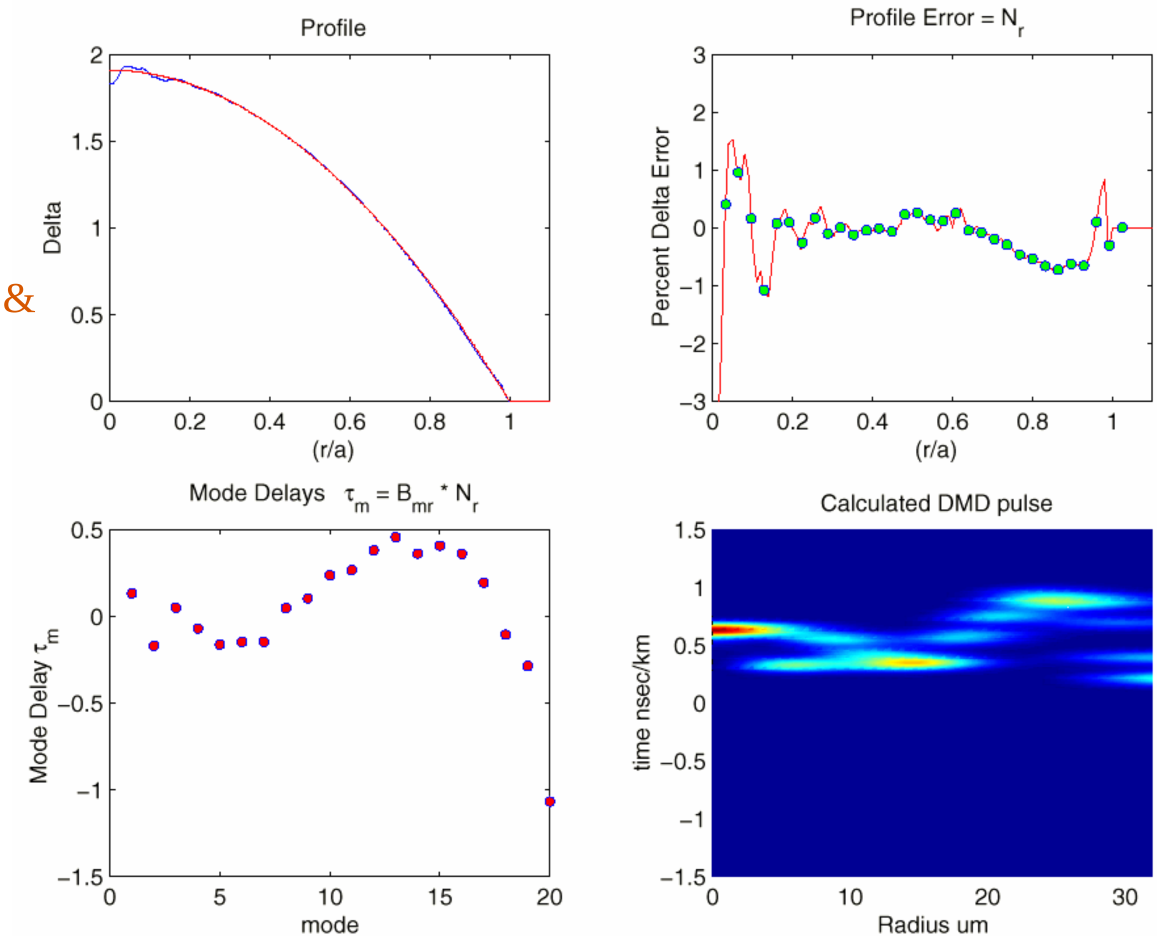
In the DMD measurement a specific launch is scanned across the fiber. At an offset x the fractional power going into mode group m can be calculated and denoted by Cxm . Then the DMD centroid vs. offset is given by

$$T_x = \sum_m B_{xm} \delta\tau_m \quad \text{, hence} \quad \boxed{T_x = B_{xm} C_{mr} \delta n_r = A_{xr} \delta n_r}$$

Worked Example

Using index profile data we estimate the index error δn_r, the mode delays $\delta\tau_m$, and DMD T_x

Measured Profile & Target Profile



Index Error

Mode Delays

DMD contours

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Estimating δn(r) from DMD data T_x

Because of the finite DMD spot size, the DMD centroid function T_x is a smooth curve. To estimate a smooth index error δn(r) we can use a least squares approach forcing some smoothness in the index perturbation (but not the mode delays). We find $\hat{N}_r$ which minimizes S where

$$S = \sum (T_x - \hat{T}_x)^2 + \lambda_a \sum \left(\frac{d^2 \hat{N}}{dr^2}\right)^2$$

Here $\hat{T}_x$ is the predicted value of T_x, i.e. $A_{xr}\hat{N}_r$

Converting DMD inversion to matrix form

We can turn this least squares formulation into an easy-to-implement matrix form by augmenting the matrix A_{xr} :

$$\begin{pmatrix} T_x \\ 0_r \end{pmatrix} = \begin{vmatrix} A_{xr} \\ \lambda_a D_{rr} \end{vmatrix} \begin{pmatrix} N_r \end{pmatrix}$$

This can then be inverted into a final matrix equation

$$N_r = G_{r(x+r)} T'_{(x+r)}$$

using singular value decomposition (SVD), which simplifies to

$$\boxed{N_r = G_{rx} T_x}$$

$$D_{rr} = \begin{vmatrix} 1 & 0 & 0 & 0 & \ldots \\ -1 & 2 & -1 & 0 & \ldots \\ 0 & -1 & 2 & -1 & \ldots \\ 0 & 0 & -1 & 2 & \ldots \\ 0 & 0 & 0 & \ldots & \ldots \end{vmatrix}$$

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Extensions of this approach

One extension of this approach with the full DMD data (not just the centroid curve) is to use the centroid to get a first approximation to $\delta n(r)$ and then calculate the full DMD pattern and generate iterations on $\delta n(r)$ which gradually improve the agreement.

Another approach is to use basis functions to represent $\delta n(r)$ and do the optimization using a limited number of free parameters.

The point in fiber manufacturing for estimating $\delta n(r)$ is to make a correction to the dopant flows in subsequent blanks to control the index profile.

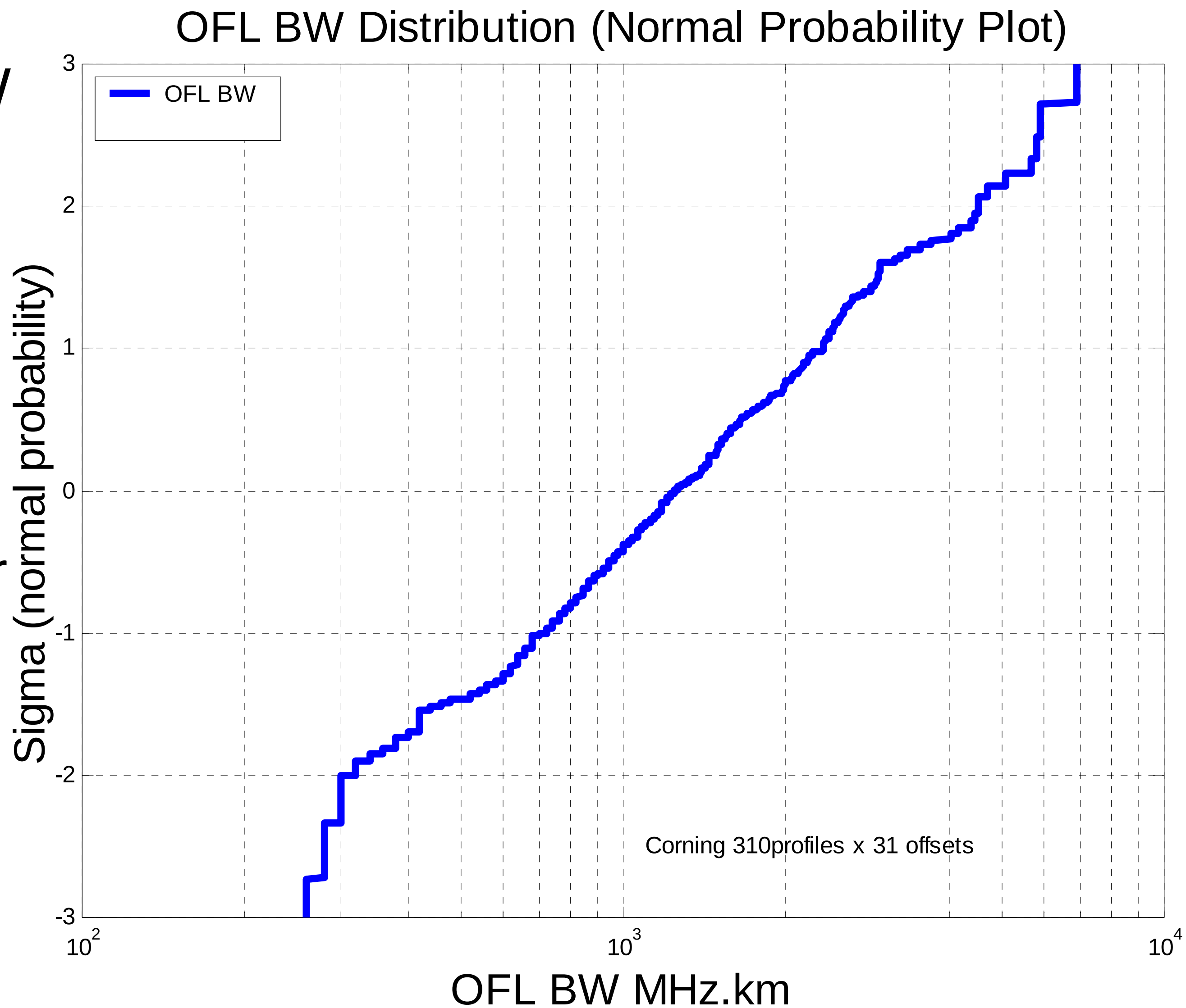
Topic 2 -- real index errors

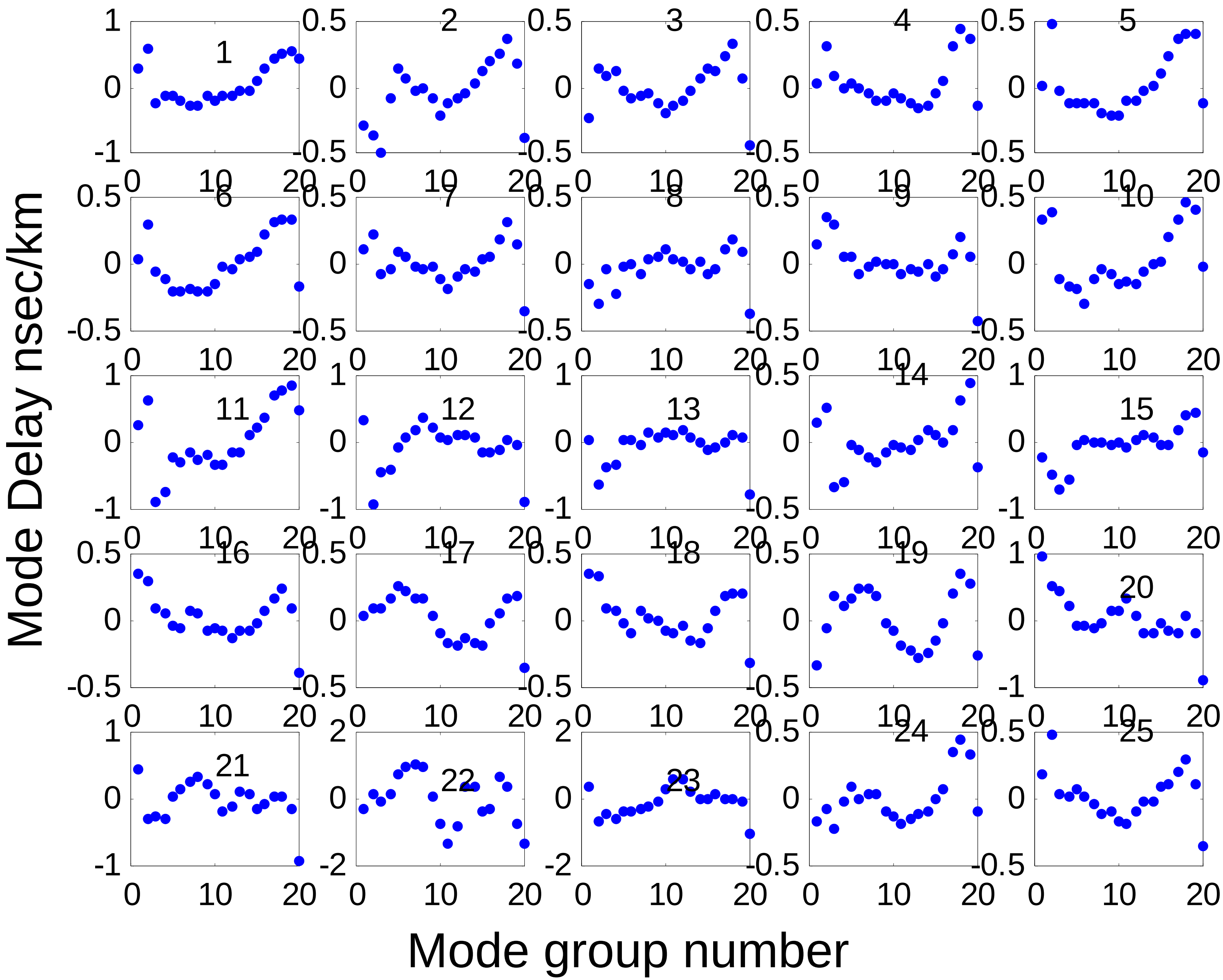
In the 1GbE MBI working group an analysis was made by Corning of the sensitivity of BW to launch conditions using a set of measured index profiles available at the time. The results were incorporated into the MBI recommendations and presented at IWCS 1998, and generally supported the concept of an offset launch.

The mode delays from these MBI profiles were studied to compare them to the Cambridge 81 fibers and current high res DMD information

modeled -3dB OFL BW distribution-MBI profiles

Calculated BW from index profile data shows a log normal distribution (not true sample of fiber distribution)




Mode Delay nsec/km
Mode group number
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References

[1] J.S. Abbott, D.E. Harshbarger, D. G. Cunningham, C.T. Di Minico, I.H. White, "Analysis of Multimode Fiber Behavior with Laser Sources in the Development of the Gigabit Ethernet Fiber Optic Specifications", **1998 IWCS Proceedings**

[2] J.S. Abbott, "Characterization of Multimode Fiber for 10Gbps Operation", **NIST Symposium on Optical Fiber Measurements**, Sept 2000.

[3] J.S. Abbott, "Light Propagation in Gbit LANS", **IMA Workshop on Analysis and Modeling of Optical Devices,** Sept 1999**.**

[4] P. Pepeljugoski, M.J. Hackert, J.S. Abbott, S.E. Swanson, S.E. Golowich, A.J. Ritger, P. Kolesar, Y. C. Chen, and P. Pleunis, "Development of System Specification for Laser-Optimized 50-um Multimode Fiber for Multigigabit Short-Wavelength LANS**", J. Lightwave Technology**, Vol. 21 No. 5 May 2003 p.1256.

[5] Petermann, K.,'simple relationship between Differential Mode Delay in Optical Fibres and the deviation from optimum profile', **Elec. Lett**. 14(1978) 24 pp.793-794.

[6] Snyder, A.W., and Love, J.D., **Optical Waveguide Theory**. New York: Chapman and Hall, 1983.

Annex1 –

BW distributions for MBI Field Test measurements &

TIA OM3 model fiber distribution

1^{st} -- Field Test Data from 1GbE MBI study

Measured OFL BW Data is on IEEE website at

http://www.ieee802.org/3/z/mbi/index.html

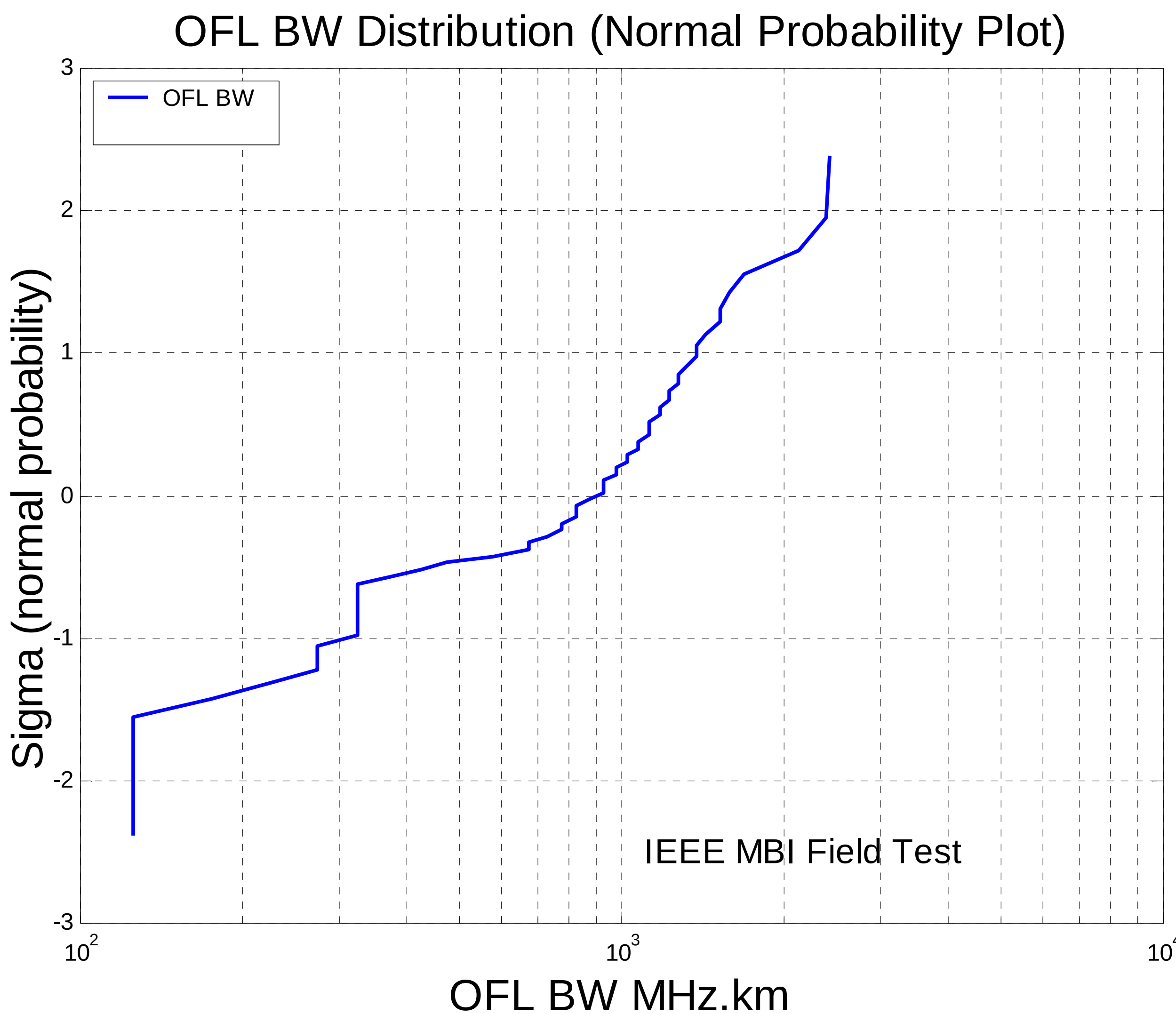
Excel spreadsheet

http://www.ieee802.org/3/z/mbi/fldbnd95.xls

MBI Field Test OFL BW on Normal Probability Scale

Measured BW distribution is lognormal – an inherently broad distribution.

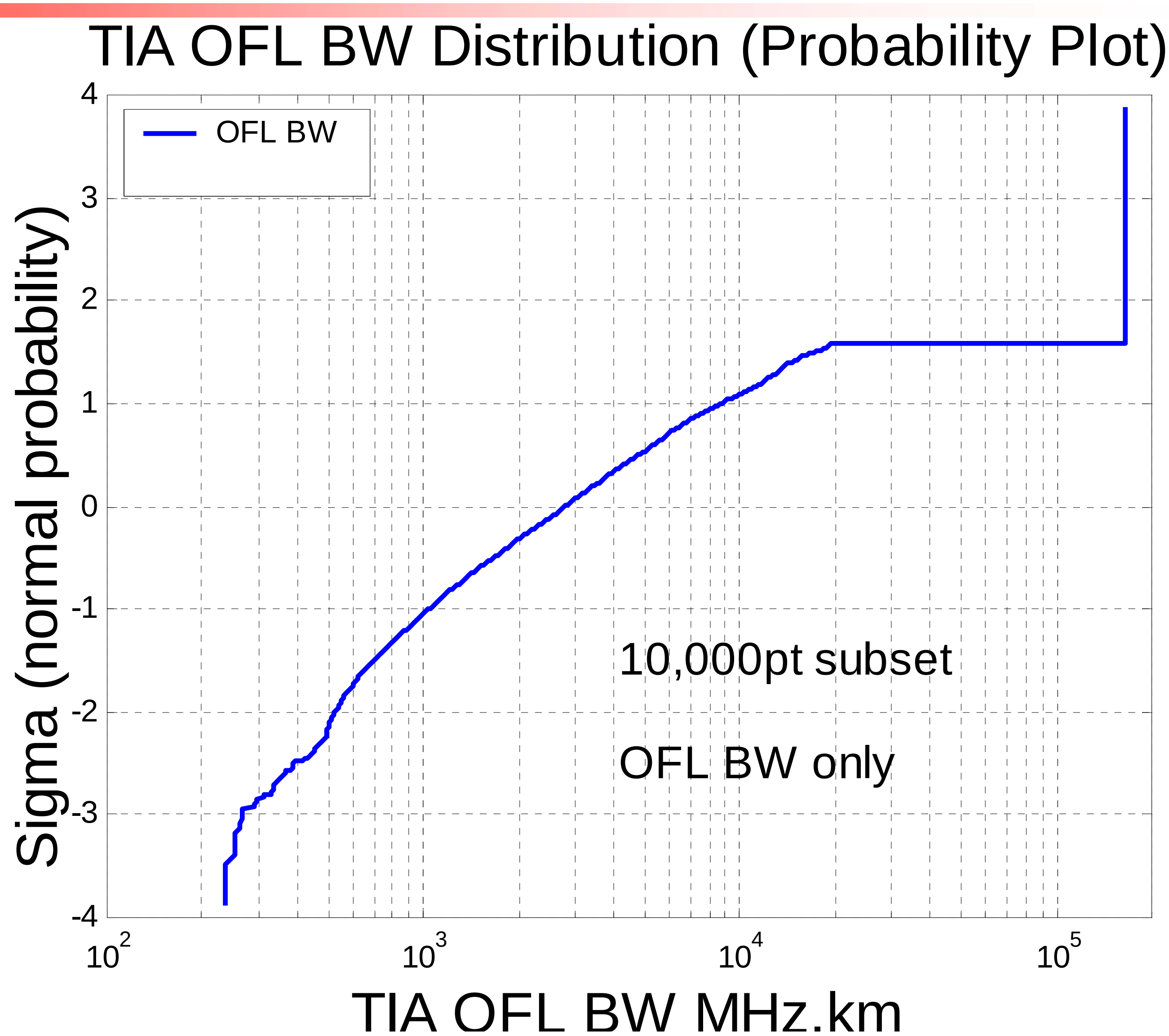
Median is 900MHz.km



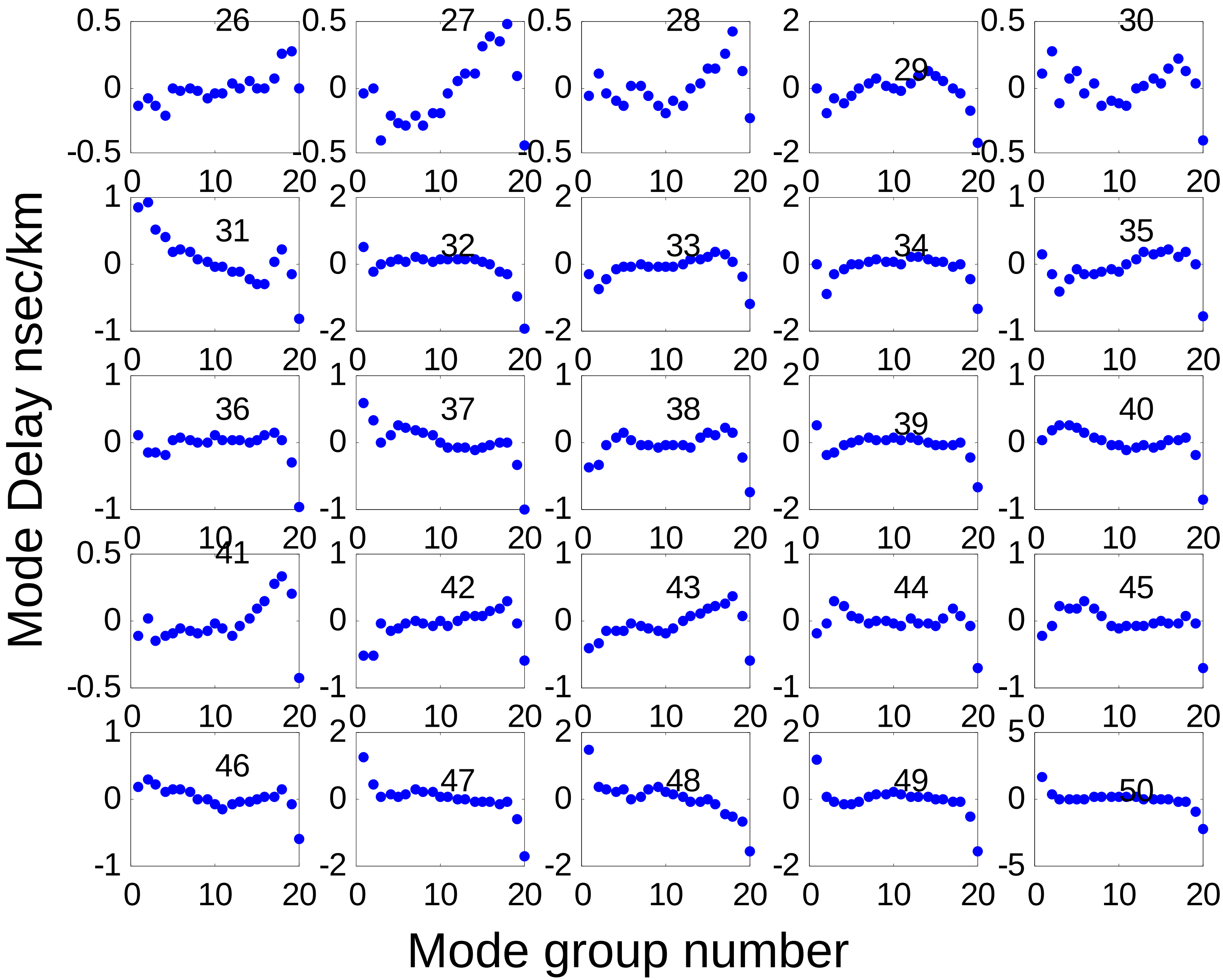
TIA 300m 10GbE: OM3 modeled OFL

TIA modeling for OM3 development generated a model fiber distribution from which estimated failure rates could be calculated as a spec was developed.

median ~3GHz.km



Annex2 – additional Mode Delays


Mode Delay nsec/km
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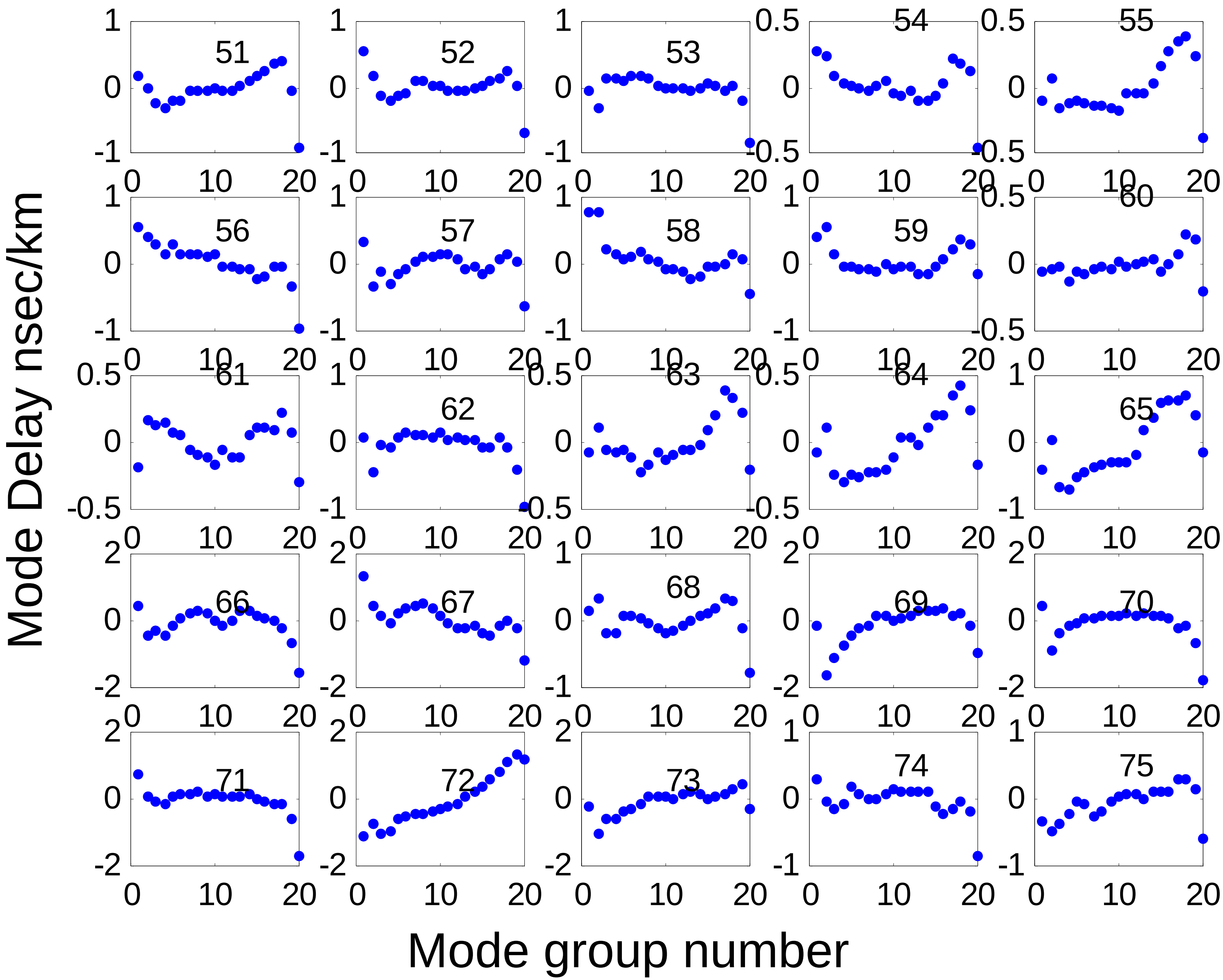
Mode group number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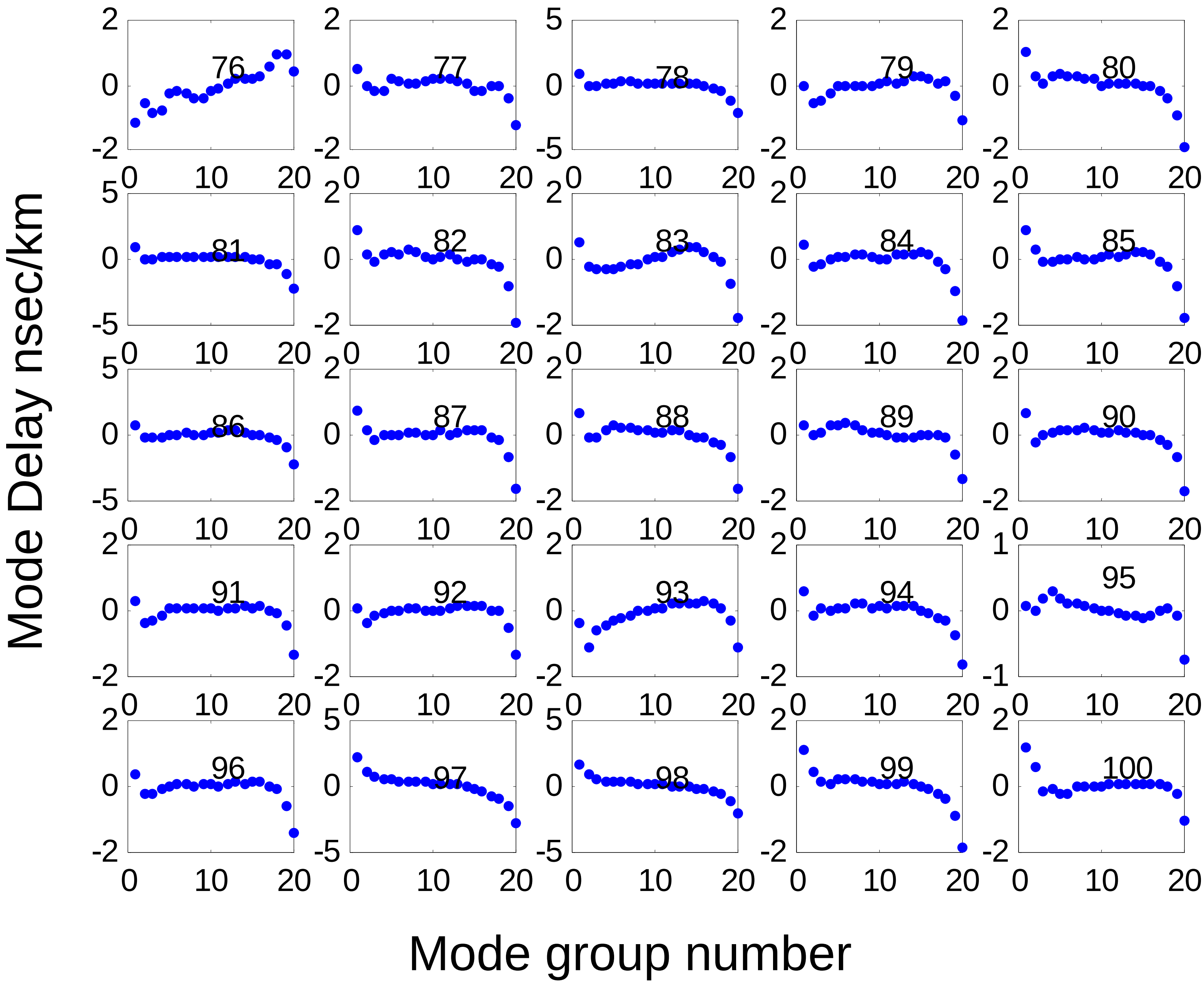
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