PRQS Test Patterns for PAM4

IEEE 802.3bs 400GbE Task Force
Coconut Point, Florida
Sept. 14-16, 2015

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Outline

- Objectives
- PRQS Generation Techniques
- Analysis and Discussion
- Conclusion
Objectives

- Propose the pseudo random quaternary sequence (PRQS) as a test pattern for PAM4
- Describe efficient methods for generating PRQS
- Describe the statistical properties of PRQS
Why PRQS for PAM4?

- PRBS patterns mapped to PAM4 can result in poor baseline wander and clock content characteristics (see analysis of PRBS15 in anslow_3bs_03_0714)
- SSPR pattern has good baseline wander characteristics but may not possess ideal “random” characteristics
- PRQS patterns are a natural generalization of PRBS to quaternary sequences, having similar random properties
- PRQS patterns of modest length can provide good baseline wander and clock content characteristics for a test pattern (anslow_01_0915_smf)
- PRQS patterns are generated algorithmically using linear feedback shift registers, providing efficient implementation and a flexible design
The Algebra of PRBS Generation

![Diagram of PRBS generator with polynomial \( P(x) = x^3 + x + 1 \)]

States cycle through all nonzero elements of \( GF(2^3) \) constructed from binary primitive polynomial \( P(x) = x^3 + x + 1 \):

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & 0 & 1 & 2 \\
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 4 \\
0 & 0 & 1 & 5 \\
1 & 1 & 0 & 6 \\
0 & 1 & 1 & 7 \\
1 & 1 & 1 & 1 \\
\end{array}
\]

PRBS generator is based on a primitive polynomial with coefficients in \( GF(2) \),

\( \Rightarrow \) Similarly PRQS requires a primitive polynomial but with coefficients in \( GF(4) \).
**GF(4) Arithmetic**

Let \( GF(4) = \{0, 1, \beta, \delta\} \), where 0, 1 are additive and multiplicative identities.

The field axioms allow only these operation tables:

\[
\begin{array}{c|cccc}
+ & 0 & 1 & \beta & \delta \\
\hline
0 & 0 & 1 & \beta & \delta \\
1 & 1 & 0 & \delta & \beta \\
\beta & \beta & \delta & 0 & 1 \\
\delta & \delta & \beta & 1 & 0 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\times & 0 & 1 & \beta & \delta \\
\hline
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & \beta & \delta \\
\beta & 0 & \beta & \delta & 1 \\
\delta & 0 & \delta & 1 & \beta \\
\end{array}
\]

Multiplication table for \( GF(4) \) in binary (lsb first) and 4-ary:

<table>
<thead>
<tr>
<th>×</th>
<th>00</th>
<th>10</th>
<th>01</th>
<th>11</th>
</tr>
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<table>
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<th>2</th>
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<td>1</td>
</tr>
</tbody>
</table>

\[\equiv\]
# Primitive Polynomials over GF(4)

## Table 5

<table>
<thead>
<tr>
<th>n</th>
<th>f(x)</th>
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<th>f(x)</th>
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<tbody>
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<td></td>
<td>11</td>
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</table>

Note in the notation above GF(4) = {0, 1, A, B}

Example: PRQS2 Generator (length $4^2-1$)

$P(x) = x^2 + x + 2$

\[
P(x) = x^2 + x + 2
\]

Output Sequence $b_n$

<table>
<thead>
<tr>
<th>$s_2$</th>
<th>$s_1$</th>
<th>$b_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

GF(4) Arithmetic Operations of Addition and Multiplication

\[
\begin{array}{c|c|c|c|c}
+ & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 1 & 2 & 3 \\
1 & 1 & 0 & 3 & 2 \\
2 & 2 & 3 & 0 & 1 \\
3 & 3 & 2 & 1 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
x & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 2 & 3 \\
2 & 0 & 2 & 3 & 1 \\
3 & 0 & 3 & 1 & 2 \\
\end{array}
\]
Connection Between PRQS and PRBS

Factor PRBS4 generator polynomial $D^4+D^3+1 = (2D^2+D+1)(3D^2+D+1)$

=> $b(D) = (s_1+s_2D)(3D^2+D+1) / (D^4+D^3+1)$

=> $b(D) = [3s_2D^3+(3s_1+s_2)D^2+(s_2+s_1)D+s_1] / (D^4+D^3+1)$

Let initial seed $s_2 = 1$, $s_1 = 0$, then $b(D) = (3D^3+D^2+D) / (D^4+D^3+1)$

Go to binary notation for GF(4): $0 = [0 0]$, $1 = [0 1]$, $2 = [1 0]$, $3 = [1 1]$

=> $b(D) = ( [1 1]D^3+[0 1]D^2+[0 1]D ) / (D^4+D^3+1)$

Note: $D$ is the unit delay operator, $b(D)$ output sequence polynomial, initial seed $s_1$, $s_2$
Connection Between PRQS and PRBS

\[ b(D) = \frac{[1 1]D^3 + [0 1]D^2 + [0 1]D}{D^4 + D^3 + 1} \]

Separate \( b(D) \) into LSB and MSB

**LSB:** \( \frac{D^3 + D^2 + D}{D^4 + D^3 + 1} \)

**MSB:** \( \frac{D^3}{D^4 + D^3 + 1} \)
PRQS2 Generator by PRBS4 Multiplexing

PRBS4

1 1 1 0

PRBS4 (different seed)

1 0 0 0

Dibit-to symbol mapper

LSB

PRQS2

0 1 3 1 0 2 2 1 2 0 3 3 3 3
To generate PRQSm of length $4^m-1$, need PRBSn with $n=2m$
Shift MSB PRBS by $m$ or $(4^m-1)/3$ or $(4^m-1)2/3$
Dibit-to-symbol mapper can use Natural or Gray Coding

Proposed PAM4 Test Pattern: PRQS10

- Algorithmically generated based on multiplexing two PRBS20 patterns
- True maximum length quaternary sequence, i.e. contains all 10 length symbol patterns with equal probability (except for all 10 zeros)
- Pattern length = $4^{10} - 1 = 1,048,575 \sim 1$ M (short enough for DCAs to support)
- Good “random” statistical properties (see also next slide):

<table>
<thead>
<tr>
<th></th>
<th>P0</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>Transition Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRQS10</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.7500</td>
</tr>
<tr>
<td>SSPR</td>
<td>0.2573</td>
<td>0.2279</td>
<td>0.2575</td>
<td>0.2573</td>
<td>0.7101</td>
</tr>
<tr>
<td>PRBS13</td>
<td>0.2499</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.7501</td>
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</tbody>
</table>
Simulated Autocorrelation

PAM4 from PRQS10
Shift = \(\frac{(4^{10}-1)}{3}\)

PAM4 from PRQS10
Shift = \(\frac{(4^{10}-1)}{3}\)
Gray Coding

PAM4 from SSPR

PAM4 from PRBS13
Baseline Wander Characteristics

Source: anslow_01_0915_smf

Source: anslow_3bs_03_0714
Conclusion

- We proposed a PRQS10 test pattern for PAM4 as a natural generalization of PRBS to quaternary sequences.
- PRQS patterns can be generated algorithmically using either GF(4) arithmetic based LFSRs or by multiplexing 2 appropriate PRBS patterns.
- The proposed PRQS10 pattern has desirable statistical properties for emulating random PAM4 data, provides good baseline wander characteristics, and has modest length ~ 1M symbols.