PRQS Test Patterns for PAM4

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Outline

- Objectives
- PRQS Generation Techniques
- Analysis and Discussion

Objectives

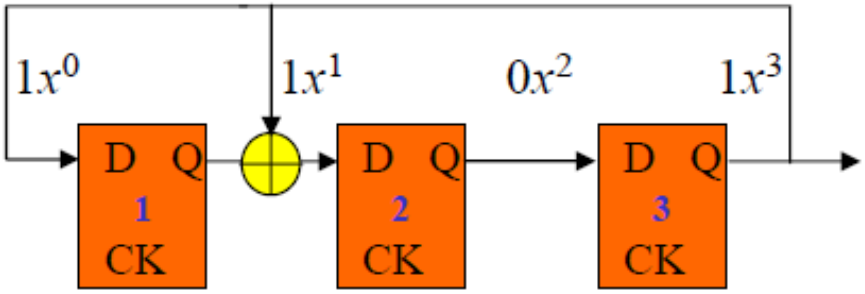
- Propose the pseudo random quaternary sequence (PRQS) as a test pattern for PAM4
- Describe efficient methods for generating PRQS

Why PRQS for PAM4?

- PRQS patterns provide many of the same advantages for PAM4 systems as PRBS patterns for NRZ systems
- PRQS patterns closely match the properties of random PAM4 data
- PRQS patterns are generated algorithmically using linear feedback shift registers

The Algebra of PRBS Generation

$$P(x) = x^3 + x + 1$$



States cycle through all nonzero elements of GF(2^3) constructed from binary primitive polynomial P(x) = x^3+x+1

1	1	1	1
1	0	1	2
1	0	0	3
0	1	0	4
0	0	1	5
1	1	0	6
0	1	1	7
1	1	1	

PRBS generator is based on a primitive polynomial with coefficients in GF(2)
=> Similarly PRQS requires a primitive polynomial but with coefficients in GF(4)

GF(4) Arithmetic

Let $\mathrm{GF}(4) = \{0, 1, \beta, \delta\}$, where $0, 1$ are additive and multiplicative identities.

The field axioms allow only these operation tables:

$+$	0	1	β	δ
0	0	1	β	δ
1	1	0	δ	β
β	β	δ	0	1
δ	δ	β	1	0

$\times$	0	1	β	δ
0	0	0	0	0
1	0	1	β	δ
β	0	β	δ	1
δ	0	δ	1	β

Multiplication table for $\mathrm{GF}(4)$ in binary (lsb first) and 4-ary:

$\times$	00	10	01	11
00	00	00	00	00
10	00	10	01	11
01	00	01	11	10
11	00	11	10	01

$\equiv$

$\times$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

Primitive Polynomials over GF(4)

TABLE 5

SOME PRIMITIVE POLYNOMIALS

GF(4)		GF(8)		GF(9)		GF(16)	
n	f(x)	n	f(x)	n	f(x)	n	f(x)
1	1A	1	1A	1	1A	1	1A
2	11A	2	1AA	2	11A	2	11G
3	111A	3	101A	3	101A	3	101G
4	101AB	4	1001C	4	1001E	4	101AB
5	10001A	5	10011C	5	10010A	5	1000AB
6	100011A	6	100001A	6	10001AA		
7	100001AB	7	100001AC	7	1000001A		
8	10000101A						
9	100000011A						
10	10000001AAA						
11	100000000001A						

$X^2 + X + 2$ → row 2

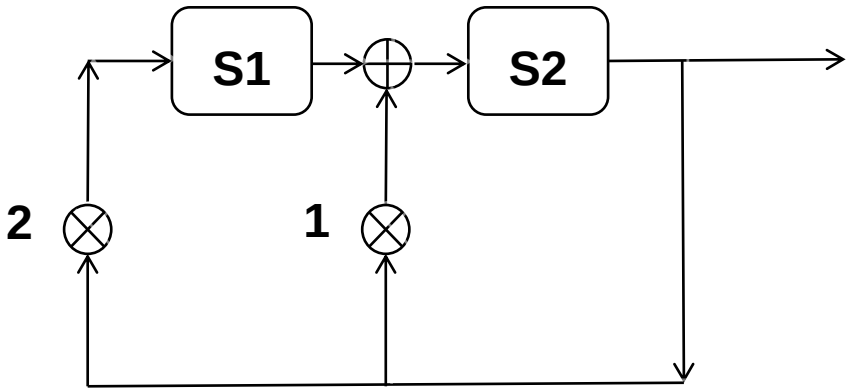
$X^7 + X^2 + 2X + 3$ → row 7

$X^{10} + X^3 + 2X^2 + 2X + 2$ → row 10

Note in the notation above GF(4)= {0, 1, A, B}

Source: D. H. Green, et. al. "Irreducible Polynomials over Composite Galois Fields and their Applications in Coding Techniques," Proc. IEE, vol. 121, no. 9, pp. 1935-1939,1974

Example: PRQS2 Generator (length 4^2-1)



$P(x)= x^2+x+2$

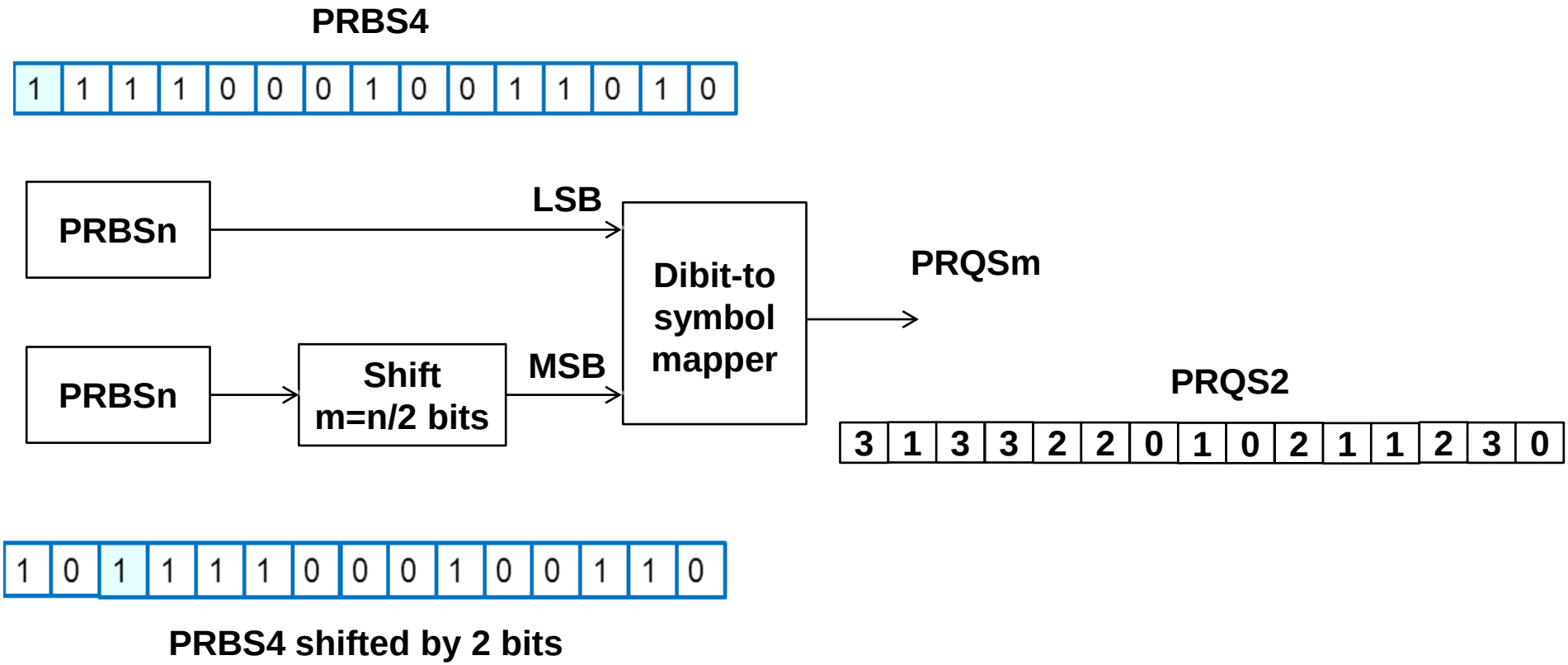
S1	S2	Output
1	0	0
0	1	1
2	1	1
2	3	3
1	1	1
2	0	0
0	2	2
3	2	2
3	1	1
2	2	2
3	0	0
0	3	3
1	3	3
1	2	2
3	3	3
1	0	

GF(4) ARITHMETIC OPERATIONS OF ADDITION AND MULTIPLICATION

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

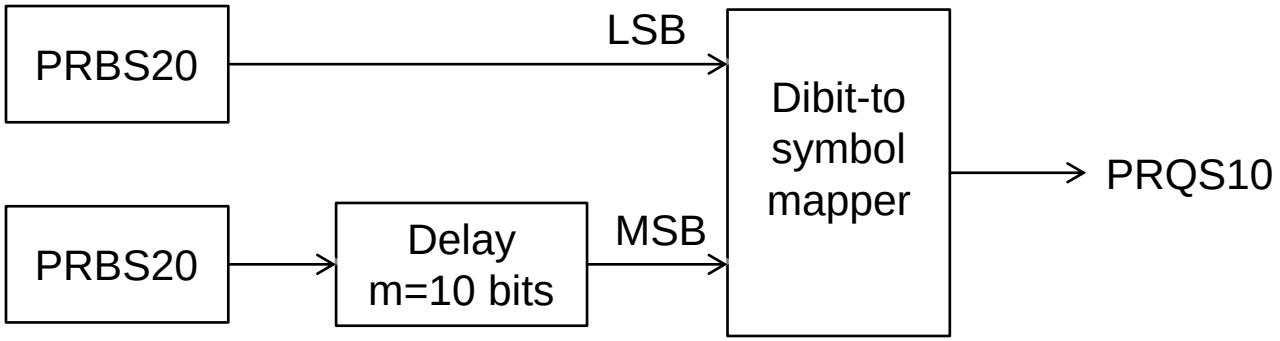
x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

PRQS Generator by PRBS Multiplexing



Source: D. van den Borne, et. al., "Pseudo Random Sequences for Modeling of Quaternary Modulation Formats," proceedings OECC/IOOC, Pacifico Yokohama, July, 2007

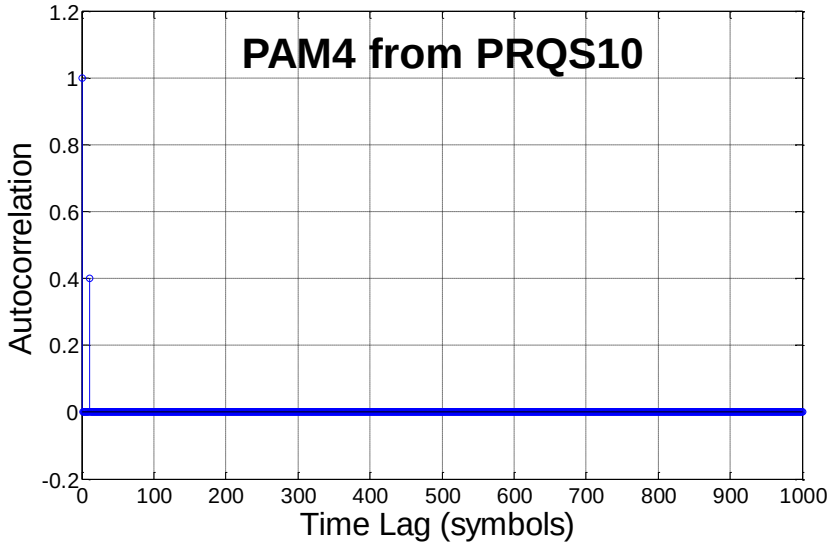
Proposed PAM4 Test Pattern: PRQS10

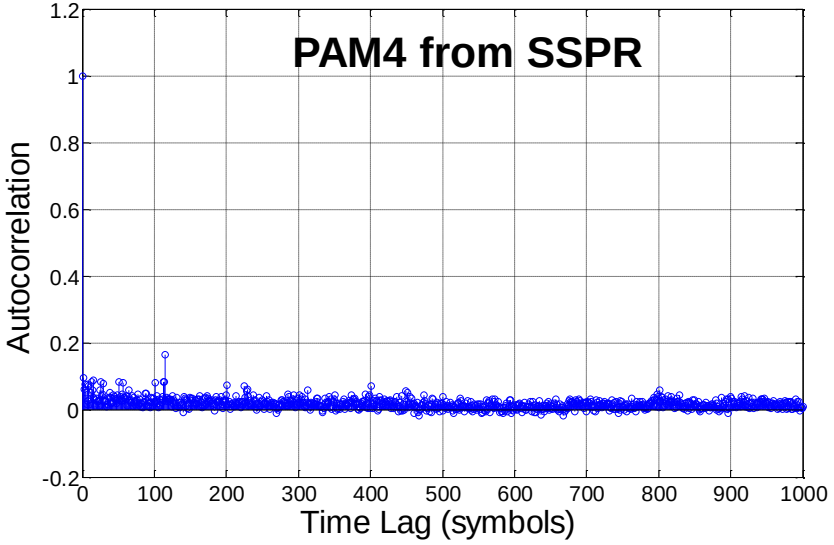


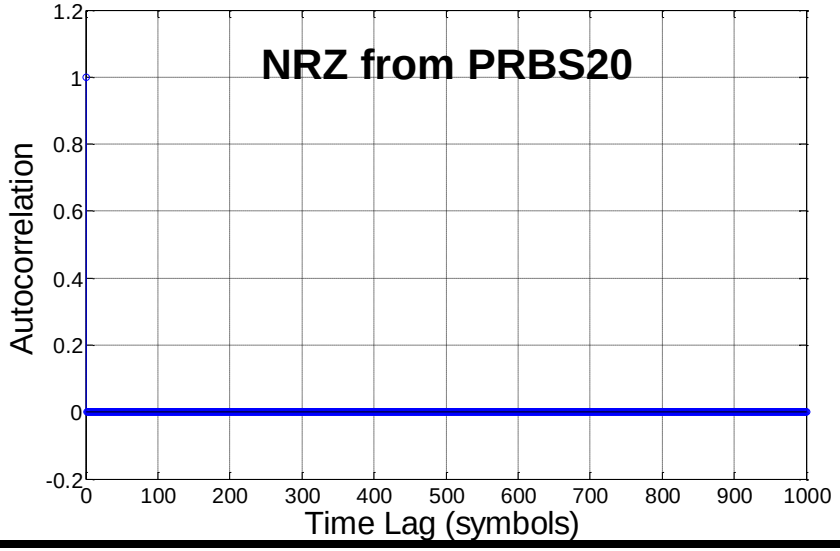
- Algorithmically generated based on multiplexing two PRBS20 patterns
- True PRQS10 pattern, i.e. contains all 10 length symbol patterns with equal probability (except for all 10 zeros)
- Pattern length = 4^{10}-1 = 1,048,575 ~ 1 M (short enough for DCAs to support)
- Better “random” statistical properties compared with PAM4 from SSPR:

	P0	P1	P2	P3	Transition Density
PRQS10	0.25	0.25	0.25	0.25	0.75
SSPR	0.2573	0.2279	0.2575	0.2573	0.71

Simulated Autocorrelation







Potential PRQS Test Patterns for PAM4

PRQS-m Pattern	Length (4^m-1)	Required PRBS-2m Generators	Application
PRQS7	16,383	PRBS14	Embed in PAM4 chips/modules
PRQS10	1,048,575	PRBS20	R&D to support DCAs and real-time scopes
PRQS14	268,435,455	PRBS28	Longer pattern for production testing

PRQS Test Patterns for PAM4

Thank you