

**The IEEE 802.3z Worst Case Link Model for Optical Physical Media Dependent Specification De**

**Summary**

***This document describes the IEEE 802.3z worst case link model for optical Physical Media Dependent Specification. The model is an extension of a previous worst case link model developed by Del (L) for the development of LED based links. The original model has been modified to include the effective extinction ratio, relative intensity noise (RIN), mode partition noise (MPN) and intersymbol interference.***

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## Introduction

In this document we develop a simple model which predicts the performance of laser based multimode optical fiber data communication links. The model has been developed as a tool to assist the IEEE 802.3z understand potential trade offs between the various link penalties and as a baseline for discussions on link specification. The model is an extension of previously reported models for LED based links [1,2]. Power penalties are calculated to account for the effects of intersymbol interference [3], mode partition noise [4], extinction ratio and relative intensity noise (RIN). In addition, a power penalty allocation is made for modal noise [5] and the power losses due to fiber attenuation, connectors and splices are considered.

In the model we assume that the laser and multimode fiber impulse responses are Gaussian [2]. However, we assume that the optical receiver is non-equalized with a single pole filter having a 3 dB electrical bandwidth of  $BW_r(3dB)$ . In this paper we analyse the case where the receiver has an exponential impulse response. However, we also state the results for a non-equalized receiver having a raised cosine response [2]. The model includes expressions that convert the rms impulse width of the laser, fiber and optical receiver to rise times, fall times and bandwidths. These calculated rise times, fall times and bandwidths are used to determine the fiber and composite channel exit response and the ISI penalty of the optical communications link. We assume that rise times and fall times are equal and shall only refer to rise time throughout the the rest of the paper. For real components the larger of the experimentally measured rise or fall time should be used as the input parameter.

For the maximum rms laser spectral linewidths being considered by IEEE 802.3z, 0.85 nm and 4 nm rms for the short wavelength and long wavelength specifications respectively, the worst case modal bandwidths of the multimode fiber, 160 MHz.km or 500MHz.km depending on wavelength and core diameter, the maximum link length is determined primarily by ISI and to a lesser extent by fiber attenuation. This is because the other power penalties are considered to be independent of link length. We have found that the model predicts the ISI power penalties to within experimental error for power penalties up to 3 dB. Since the maximum allocation for ISI in the IEEE 802.3z link budget would be approximately 3 dB, the model can be used to accurately estimate the worst case power budgets for all IEEE 802.3z links. For power penalties above 3 dB the model estimates the ISI power penalty with an uncertainty of approximately 10% in link length when compared to our experimental results. However, for power penalties above 3 dB the uncertainty of the measured eye center power penalty increase significantly due to increased timing jitter.

## RMS pulse width, rise time and bandwidth

It has been shown [6] that if  $h_1(t)$  and  $h_2(t)$  are positive pulses and if  $h_3(t)=h_1(t)*h_2(t)$  (\* represents the convolution operation) then:

$$(1) \quad \sigma_3 = \sqrt{\sigma_1^2 + \sigma_2^2}$$

where  $\sigma_i$  is the rms pulse width of the individual components. The 10% to 90% rise time,  $T_i$ , and bandwidth of individual components,  $BW_i$ , are related by constant conversion factors,  $a_i$  and  $b_i$ , so that:

$$(2) \quad \sigma_i(BW) = \frac{a_i}{BW_i}$$

and

$$(3) \quad \sigma_i(T) = \frac{T_i}{b_i}$$

therefore

$$(4) \quad T_i = \frac{a_i \cdot b_i}{BW_i(6dB)}$$

Equation 1 can be generalised for an arbitrary number of components:

$$(5) \quad \sigma_s^2 = \sum_i \sigma_i^2$$

The rms pulse widths of the individual components may therefore be used to calculate the bandwidth or the 10% to 90% rise time of the composite system if the appropriate conversion factors for each individual component are known [2]. For example the overall system rise time,  $T_s$ , may be calculated using:

$$(6) \quad T_s^2 = \sum_i \left( \frac{b_s \cdot T_i}{b_i} \right)^2 = \sum_i \left( \frac{a_i \cdot b_s}{BW_i(6dB)} \right)^2 = \sum_i \left( \frac{C_i}{BW_i(6dB)} \right)^2$$

The Central Limit Theorem has been used to show that the composite impulse response of multimode fiber optic links tend to a Gaussian impulse [2].

**Conversion Factors for the Laser and Fiber Components: Gaussian Impulse Response**

For systems or components having a Gaussian impulse response the conversion factors  $a$  and  $b$  are equal to 0.187 and 2.563 respectively so that  $C = 0.48$  [2]. Hence the relationships between the rms impulse width ( $\sigma$ ), rise time ( $T$ ) and bandwidth are:

$$(7) \quad T = 2.563 \cdot \sigma$$

and

$$(8) \quad BW(6dB) = \frac{0.187}{\sigma}$$

where  $BW(6dB)$  is the 6dB electrical bandwidth (3dB optical bandwidth) and:

$$(9) \quad T = \left( \frac{0.187 \cdot 2.563}{BW(6dB)} \right) = \frac{0.48}{BW(6dB)}$$

**Conversion Factor for the Non-equalized Optical Receiver: Exponential Impulse Response**

The simplest form of optical receiver is a non-equalised receiver with a single pole filter. This type of receiver can be modeled by an exponential impulse response of the form [2]:

$$(10) \quad hr(t) = \frac{1}{\tau} \cdot \exp\left(-\frac{t}{\tau}\right) \text{ for } t \geq 0, \quad hr(t) = 0 \text{ otherwise.}$$

where  $\tau$  is called the rise time constant. This impulse response has an rms width of  $\tau$ . If the receiver is excited by a step function then the 10% to 90% rise time of the source is [2]:

$$(11) \quad tr = \ln(9) \cdot \tau$$

and the 3 dB bandwidth is [2]:

$$(12) \quad BW_r(3dB) = \frac{0.1588}{\tau} = \frac{0.1588}{\sigma}$$

Since  $\sigma = \frac{0.1588}{BW_r(3dB)}$  by substitution we have:

$$(13) \quad tr = \ln(9) \cdot \left( \frac{0.1588}{BW_r(3dB)} \right) = \frac{0.35}{BW_r(3dB)}$$

Therefore,  $a = 0.1588$  and  $b = \ln(9)$ , for a component or system with an exponential impulse response.

### Fiber Exit and Channel Response Time

With the assumption that the fiber exit impulse response is Gaussian, equation (6) can be used to calculate the fiber 10% to 90% exit response time ( $T_e$ ):

$$(14) \quad T_e = \sqrt{\left(\frac{C1}{BW_m}\right)^2 + \left(\frac{C1}{BW_{ch}}\right)^2 + T_s^2}$$

where  $BW_m$  is the 3 dB optical modal bandwidth of the fiber link,  $BW_{ch}$  is the 3 dB chromatic bandwidth of the fiber link and  $T_s$  is the 10% to 90% laser rise time. Since we are assuming that the fiber has a Gaussian response,  $C1=0.48$ .

The approximate 10% to 90% composite channel exit response time ( $T_c$ ) is then:

$$(15) \quad T_c = \sqrt{\left(\frac{C1}{BW_m}\right)^2 + \left(\frac{C1}{BW_{ch}}\right)^2 + T_s^2 + \left(\frac{0.4}{BW_r}\right)^2}$$

If a raised cosine receiver is used the last term in equation 15 would be  $(0.35/BW_r)$ .

### ISI Penalty

In appendix A and B it is shown that the ISI penalty,  $P_{isi}$ , for a channel having a Gaussian impulse response is approximated by:

$$(16) \quad P_{isi} = \frac{1}{1 - 1.425 \cdot \exp\left[-1.28 \cdot \left(\frac{T}{T_c}\right)^2\right]}$$

where  $T$  is the bit period. This equation is useful for spread sheet implementations of the model. It is accurate to within 0.3 dB of the exact solution for ISI penalties up to 5 dB and to within 1dB for ISI penalties less than 20 dB (see graph in appendix B).

### Mode Partition Noise (MPN)

The various wavelength components of a laser output will travel at slightly different velocities through a fiber. If the power in each laser mode remained constant, then  $BW_{ch}$ , due to the laser time averaged spectrum, would accurately account for chromatic dispersion induced ISI. However, in a multimode laser, although the total output power is constant, the power in each laser mode is not constant. As a result, power fluctuations between laser modes leads to an additional ISI component. This is usually referred to as mode partition noise [4]. The MPN-induced power penalty has been shown to be [4]:

$$(17) \quad P_{mpn} = \frac{1}{\sqrt{1 - (Q \cdot \sigma_{mpn})^2}}$$

where the value of  $Q$  is determined by the maximum acceptable bit error rate (BER) using [4]:

$$(18) \quad BER = \frac{1}{Q \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{Q^2}{2}\right)$$

and

$$(19) \quad \sigma_{mpn} = \frac{k}{\sqrt{2}} \cdot \left[1 - \exp\left[-(\pi \cdot B \cdot D \cdot L \cdot \sigma_\lambda)^2\right]\right]$$

where  $k$  is the laser mode partition factor ( $0 \leq k \leq 1$ ),  $B = \frac{1}{T}$  in  $ps^{-1}$ ,  $D$  the dispersion in  $\left(\frac{ps}{km \cdot nm}\right)$ ,  $L$  is the link length in km and  $\sigma_\lambda$  is the rms width of the total laser spectrum in nanometers.

### Multimode Fiber Chromatic Bandwidth Model

The chromatic dispersion of the multimode fiber, in MHz, is [1,2]:

$$(20) \quad BW_{ch} = \frac{0.187}{L \cdot \sigma_\lambda} \cdot \frac{1}{\sqrt{D1^2 + D2^2}}$$

where:

$$(21) \quad D1 = \frac{S0}{4} \cdot \left( \lambda_c - \frac{\lambda_0^4}{\lambda_c^3} \right)$$

and

$$(22) \quad D2 = 0.7 \cdot S0 \cdot \sigma_\lambda$$

and  $\lambda_0$  is the zero dispersion wavelength, in nm, of the fiber,  $\lambda_c$  is the laser center wavelength, in nm,  $S0$  is the dispersion slope parameter at  $\lambda_0$  in  $\left( \frac{\text{ps}}{\text{km} \cdot \text{nm}^2} \right)$  and the other terms are as previously defined.

### Extinction Ratio Penalty

The power penalty associated with transmitting a non zero power level for a zero is [6]:

$$(23) \quad P_e = \frac{1 + \epsilon}{1 - \epsilon}$$

where  $\epsilon$  is the laser extinction ratio of the power on "zero's" and the power on "ones".

### Relative Intensity Noise (RIN)

The worst case noise variance,  $\sigma_{\text{rin}}^2$ , due to laser RIN can be calculated using the following equation:

$$(24) \quad \sigma_{\text{rin}}^2 = 4 \cdot BWr(3\text{dB}) \cdot 10^{\frac{\text{RIN}}{10}}$$

where it has been assumed that the RIN is worst during "ones" so that the peak laser power is used to calculate the noise variance. Assuming equi - probable symbols, zero transmitted power on "zeros" and unity photodiode responsivity the peak detected electrical power will be four times the average detected electrical power due to square law detection; hence the factor of four in equation (24) for the RIN variance. In equation (24),  $BWr(3\text{dB})$  is the 3 dB electrical bandwidth of the optical receiver and RIN is the laser RIN in dB/Hz.

The worst case RIN induced power penalty is then:

$$(25) \quad P_{\text{rin}} = \frac{1}{\sqrt{1 - (Q \cdot \sigma_{\text{rin}})^2}}$$

Q as previously defined.

### Fiber Attenuation

The attenuation of optical fiber decreases as a power of the wavelength. This is modeled by the following equation [1]:

$$(26) \quad \text{Attenuation}(\lambda_c) = \frac{A_{\text{ref}} \cdot L}{\lambda_c^{3.2}}$$

where  $A_{\text{ref}}$  is the fiber attenuation (in dB) at the wavelength  $\lambda_{\text{ref}}$ ,  $\lambda_c$  is the laser center wavelength in nm and  $L$  is length in km.

### **Worst Case Power Budget, Modal Noise Allocation and Link Length**

The worst case power budget,  $P_b$ , is the difference between the minimum allowed laser launch power and the maximum allowed receiver sensitivity at the specified BER. If the summation of the worst case power losses and penalties is less than  $P_b$  then the link will remain within specification. Since some of power penalties and losses vary with link length there will be a maximum link length that can be supported when all penalties and losses are set to their worst case values.

When calculating the worst case link length an allocation for worst case modal noise [5] and worst case connector loss must be made.

### **Conclusion: *Theory and Experiment***

We have documented the IEEE 802.3z worst case link model. The model is a simulation tool developed for IEEE 802.3z that provides a baseline for discussions on optical link specification. It can be used to illuminate the potential impact of the various link impairments and identify possible trade offs between them. Although the individual components of the optical link may not have Gaussian impulse responses we assumed that the composite link impulse response was Gaussian, as expected from the Central Limit Theorem [2].

We have experimentally measured the fiber and component input parameters required for the link model. In addition, we measured the ISI and MPN power penalties as a function of link length and baud rate for the three wavelengths of interest to IEEE 802.3z. Experimental results and theory are plotted in the six figures at the end of the paper. The laser and link parameters were typical rather than worst case and are documented with the figures. Use of typical components does not effect the accuracy of the comparison between theory and experiment as a wide range of conditions: three wavelengths, variable ISI ( various link lengths, baud rates, two sets of fiber having widely different modal and chromatic bandwidths) have been tested. The maximum link lengths due to ISI predicted by the model are within approximately 10% of those observed. However, the experimental error associated with measured power penalties greater than 3 dB is large due the the difficulty in finding eye center because of increased timing jitter. For power penalties smaller than 3 dB theory and experiment agree to within the experimental uncertainty associated with the experimental data.

## **References**

[1] ANSI T1.646-1995, Broadband ISDN-Physical Layer Specification For User-Network Interfaces, Appendix B.

[2] Gair D. Brown, "Bandwidth and Rise Time Calculations for Digital Multimode Fiber-Optic Data Links", Journal of Lighthwave Technology, VOL. 10, No. 5, May 1992, pp 672-678.

[3] James L. Gimlett and Nim K. Cheung, "Dispersion Penalty Analysis for LED/Single-Mode Fiber Transmission Systems", Journal of Lighthwave Technology, VOL., LT-4, No. 9, Sept., 1986, pp 1381-1392.

[4] Govind P. Agrawal, P. J. Anthony and T. M. Shen, "Dispersion Penalty for 1.3-mm Lightwave Systems with Multimode Semiconductor Lasers", Journal of Lightwave Technology, VOL., 6, No. 5, May., 1988, pp 620-625.

[5] Richard J. S. Bates, Daniel M. Kuchta, Kenneth P. Jackson, "Improved Multimode Fiber Link BER Calculations due to Modal Noise and Non Self-Pulsating Laser Diodes", Optical and Quantum Electronics, **27** (1995) pp 203-224.

[6] "Receiver Design for Optical Communication Systems" by R. G. Smith and S. D. Personick in Topics in Applied Physics, Volume 39, Semiconductor Devices for Optical Communications, Editer: H. Kressel, Published by Springer-Verlag, 1982 (ISBN 0-387-11348-7).

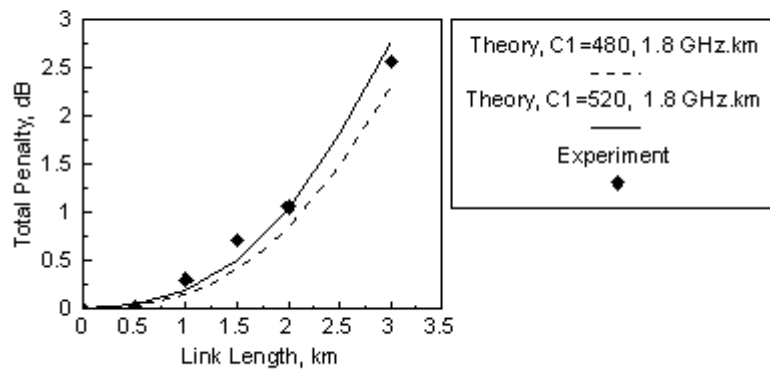
Note: C1 = 480 in all plots because this is the input to the spread sheet model. This is equivalent to setting C1= 0.480 in the units used in *this* paper.

Figure 1:

### Comparison of Theory & Experiment:

*Long Wavelength Link With Typical Components*

Power Penalty Versus Link Length

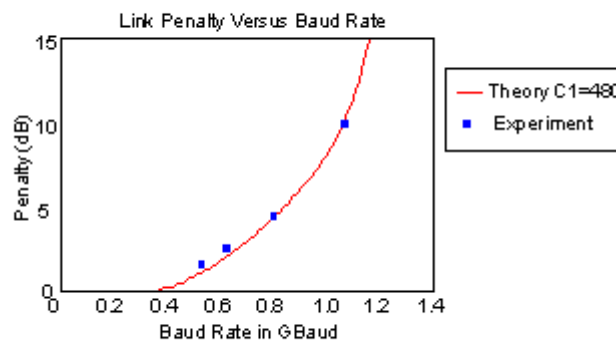


HP coaxial laser, 1318 nm, 1.5 nm rms width, k=0.5, Ts=350 ps, RIN = -137 dB/Hz  
62MMF, 2 GHz.km modal bandwidth, Zero Dispersion=1377nm, So=0.084 ps/(nm<sup>2</sup>.km)

Figure 2:

### Comparison of Theory & Experiment:

*Long Wavelength Link With Typical Components*



HP coaxial laser, 1318nm, 1.5 nm rms width, k=0.5, Ts=350 ps, RIN = -137 dB/Hz, 62MMF, L=2000m,  
700 MHz.km modal bandwidth, zero dispersion=1398nm, dispersion slope=0.073 ps/nm<sup>2</sup>.km

Figure 3:

**Comparison of Theory & Experiment:**  
*Short Wavelength (857 nm) Link With Typical Components*  
 Power Penalty Versus Link Length

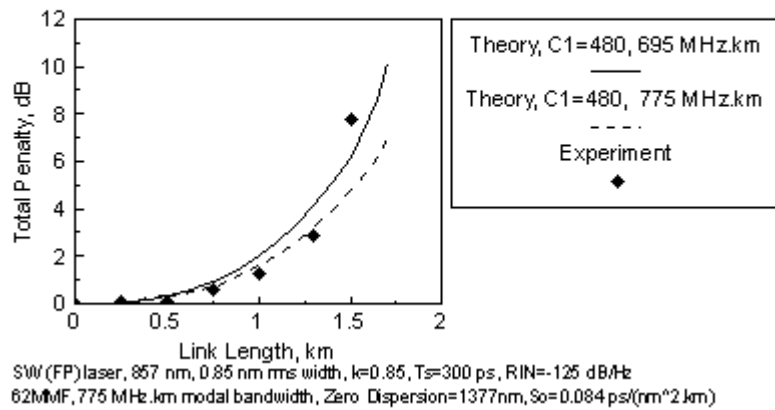


Figure 4:

**Comparison of Theory & Experiment:**  
*Short Wavelength (857 nm) Link With Typical Components*  
 Link Penalty Versus Baud Rate

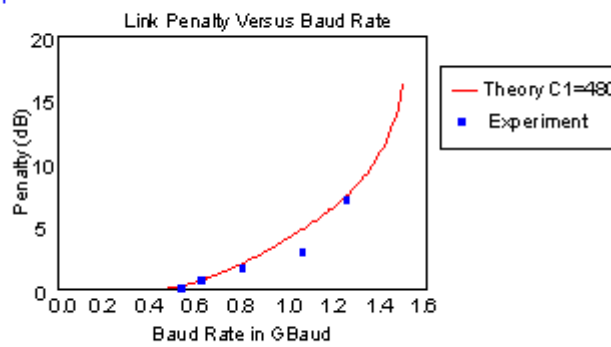
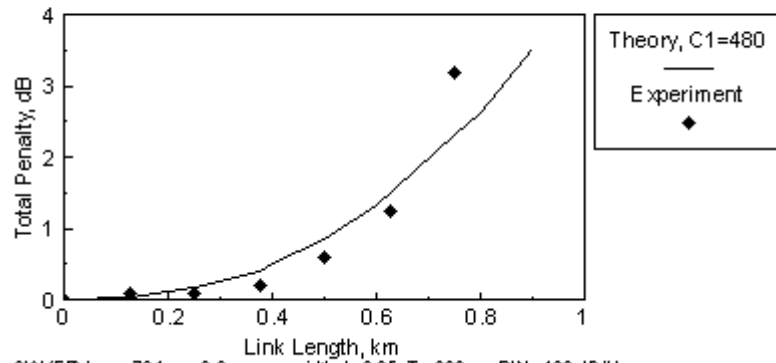


Figure 5:

**Comparison of Theory & Experiment:**

*Short Wavelength (770 nm) Link With Typical Components*

Power Penalty Versus Link Length



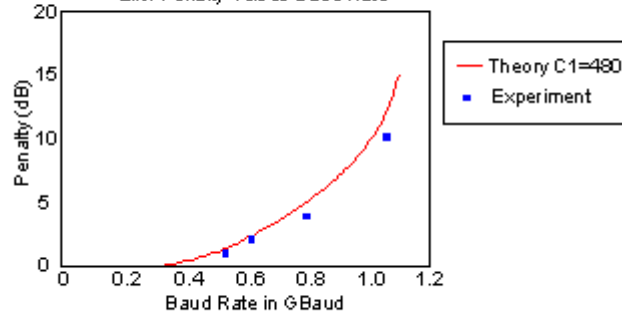
SW (FP) laser, 794 nm, 0.8 nm rms width,  $k=0.85$ ,  $T_s=300$  ps,  $RIN=-120$  dB/Hz  
 62MMF, 525 MHz.km modal bandwidth, Zero Dispersion=1377nm,  $S_o=0.084$  ps/(nm<sup>2</sup>.km)

Figure 6:

**Comparison of Theory & Experiment:**

*Short Wavelength (794 nm) Link With Typical Components*

Link Penalty Versus Baud Rate



SW (FP) laser, 794nm, 0.79 nm rms width,  $k=0.85$ ,  $T_s=300$  ps,  $RIN = -120$  dB/Hz, 62MMF,  $L=500$ m,  
 166 MHz.km modal bandwidth, zero dispersion=1398nm, dispersion slope=0.073 ps/nm<sup>2</sup>.km

### **Appendices: Introduction**

In the following appendices we outline the derivation of the relationships between the rms impulse width, the bandwidth and the rise time for systems or components having Gaussian impulse responses. We also plot the graph of the step response of a system having a Gaussian impulse response to illustrate the fact that it follows an error function dependence. The worst case eye, produced by superimposing the time response of the system to a "one" isolated in a train of "zeros" and an isolated "zero" in a train of "ones" together with the worst case inner eye opening are also plotted.

Finally, the ISI penalty, calculated in the time domain, is compared to a published equation [3] obtained from a frequency domain analysis. Exact agreement between the two results is found as would be expected. These equations are compared to a proposed approximate equation useful for spread sheet analysis. The approximate equation is accurate to within 0.3 dB of the exact solution for ISI penalties up to 5 dB and to within 1dB for ISI penalties less than 20 dB (see graph in appendix B).

**Appendix A (Mathcad format)**  
Rise time, Eye Opening and ISI Power Penalty for Gaussian Systems: Time Domain Method

Assuming that the normalised composite impulse response  $hc(t)$  of the laser, multimode fiber and optical receiver channel is Gaussian we may write:

$$hc(t) = \frac{1}{\sigma t \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{t^2}{2 \cdot \sigma^2}\right)$$

where  $\sigma t$  is the rms width of the impulse response of the channel. The Fourier transform of  $hc(t)$  will then be given by:

$$\int_{-\infty}^{\infty} \frac{1}{\sigma t \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{t^2}{2 \cdot \sigma^2}\right) \cdot \exp(i \cdot 2 \cdot \pi \cdot f \cdot t) dt = \exp\left(2 \cdot i^2 \cdot \pi^2 \cdot f^2 \cdot \sigma^2\right)$$

so that the normalised frequency response is  $Hc(f)$ :

$$Hc(f) = \exp\left(\frac{2^2 \cdot i^2 \cdot \pi^2 \cdot f^2 \cdot \sigma^2}{2}\right)$$

Therefore the 3 dB and 6 dB electrical bandwidths are as stated earlier in paper:

$$BW(3dB) = \frac{1}{2} \cdot \frac{1}{(\pi \cdot \sigma t)} \cdot \sqrt{\ln(2)} = \frac{0.132}{\sigma t}$$

and

$$BW(6dB) = \frac{1}{2} \cdot \frac{1}{(\pi \cdot \sigma t)} \cdot \sqrt{2 \cdot \ln(2)} = \frac{0.187}{\sigma t}$$

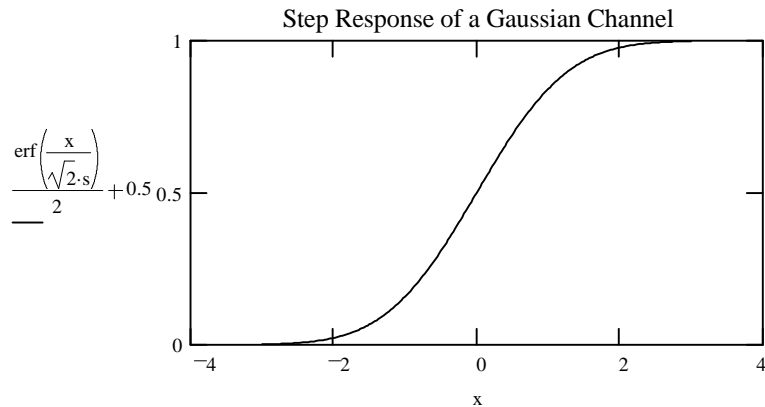
Let  $i := \sqrt{-1}$

Letting  $s = \sigma t$  then the convolution of the unit step and a Gaussian impulse response is:

$$\int_{-x}^{\infty} \frac{1}{\sqrt{2 \cdot \pi \cdot s}} \cdot \exp\left(-\frac{t^2}{2 \cdot s^2}\right) dt = \frac{\operatorname{erf}\left(\frac{x}{\sqrt{2 \cdot s}}\right)}{2} + 0.5$$

We can plot the response of the Gaussian impulse channel to a unit step:

$s := 1$      $x := -3, -2.975 \dots 3$



The rise time has an **erf** dependence having 10% and 90% values at  $x$  equal to  $\pm 1.28155$ . The channel 10% to 90% rise time,  $T_c$ , is therefore  $T_c = 2.563s$ .

### Worst Case Inner Eye for a Gaussian System

The response of the Gaussian channel to an isolated "zero" of duration  $T$  in a string of "ones" is then:

$$1 - \int_{-(x+T)}^{\infty} \frac{1}{\sqrt{2\pi}\cdot s} \cdot \exp\left(-\frac{t^2}{2\cdot s^2}\right) dt + \int_{-x}^{\infty} \frac{1}{\sqrt{2\pi}\cdot s} \cdot \exp\left(-\frac{t^2}{2\cdot s^2}\right) dt$$

which equals:  $1 - \frac{1}{2} \cdot \text{erf}\left[\frac{1}{2} \cdot \sqrt{2} \cdot \frac{(x+T)}{s}\right] + \frac{1}{2} \cdot \text{erf}\left(\frac{1}{2} \cdot \frac{\sqrt{2}}{s} \cdot x\right)$

The eye opening as a function of time is given by the response to an isolated "zero" minus the decision threshold all multiplied by two. Assuming the response is normalised to one the threshold will be at 0.5 and the eye opening is:

$$\text{erf}\left[\frac{1}{2} \cdot \sqrt{2} \cdot \frac{(x+T)}{s}\right] - \text{erf}\left(\frac{1}{2} \cdot \frac{\sqrt{2}}{s} \cdot x\right) - 1$$

The turning point of the function describing the eye opening as a function of  $x$  will be at the value of  $x$  for which the eye opening is a maximum. This turning point is given by:

$$\frac{d}{dx} \left[ \text{erf}\left[\frac{1}{2} \cdot \sqrt{2} \cdot \frac{(x+T)}{s}\right] - \text{erf}\left(\frac{1}{2} \cdot \frac{\sqrt{2}}{s} \cdot x\right) - 1 \right] = 0$$

so that:  $x = \ln \left[ \frac{1}{\sqrt{\exp\left(\frac{1}{s^2} \cdot T^2\right)}} \right] \cdot \frac{s^2}{T}$  at maximum eye opening.

Let the baud period,  $T$ , equal  $m \cdot s$  then substituting for  $x$  into the expression for the eye opening and simplifying gives:

$$\text{eyemax}(m) := 2 \cdot \text{erf}(0.3536 \cdot m) - 1.$$

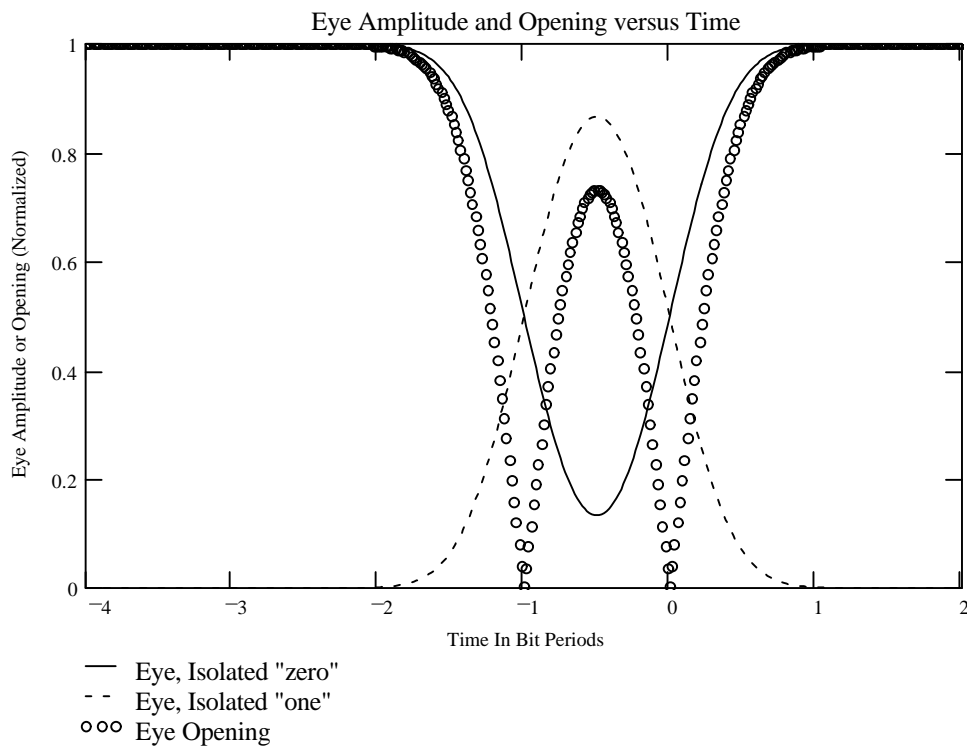
**We now plot the worst case inner eye diagram and the eye opening.**

$s := 1$  this normalises  $s$  to unity.

$m := 3$

$T := m \cdot s$  remember that the bit period equals  $m$  times  $s$ .

range of  $t$  for plot  $x := -12, -11.95 \dots 6$



The expression derived in this section for the eye opening by the time domain method will be compared to Gimlett's frequency domain expression in appendix B. It will be found that they are in exact agreement as expected.

**Appendix B (Mathcad format)**

Alternative Method for Calculating the ISI Penalty From A Nonequalized Optical Receiver[4].

Consider a multimode fiber link consisting of a laser transmitter, a length of multimode optical fiber and an optical receiver. Assume that the laser is modulated with a nonreturn-to-zero (NRZ) input signal. The frequency spectrum of the NRZ input pulse supplied to the laser is then  $H_p(f)$ :

$$H_p(f) = \frac{\sin\left(\frac{\pi \cdot f}{B}\right)}{\pi \cdot f}$$

where B is the baud rate.

Let the transfer function of the multimode fiber be  $H_f(f)$  and the transfer function of the optical receiver be  $H_r(f)$ . Then Gimlett et al[4] have shown that the maximum eye closure due to dispersion induced ISI is  $E_m$ :

$$E_m = 2 \left( 1 - \int_{-\infty}^{\infty} H_p(f) \cdot H_f(f) \cdot H_r(f) \cdot e^{-i \cdot 2 \cdot \pi \cdot f \cdot t_0} df \right)$$

where  $t_0$  is the optimum (maximum eye opening) sampling time. The maximum power penalty is  $P_d$ :

$$P_d = \frac{1}{1 - E_m}$$

Assuming that the normalised impulse response  $h_c(t)$  of the laser, multimode fiber and optical receiver channel is Gaussian we may write:

$$h_c(t) = \frac{1}{\sigma_t \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{t^2}{2 \cdot \sigma_t^2}\right)$$

where  $\sigma_t$  is the rms width of the impulse response of the channel. The normalised Fourier transform of  $h_c(t)$  will be:

$$\int_{-\infty}^{\infty} \frac{1}{\sigma_t \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{t^2}{2 \cdot \sigma_t^2}\right) \cdot \exp(i \cdot 2 \cdot \pi \cdot f \cdot t) dt = \exp\left(-\frac{1}{2} \cdot (2 \cdot \pi \cdot f \cdot \sigma_t)^2\right)$$

so that  $E_m$  may be calculated as:

$$E_m = 2 - 2 \cdot \int_{-\infty}^{\infty} \frac{\sin\left(\frac{\pi \cdot f}{B}\right)}{\pi \cdot f} \cdot \exp\left[-\frac{(2 \cdot \pi \cdot f \cdot \sigma_t)^2}{2}\right] \cdot \exp(i \cdot 2 \cdot \pi \cdot f \cdot t_0) df$$

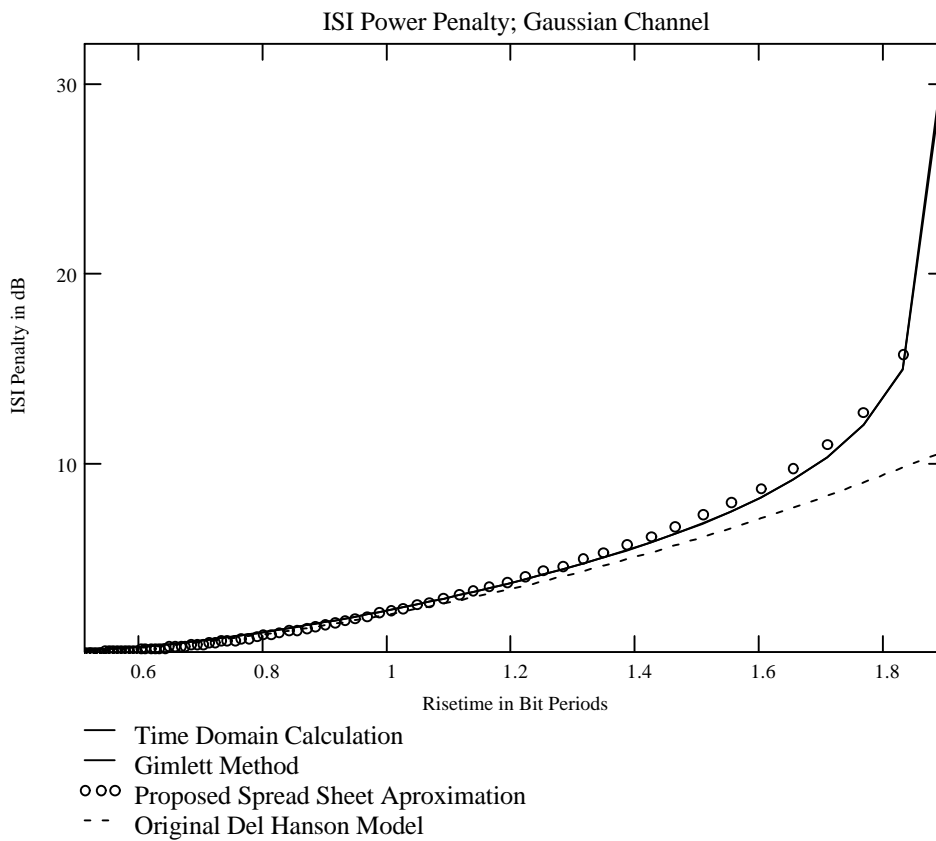
and the ISI power penalty  $P_d$  is:

$$Pd = \frac{1}{1 - 2 \cdot \int_{-\infty}^{\infty} \frac{\sin\left(\pi \frac{f}{B}\right)}{\pi f} \cdot \exp\left[-\frac{(2 \cdot \pi \cdot f \cdot \sigma t)^2}{2}\right] \cdot \exp(i \cdot 2 \cdot \pi \cdot f \cdot t) df}$$

We can now plot the ISI penalties as calculated by Gimlett's spectral method and the time domain method. These can be compared to the approximate expression proposed for the spread sheet model and to the expression originally used by Del Hanson.

t := 0

m := 5, 4.95.. 1.35    s = 1



It is clear that the proposed spread sheet equation:  $Pd = \frac{1}{1 - 1.425 \cdot \exp\left[-1.28 \cdot \left(\frac{T}{T_c}\right)^2\right]}$

is an accurate, see graph above, approximation for the ISI power penalty of a Gaussian channel.