Proposed Steady-State Limits for Turbine-Generator Torsional Response

This white paper proposes a rationale for and a standard for setting limits for sinusoidal torque excitation of turbine-generators, suitable for incorporation in the IEEE standard 519 under the heading of inter-harmonic duty.

The turbine-generator is a complex mechanical structure comprised of multiple stiff rotor-elements connected by slender shaft and journal sections. The response of this type structure to torque applied at the generator is similarly complex, and different shaft sections may reach fatigue stress limits first depending on the torque characteristics.

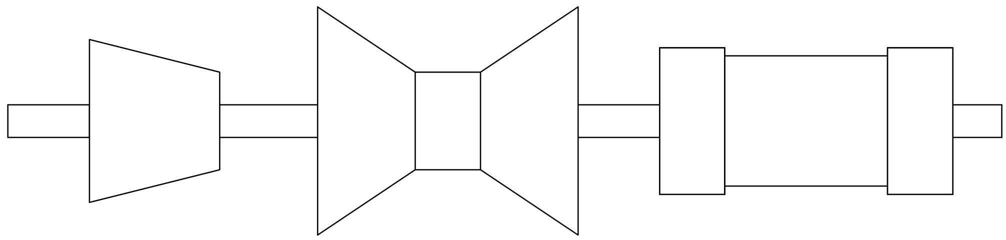

Figure 1: Turbine-Generator Shaft Line

In 1985, an IEEE-PES working group was formed to provided screening criteria for turbine-generator switching duty. From this work a consensus standard[1] was established for acceptable levels of sudden change in generator applied torque. This consensus standard provided a method for routine evaluation of switching duty by power system operating companies so as to avoid causing any fatigue damage on the turbine-generator shaft line.

Because we know the structure of the turbine-generator model, and the basis of the above consensus standard, it is possible to arrive at turbine-generator limitations for other types of duty in the spirit of the original consensus. One form of duty for which a standard is lacking is that associated with subsynchronous sinusoidal applied torque. Such torque's come about due to transmission series capacitors, AC/DC converter control action, and large-scale hot-strip steel mill operations such as cyclo-converter motor operation. We will propose here a method to establish a limit for sinusoidal applied torque that is based on the switching standard.

Before dealing with the issues it is instructive to look at the properties of the mechanical structure like figure 1. What we observe physically is that the rotors can only vibrate at certain fixed frequencies, called natural frequencies and against one another in fixed patterns. We find that any specific response can be looked upon as some linear combination of these patterns.

These patterns of response are called mode shapes and are characterized by a frequency of oscillation and a constant ratio of rotor response (velocity or displacement) relative to another in amplitude and direction. The mode shapes of a turbine-generator like figure 1, might have mode shapes like those illustrated in Figure 2.

The mode shapes are important both for a physical understanding and a mathematical understanding of the process toward mechanical response for arbitrary applied generator torque. We can see for instance that the generator location in the mode shape dictates the degree to which the particular mode can participate in the response. The frequencies of the modes tell us

[1] "Torsional Vibration & Fatigue of Turbine-Generator Shafts," DN Walker, SL Adams, RJ Placek IEEE Technical Publication TH0059-6/79/0000-017500.75 1979.

the closeness to resonance with the applied force frequencies. We will now develop the mathematical understanding of mechanical response for given applied torque description.

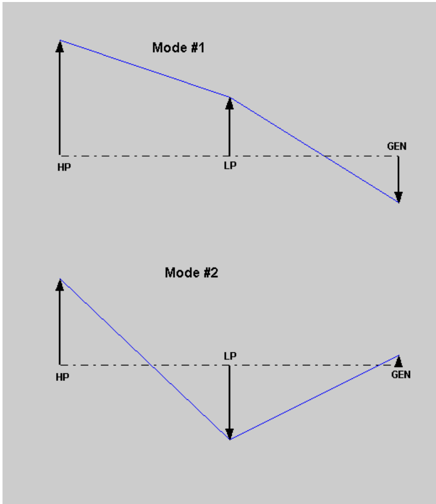

Figure 2: Characteristic Mode Shapes of Turbine-Generator

Model Structure and Equations for Turbine Torsional Dynamic Response

The multi-mass spring-mass model of a turbine-generator can be characterized for dynamic response calculation by integration of the differential equations of motion of the structure in physical coordinates. This is accomplished by writing the free-body force equations applicable to each mass. This kind of model is complex mathematically and difficult to characterize for human mental processing. A coordinate transformation is possible[2] that renders the model as a simple set of uncoupled second order differential equation whose response translation to physical coordinates is a simple linear combination of response of each mode.

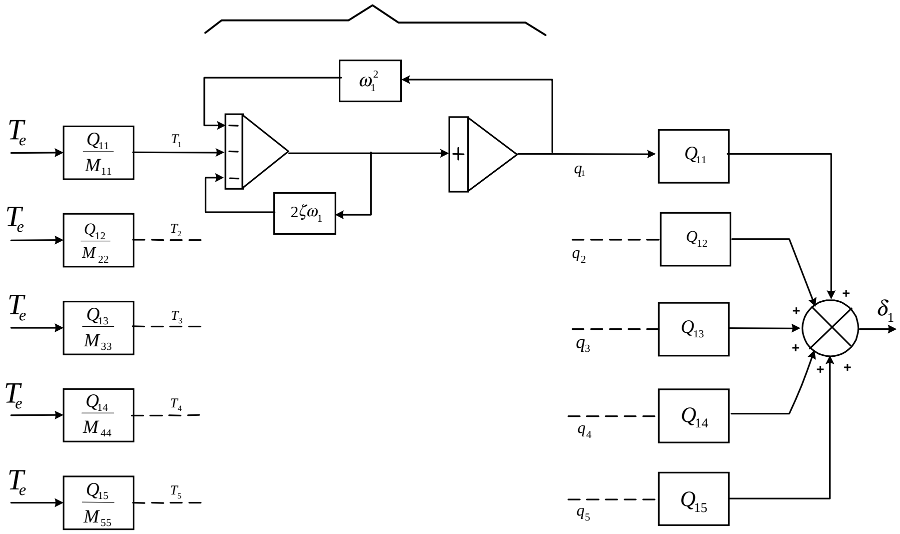

Figure 3: Modal Torsional Modeling for Dynamic Response Simulation

[2] "Understanding Sub-synchronous Resonance," CEJ Bowler, 1975 IEEE-PES SSR Symposium & Tesla Conference, San Francisco.

For simple types of applied torque such as a step, or oscillatory torque, it is relatively easy to use the modal model to construct the solution for mechanical response symbolically. Figure 3, presents the form of the so-called modal model of a turbine generator structure that relates the modal coordinate solution (τ,q) to the physical coordinate solution (Te,δ).

We can think of each mode of oscillation as a simple single spring-mass system or modal oscillator. The equations of motion can be written by applying Newton's equations of motion for the Inertia (M) in rotation (θ), resisted by viscous damping D and spring stiffness K and propelled by torque T, expressed here in the complex frequency domain ($s = j\omega$).

$$(Ms^2 + Ds + K)\theta(s) = T(s)$$
$$(s^2 + 2\zeta\omega_n s + \omega_n^2)\theta(s) = T(s)/M$$
$$((s-\sigma)^2 + \omega_{dn}^2)\theta(s) = T(s)/M$$
$$s\theta(s) = \varpi(s)$$
$$\varpi(s) = \frac{sT(s)}{M((s-\sigma_n)^2 + \omega_{dn}^2)}$$

Where $\omega_{dn} = \omega_n\sqrt{(1-\varsigma^2}$ is the damped natural frequency of oscillation and $\sigma_n \approx \varsigma\omega_{dn}$ is the damping decrement factor, $\omega_n = \sqrt{\frac{K}{M}}$ is the un-damped natural frequency (rad/sec), and ϖ is the symbol for instantaneous response velocity. The modal equations of motion can be solved for a variety of conditions of applied torque. We are interested here in the case of step response, and sinusoidal resonant response.

Step Response

Solving for the velocity time response to a step of applied torque amplitude $\overline{T}$ at time t=1.0, for mode 'n', see Figure 4 top left inset, we have:

$$\varpi(s) = \frac{\overline{T}Q_{gn}}{M_n((s-\sigma_n)^2 + \omega_{dn}^2)}$$

$$\varpi(t) = \frac{\overline{T}Q_{gn}}{\omega_{dn}M_n}\varepsilon^{-\sigma t}\sin(\omega_{dn}t + \phi) = \frac{\overline{T}C_n}{\omega_{dn}}\varepsilon^{-\sigma t}\sin(\omega_{dn}t + \phi) \qquad 1.0$$

Sinusoidal Response

Solving for resonant response for sinusoidal applied torque excitation at a natural frequency, with torque amplitude $\tilde{T}$ at time t=1.0, see Figure 4 top right, we have:

$$\varpi(t) = \tilde{T}\frac{C_n}{2\sigma_n}(1-\varepsilon^{-\sigma_n t})\sin(\omega_{dn}t + \phi)$$

$$\varpi^{\max}{}_{(s=j\omega_{dn})} = \frac{\tilde{T} C_n}{2\sigma_n} \qquad 2.0$$

Where $\varpi^{\max}$ is defined to be the resonant gain factor per unit of applied torque for the mode in question as time increases much beyond about five time-constants.

We can equate the result of equation 1 and 2 to establish the limit in oscillatory torque, given the limit in step applied torque (the consensus standard), as follows:

$$\tilde{T} = \frac{\overline{T}\, 2\sigma}{\omega_{dn}} \qquad 3.0$$

Application Considerations of Consensus Standard for Sinusoidal Perturbation

An example of step and resonant response calculation is shown in Figure 4, illustrating the behavior of a real example turbine-generator excited at mode-1 frequency, at approximately 18 Hz for torsional damping representative of no-load and full-load generator operation.

We can see the relationship implied by equation 3, in the peak responses calculated in Figure 5. What is clear is that the influence of mechanical damping is overwhelming in the response levels that are achieved during resonant conditions. This is expected, but now we have a way of calibrating the severity of the applied torque in this condition, which was previously not considered.

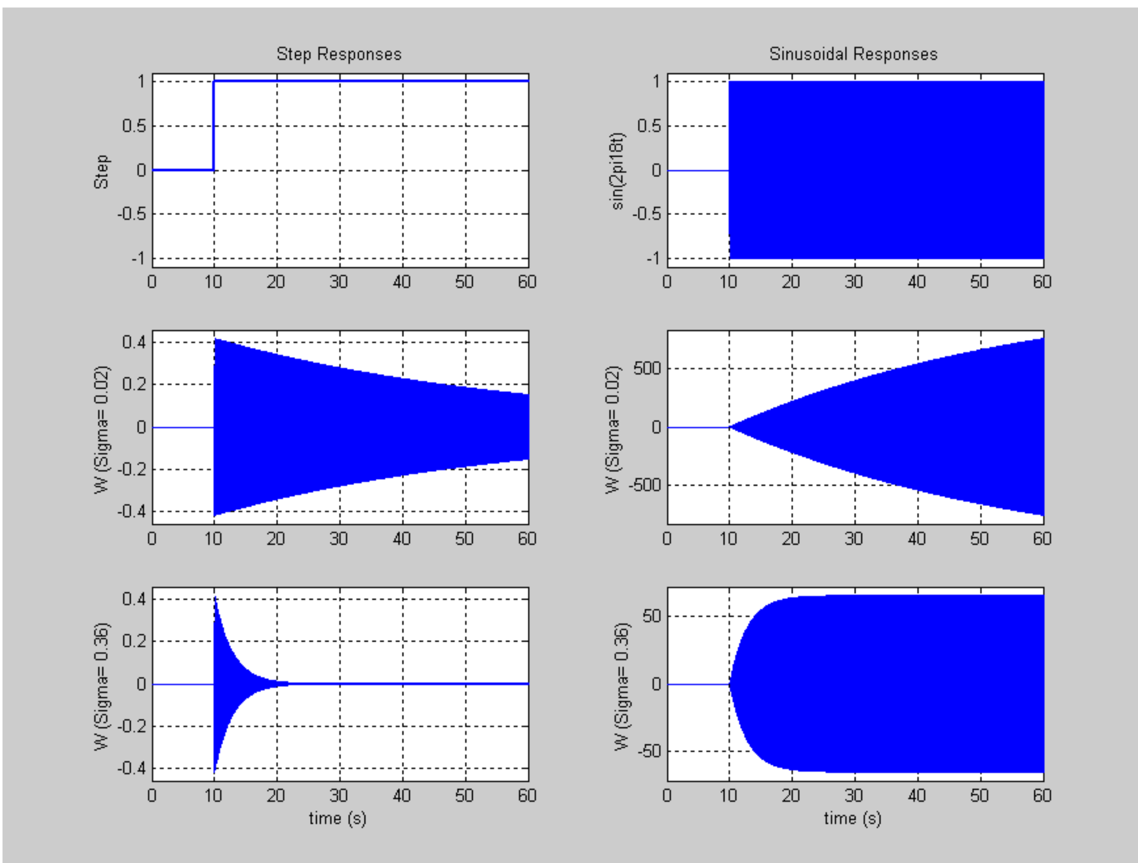

Figure 4: Typical Unit Torsional Response Illustrative Examples One Per Unit Step Torque & One Per Unit Resonant Sinusoid, Mode 1 Frequency at No-Load and Full-Load Damping

What is implied here is that response levels that are permissible in one condition are also permissible in a completely different condition? While this idea is reasonably obvious it becomes useful when we can relate response to the characteristics of the applied torque between the two cases. This is what equation 3 provides between the two cases of step response (transmission line switching) and subsynchronous response (from cyclo-convertors etc.).

Knowing the permissible torque level may not be that useful for engineers who are more concerned with limits on network voltage and current. We can transform the limit on torque to either current or voltage level, noting that the torque in question is not synchronous and is really an interaction between the armature flux and the synchronous rotor flux of the generator. Since the rotor flux is constant at 1 per unit, the perturbation current and resulting torque are identical on a per-unit basis. Similarly the current is limited by the generator reactance which in the frequency range of interest is given by $\omega L''$, the generator sub-transient reactance at the stator current frequency that corresponds to the given rotor oscillation frequency.

In other words the limit on voltage perturbation is given by:

$$V = L''\left(\frac{2\pi 60 - \omega_d}{2\pi 60}\right)\overline{T}\,\frac{2\sigma}{\omega_d} \qquad 4.0$$

The implication of Equation 3.0 and 4.0 is shown for our example turbine-generator in Table I below for the consensus standard that allows a 1.0 per unit sudden change in generator electrical torque.

Mode Frequency Hz	18	**X"**	0.25
Load	Damping sigma rad/second	Torque Limit	Voltage Limit
FL	0.360	0.6366%	0.1114%
	0.300	0.5305%	0.0928%
	0.200	0.3537%	0.0619%
	0.100	0.1768%	0.0309%
NL	0.025	0.0442%	0.0077%

Table I: Limit of Perturbation Torque and Voltage

In the case of a generator with a first mode frequency of 18 Hz, that also meets the consensus standard for sudden change in torque of 1.0 per unit, can withstand a sinusoidal applied torque of 0.64% at a full-load damping of 0.36 radians/sec. However at no-load operation where the torsional damping is minimal, the corresponding torque limit reduces to 0.044%. The corresponding voltage limits are 0.11% at full-load reducing to 0.0077% at no-load operation.

The limits for no-load operation are rather severe, and maybe difficult to guaranteed. What must be remembered however is that applications that are subject to such sinusoidal perturbation must consider their location relative to the source of such perturbation to determine severity, and also consider the duration of such effects. What we see at resonance is that the response, which can be very large takes a very long time to be achieved. This has two implications, 1) it offers the opportunity for protection systems to operate in a very secure manner, and 2) because neither the frequency of the system, nor the perturbation frequency are exactly constant, then resonant response is less likely. The latter implies reasonably low probability of true resonance, which is a requirement for successful backup protection.

The proposed standard provides a simple method for an initial evaluation of a particular application without resort to complex simulations. Furthermore the required parameters of damping and frequency are easily measured with simple tests of the unit electrically.