A METHOD FOR CHARACTERIZATION OF THREE-PHASE UNBALANCED DIPS FROM RECORDED VOLTAGE WAVESHAPES

M.H.J. Bollen, L.D. Zhang
Dept. Electric Power Engineering
Chalmers University of Technology, Gothenburg, Sweden

Abstract: A proposal is presented for characterization of voltage dips as experienced by three-phase load. The authors propose this method for inclusion in international standards and recommendations. The method is based on the well-proven theory of symmetrical components. The primary result of the method is a so-called "characteristic magnitude" which corresponds to the magnitude (remaining voltage) as used for the existing methods to characterize dips experienced by single-phase load. The proposed method may be extended by adding additional parameters where further accuracy is needed for characterization. For three-phase balanced dips the proposed method corresponds to the methods currently in use and recommended by international standards.

1. Introduction

All existing standard documents on voltage dips (sags) characterize a dip through one magnitude (remaining voltage or voltage drop) and one value for the duration [6,7,8]. There are obvious limitations to this method as one e.g. neglects the phase-angle jump [1] and the post-fault dip [2]. For the majority of sensitive single-phase equipment, the existing characterization enables a prediction of the behavior of the equipment during and after the event. Further, the phase-angle jump can be incorporated by using a complex dip voltage; the post-fault dip can be incorporated by giving the magnitude as a function of time.

Three-phase equipment will typically experience three different magnitudes, as the majority of dips are due to single-phase or phase-to-phase faults. The existing method of characterization uses the lowest of the three voltages and the longest duration. An example of a three-phase unbalanced dip is shown in Figure 1.

![Figure 1, example of a three-phase unbalanced dip.](image)

Dip characterization is often part of the voltage characteristics / power quality in general. In that case, the results should be applicable both to single-phase and three-phase equipment. Using the
lowest of the three voltages to characterize the dip will result in erroneous results for both single-phase and three-phase equipment. An alternative technique is proposed in this document, which enables a characterization through one complex voltage, without significant loss of information. The method is based on the decomposition of the voltage phasors in symmetrical components.

An additional characteristic is introduced to enable exact reconstruction of the three complex voltages. The mathematics behind the method and additional examples are described in [3,4,5,9,10,11].

2. Background

2.1. Basic Classification

A classification of three-phase unbalanced dips was proposed in [5]. The classification considers three-phase, single-phase and phase-to-phase faults, star and delta-connected equipment and all types of transformer connection. It was further assumed that positive and negative-sequence source impedances are equal. This resulted in four types of three-phase unbalanced sag, shown as a phasor diagram in Figure 2. Type A is due to three-phase faults, types B, C and D are due to single-phase and phase-to-phase faults. Type B contains a zero-sequence component which is rarely transferred down to the equipment terminals. Three-phase equipment is normally connected in delta or in star without neutral connection. Single-phase low-voltage equipment is connected between phase and neutral, but the number of dips originating in the low-voltage system is small. Therefore the vast majority of three-phase unbalanced dips at the equipment terminals are of type C or type D, so that a distinction between type C and D is sufficient, together with a characteristic magnitude and phase-angle jump. The definition of characteristic magnitude and phase-angle jump is such that these do not change when the sag transfers from one voltage level to the other. The characteristic magnitude and phase-angle jump are defined as the absolute value and the argument of the complex phasor representing the voltage in the lowest phase for a type D dip, and the voltage between the two lowest phases for a type C dip.

![Figure 2, four types of three-phase unbalanced voltage dips in phasor-diagram form.](image)

The complex voltages for a three-phase unbalanced dip of type C with characteristic voltage \( \mathcal{V} \) are as follows:
\[V_a = 1\]
\[V_b = -\frac{1}{2} - \frac{1}{2} j \sqrt{3}\]
\[V_c = -\frac{1}{2} + \frac{1}{2} j \sqrt{3}\]  

(1)

For a dip of type D the complex voltages are:
\[V_a = 1\]
\[V_b = -\frac{1}{2}V - \frac{1}{2} j \sqrt{3}\]
\[V_c = -\frac{1}{2}V + \frac{1}{2} j \sqrt{3}\]  

(2)

2.2. Generalization

A sound mathematical basis for the above classification is given in [3,4,10], including a generalization that holds when positive and negative-sequence source impedances are different. This extension of the classification is based on the theory of symmetrical components. The three (complex) phase voltages in an unbalanced three-phase system can be completely described through three component voltages, known as symmetrical components. Positive-sequence voltage \(V_1\), negative-sequence voltage \(V_2\) and zero-sequence voltage \(V_0\) are calculated from the complex phase voltages \(V_a\), \(V_b\) and \(V_c\) as follows:

\[
\begin{bmatrix}
V_0 \\
V_1 \\
V_2
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\]

(3)

where
\[a = -\frac{1}{2} + \frac{1}{2} j \sqrt{3}\]  

Knowing the complex sequence voltages, the voltages in the three phases can be calculated from:

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 0 \\
1 & -\frac{1}{2} & -\frac{1}{2} j \sqrt{3} \\
1 & -\frac{1}{2} & \frac{1}{2} j \sqrt{3}
\end{bmatrix} \begin{bmatrix}
V_0 \\
V_1 + V_2 \\
V_1 - V_2
\end{bmatrix}
\]

(4)

Comparing (4) with (1) and (2), shows that the following relations hold for a dip of type C:
\[V_0 = 0\]
\[V_1 + V_2 = 1\]
\[V_1 - V_2 = \overline{V}\]  

(5)

The equivalent expressions for a dip of type D are:
\[V_0 = 0\]
\[V_1 + V_2 = \overline{V}\]
\[V_1 - V_2 = 1\]  

(6)
In the proposed characterization method these relations are used to obtain the characteristic complex voltage $\bar{V}$. The underlying assumption for (5) and (6) is that positive and negative-sequence source impedances are identical. As this is not exactly the case in reality, a second dip characteristic is introduced: the PN-factor $\bar{F}$. For a dip of type C the definitions are as follows:

$$\bar{V} = \bar{V}_1 - \bar{V}_2$$
$$\bar{F} = \bar{V}_1 + \bar{V}_2$$

(7)

For a dip of type D the definitions are as follows:

$$\bar{V} = \bar{V}_1 + \bar{V}_2$$
$$\bar{F} = \bar{V}_1 - \bar{V}_2$$

(8)

A method to obtain the type of dip from the recorded voltages will be discussed below.

2.3. Symmetrical Phase

Expression (4) gives the symmetrical components with reference to phase a. Expressions (1) and (2) are valid for a fault in phase a or between phases b and c, i.e. with phase a as the symmetrical phase. Including all three possible symmetrical phases results in six (sub)types of three-phase unbalanced dips: C_{a}, C_{b}, C_{c} and D_{a}, D_{b}, D_{c}. Expression (1) describes a dip of type C_{a}; expression (2) describes a dip of type D_{a}. Expressions for the complex voltages of the all six types are given in Table 1. It has been assumed that in all cases, the positive real axis is along the a-phase pre-event voltage.

For phasor diagrams as shown in Figure 2, the selection of the symmetrical phase is straightforward: the lowest phase for type D, the highest phase for type C. For a non-unity PN-factor and a non-zero phase-angle jump, it may become less obvious which phase is the symmetrical phase. The method proposed in [3,4] transforms phase voltage to symmetrical components to obtain the symmetrical phase.

Transforming the 6 different three-phase unbalanced dips to symmetrical components, by using (3), results in the expressions for positive-sequence voltage $V_1$ and negative-sequence voltage $V_2$ summarized in Table 2. Note that in all cases the positive real axis is along the a-phase pre-event voltage and in all cases expression (3) has been used to obtain the positive and negative-sequence voltages (i.e. the symmetrical component transformation with phase a as reference phase).

From Table 2 it follows that the positive-sequence voltage is type independent. The direction of the positive-sequence voltage is along the reference axis (phase a pre-event voltage in this case) if the argument of the characteristic complex voltage is neglected. The direction of the negative-sequence voltage depends on the type of dip. By rotating the negative-sequence voltage over an integer multiple of 60º all dip types can be obtained from one prototype dip. Dip type C_{a} has been chosen as prototype dip. From Table 2, the following relation between positive and negative sequence voltage is obtained for the prototype dip:

$$V_{2,\text{ref}} = 1 - V_1$$

(9)
### TABLE 1, COMPLEX VOLTAGES FOR THREE-PHASE UNBALANCED DIPS

<table>
<thead>
<tr>
<th>Type</th>
<th>( V_a )</th>
<th>( V_b )</th>
<th>( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_a )</td>
<td>( 1 )</td>
<td>( -\frac{1}{2} - \frac{j}{2} \sqrt{3} )</td>
<td>( -\frac{1}{2} + \frac{j}{2} \sqrt{3} )</td>
</tr>
<tr>
<td>( C_b )</td>
<td>( \frac{1}{4} + \frac{3}{4} j \sqrt{3} + \frac{1}{4} j \sqrt{3} )</td>
<td>( -\frac{1}{2} - \frac{j}{2} \sqrt{3} )</td>
<td>( -\frac{1}{2} + \frac{j}{2} \sqrt{3} )</td>
</tr>
<tr>
<td>( C_c )</td>
<td>( \frac{1}{4} - \frac{3}{4} j \sqrt{3} + \frac{1}{4} j \sqrt{3} )</td>
<td>( -\frac{1}{2} - \frac{j}{2} \sqrt{3} )</td>
<td>( -\frac{1}{2} + \frac{j}{2} \sqrt{3} )</td>
</tr>
<tr>
<td>( D_a )</td>
<td>( \frac{1}{4} + \frac{3}{4} j \sqrt{3} - \frac{1}{4} j \sqrt{3} )</td>
<td>( -\frac{1}{2} + \frac{j}{2} \sqrt{3} )</td>
<td>( -\frac{1}{2} - \frac{j}{2} \sqrt{3} )</td>
</tr>
<tr>
<td>( D_b )</td>
<td>( \frac{1}{4} + \frac{3}{4} j \sqrt{3} - \frac{1}{4} j \sqrt{3} )</td>
<td>( -\frac{1}{2} + \frac{j}{2} \sqrt{3} )</td>
<td>( -\frac{1}{2} - \frac{j}{2} \sqrt{3} )</td>
</tr>
</tbody>
</table>

### TABLE 2, SYMMETRICAL COMPONENTS FOR THREE-PHASE UNBALANCED DIPS.

<table>
<thead>
<tr>
<th>Type</th>
<th>( \bar{V}_1 )</th>
<th>( \bar{V}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_a )</td>
<td>( \frac{1}{2} + \bar{V} )</td>
<td>( \frac{1}{2} - \bar{V} )</td>
</tr>
<tr>
<td>( C_b )</td>
<td>( \frac{1}{2} + \bar{V} )</td>
<td>( \alpha \cdot \frac{1}{2} - \bar{V} )</td>
</tr>
<tr>
<td>( C_c )</td>
<td>( \frac{1}{2} + \bar{V} )</td>
<td>( \alpha^2 \cdot \frac{1}{2} - \bar{V} )</td>
</tr>
<tr>
<td>( D_a )</td>
<td>( \frac{1}{2} + \bar{V} )</td>
<td>( -\frac{1}{2} - \bar{V} )</td>
</tr>
<tr>
<td>( D_b )</td>
<td>( \frac{1}{2} + \bar{V} )</td>
<td>( -\alpha \cdot \frac{1}{2} - \bar{V} )</td>
</tr>
<tr>
<td>( D_c )</td>
<td>( \frac{1}{2} + \bar{V} )</td>
<td>( -\alpha^2 \cdot \frac{1}{2} - \bar{V} )</td>
</tr>
</tbody>
</table>

The dip type may be obtained from the angle between the negative-sequence voltage of the measured dip and the negative-sequence voltage of the prototype dip. Due to various approximations made and measurement errors, this angle is not exactly an integer multiple of 60° so that the following expression may be used to obtain the dip type:

\[
k = \text{round} \left( \frac{\angle(\bar{V}_2, 1 - \bar{V}_1)}{60^\circ} \right) (10)
\]

- \( k=0 \): type Ca
- \( k=1 \): type Da
- \( k=2 \): type Cb
- \( k=3 \): type Db
- \( k=4 \): typeCc
- \( k=5 \): type Db
Knowing the dip type, the negative-sequence voltage can be calculated back to the corresponding value for the prototype dip:

$$\bar{V}_2' = \bar{V}_2 e^{-j60\degree}$$  \hspace{1cm} (11)

where k is obtained according to (10) and the negative sequence voltage of the measured dip. Characteristic voltage $\bar{V}$ and PN-factor $F$ are obtained from the expressions for the prototype dip (7):

$$\bar{V} = \bar{V}_1 - \bar{V}_2'$$

$$F = \bar{V}_1 + \bar{V}_2'$$  \hspace{1cm} (12)

### 2.4. Overview of Characterization

This method has been applied to recorded dips in both transmission (220 and 400 kV) and distribution (11 and 33 kV) systems. It was shown that the PN-factor is very close to unity in transmission systems. In distribution systems, the PN-factor is typically less than unity due to the effect of induction motor load. But even in distribution systems, the PN-factor is rarely less than 90% in absolute value [10].

The result is that the characteristic magnitude (the absolute value of the characteristic complex voltage $\bar{V}$) can be used to characterize three-phase unbalanced dips without loss of essential information. Using characteristic magnitude and duration for three-phase unbalanced dips, corresponds to the existing classification (through magnitude and duration) for single-phase equipment. Where needed the characterization for three-phase unbalanced dips may be extended in several ways:

- the characteristic phase-angle jump may be defined as the argument of the complex characteristic voltage in the same way as the phase-angle jump may be used as an additional characteristic for dips experienced by single-phase equipment.
- the PN-factor may be used as an additional characteristic in case positive and negative-sequence source impedances differ significantly. This is the case in systems with a large amount of induction motor load.
- the zero-sequence voltage is needed as an additional characteristic for specific system configurations in combination with three-phase star-connected load.
- characteristic magnitude, characteristic phase-angle jump and PN-factor may all be given as a function of time.

### 3. The Proposed Method

The proposal is to use characteristic complex voltage and PN-factor to characterize three-phase unbalanced dips. The proposed algorithm for classification and characterization consists of a number of steps. It is assumed that time-domain sampled data is available for the three phases including at least two cycles pre-event voltages.

I. Determine the voltage frequency from the pre-event voltage samples.

II. Determine voltage phasors for the three phase voltages by using a DFT (Discrete Fourier Transform) algorithm. The voltage frequency is used to obtain the phase shift between the during-event and the pre-event voltages.
III. Obtain positive, negative and zero-sequence voltages by using expression (3).

IV. Determine if the dip is balanced or unbalanced from the magnitude of the negative sequence voltage compared to the positive sequence voltage.

V. For balanced dips the dip type is A and the characteristic voltage equals the positive sequence voltage.

VI. For unbalanced dips the dip type is determined from positive and negative-sequence voltages by using expression (10), characteristic voltage and PN-factor are obtained by using expression (12).

VII. A balanced dip is fully characterized through the characteristic voltage.

VIII. An unbalanced dip is fully characterized through dip type, characteristic voltage, PN-factor and zero-sequence voltage.

IX. The characteristic magnitude is obtained as the absolute value of the characteristic voltage. The phase-angle jump is obtained as the argument of the characteristic voltage.

4. Example of Characterization

The above-proposed method has been applied to the three-phase unbalanced dip shown in Figure 1. The rms voltage as a function of time, for the three voltages, is shown in Figure 3.

![Figure 3](image3.png)

*Figure 3, rms voltage versus time for the three voltages in Figure 1.*

![Figure 4](image4.png)

*Figure 4, positive-sequence voltage (solid curve) and negative-sequence voltage (dashed curve) for the dip in Figure 1.*
Figure 4 shows positive and negative-sequence voltage, obtained by using expression (3). Fault initiation takes place about 5 cycles after the start of the recording. Fault clearing is 4.5 cycles later. Before fault initiation and after fault clearing, the negative-sequence voltage is small, in other words: the voltages are balanced. Note that the positive-sequence voltage does not immediately recover upon fault clearing. This is probably due to induction motor load taking a larger current when their speed has dropped below nominal speed.

Figure 5 plots the angle in the complex plane between the negative-sequence voltage and the drop in positive-sequence voltage, as used in (10). When the negative-sequence voltage is less than 0.02 pu, the angle is given a small negative value. This same threshold is also used to distinguish between a balanced and an unbalanced dip.

![Figure 5, angle between (complex) negative-sequence voltage and (complex) drop in positive-sequence voltage, for the dip in Figure 1.](image)

From the angular difference in Figure 5, the dip type is determined by using (10). The result is shown in Figure 6. The integer value of the dip type is according to the list below (10), where dip type 6 is a balanced dip (Type A). Note that also before the fault, the event is characterized as type A. This is due to the criterion used to detect a type A dip: negative-sequence voltage less than 0.02 pu. Therefore the normal supply is characterized as a type A dip without voltage drop (100% characteristic magnitude). From Figure 6 it can be concluded that the dip shown in Figure 1 is of type CC: a drop in phases a and b with no drop or a minor drop in phase c. This is exactly as shown in Figure 1, but would be hard for an automatic algorithm to detect. Also will it not in all cases be straightforward which is the type of a three-phase unbalanced dip. Around fault initiation and around fault clearing the algorithm indicates a different dip type. This is due to the method used to obtain the complex phase voltages, which takes about one cycle for the transition. During such a transition especially the angle of the complex voltage may obtain unrealistic values. The problem may be solved by increasing the negative-sequence value below which the dip is classified as balanced. Other methods may need to be developed for this.
The positive and negative-sequence voltages, together with the dip type, give the characteristic voltage and the PN-factor for the dip. The result is shown in Figure 7. Before fault initiation and after fault clearing, characteristic magnitude and PN-factor are almost equal because the negative-sequence voltage is very small. During the fault, both characteristic voltage and PN-factor show a decreasing trend. The PN-factor is continuous, i.e. it does not show a jump at fault initiation or at fault clearing. This behavior may be explained from the deceleration of induction motor load during the fault and their acceleration after the fault. The lower the motor speed, the lower their impedance and the more the voltage in the system is reduced. The PN factor is thus an indication of the effect of the load on the voltage dip. This relation, and the underlying mathematical models, are described in detail in [10].

5. Conclusion

A new method has been proposed for characterization of three-phase unbalanced voltage dips. The result of the characterization is a simple characteristic magnitude. This single value enables a prediction of the effect of the event on most single-phase and three-phase equipment. When more detailed characterization of the event is required, additional parameters can be added. For
three-phase balanced dips, the proposed characterization corresponds to the methods currently in use and recommended by international standards [6,7,8].

6. References