# Statistical Methods of Classifying Major Event Days in Distribution Systems

Richard D. Christie, *Member, IEEE*

*Abstract*--**This paper describes three methods currently under consideration for classifying major event days in electric power distribution systems, the Three Sigma, Three Beta and Bootstrap methods. An example using data from three anonymous utilities compares the three methods. The Three Beta is judged somewhat better than the other two.**

*Index Terms*--**power distribution system reliability, statistics, classification.**

## I. Introduction

The Distribution System Design Working Group of the Distribution Subcommittee has been discussing standard methods for classifying major event days (MEDs) for some time. Major event days are days when the reliability of the distribution system is much, much worse than normal.

Three statistical methods of classifying major event days (MEDs) are described below:

- Three Sigma
- Three (or, Two Point Five) Beta
- Bootstrap

All three assume that a statistical method of classification is desired, and that MEDs must be identified on the day they occur. This means that each method calculates a reliability threshold $R^*$ for daily System Average Interruption Duration Index (SAIDI) using historical data from previous years. When the SAIDI of a day in the current year exceeds $R^*$, that day is a major event day.

This paper is a further development of the methods described in [1], which addresses why SAIDI is used and how many years of historical data are desirable.

## II. Three Sigma Method

MEDs are large, rare events that populate the high side (right hand) tail of the probability distribution of daily SAIDI. Engineers are familiar with the concepts of average (mean) $\mu$ and standard deviation $\sigma$ of a probability distribution, and know that the more multiples of standard deviation away from the mean a value is, the rarer it is. Therefore it is natural to think of a threshold for MEDs based on mean and standard deviation. In fact, three standard deviations ($3\sigma$) is a common threshold used in many applications to identify exceptional values. This leads to the Three Sigma method of major event day identification, as follows:

1. Assemble the preceding three to five years of daily SAIDI values (including zeros).
2. Calculate the mean $\mu$ and standard deviation $\sigma$ of the data.
3. Calculate a threshold

$$R^* = \mu + 3\sigma \qquad [1]$$

This threshold determines how much of the right hand tail of the distribution consists of MEDs.

An underlying assumption of the Three Sigma method is that daily SAIDI is normally distributed, that is, it has a Gaussian distribution giving the classic bell curve, as shown in Fig. 1. Then the expected number of days in the right hand tail can be found from the probability in the right hand tail, $p$, which is a function of the number of standard deviations in the threshold formula [1]. $p$ can be found from tables or standard worksheet functions. $p$ is only a function of the standard deviation multiplier $k$ ($k$ = 3 in this case), and is independent of values of the mean $\mu$ and standard deviation $\sigma$.

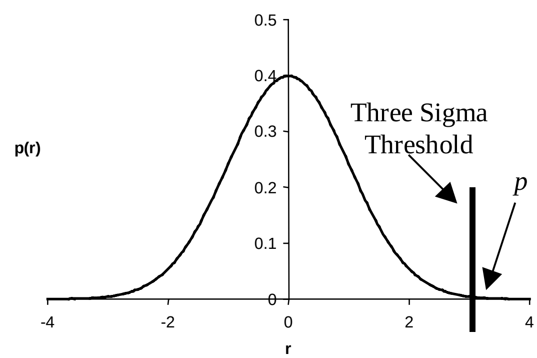


Fig. 1. Ideal normalized Gaussian probability distribution ($\mu$ = 0, $\sigma$ = 1) showing Three Sigma threshold and the probability in the tail of the distribution above the threshold.

The relationship between the standard deviation multiplier, $k$, the probability in the tail, $p$, and the expected number of MEDs/year is shown in Table 1 for different values of $k$. As can be seen, a $3\sigma$ threshold should identify, on average, one MED every two years. Alternatively, an average of 3 MEDs/year requires a $k$ value of 2.4.

The Three Sigma method is fairly simple and based on common knowledge about probability distributions, but it has one major flaw. This is best seen by examining a histogram of actual historical daily reliability data, as shown in Fig. 2.

Richard D. Christie is with the Department of Electrical Engineering, University of Washington, Seattle, WA 98195-2500 USA.
(e-mail: christie@ee.washington.edu).

TABLE 1 - THRESHOLD VS. EXPECTED MEDS FOR THREE SIGMA METHOD

| *k* | *p* | MEDs/yr |
|---|---|---|
| 1 | 0.15866 | 57.9 |
| 2 | 0.02275 | 8.3 |
| 2.4 | 0.00822 | 3.0 |
| 2.5 | 0.00621 | 2.3 |
| 3 | 0.00135 | 0.5 |
| 6 | $9.9 \times 10^{-10}$ | 3.6E-07 |

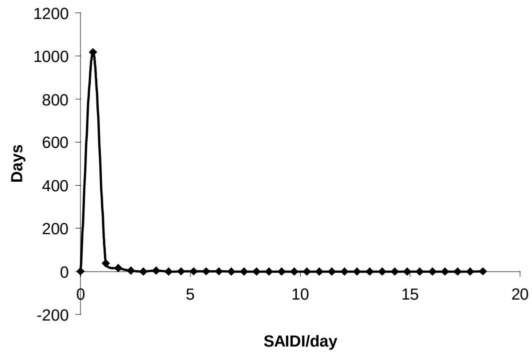


Fig. 2. Histogram of three years of daily SAIDI data from anonymous Utility 2 supplied by the Distribution Design Working Group. Daily SAIDI is clearly not normally distributed.

The flaw is that the actual daily SAIDI values are not normally distributed. Instead they have a large peak on left and a long tail on the right, and exhibit strong kurtosis, or skew. Since the distribution is not normal, all of the characteristics derived from the assumption that the distribution is normal are invalid. The relationship between expected MEDs and standard deviation multiplier shown in Table 1 no longer holds. The expected number of MEDs will vary with the values of mean $\mu$ and standard deviation $\sigma$, which makes comparisons between different utilities or between different years in the same utility inequitable.

The shape of the actual distribution of SAIDI, and a desire to find a more equitable threshold calculation method, leads to the Three Beta method described next.

## III. THREE BETA METHOD

The Three Beta method starts from the identification of actual probability distributions of daily SAIDI as log-normal distributions. In a log-normal probability distribution, the *natural logs* of the daily SAIDI values form a normal, or Gaussian, distribution. (Note that $\log_{10}$ of the daily reliability could also be used.) A discussion of the log-normal character of distribution reliability is found in [1].

The log-normal nature of daily reliability data is illustrated by plotting a histogram of the natural log of the data used for Fig. 2. The results are in Fig. 3. Even with some fluctuation in the middle of the distribution, the log of the samples is recognizable as having a normal, or Gaussian, distribution. Thus the assumptions discussion in the previous section hold for the logs of the sample values, not for the sample values themselves.

The same thresholding process used in the Three Sigma method can then be applied to the natural logs of the daily reliability, as follows:

1. Assemble the preceding three to five years of daily SAIDI values.
2. Take the natural log (ln) of each of the values. (For days with SAIDI of zero, use the lowest SAIDI in the data set.)
3. Calculate the mean $\alpha$ and standard deviation $\beta$ of the natural logs of the values. $\alpha$ is sometimes called the log- mean, and $\beta$ is sometimes called the log-standard deviation.
4. Calculate a threshold

$$R^* = e^{(\alpha+3\beta)} \qquad [2]$$

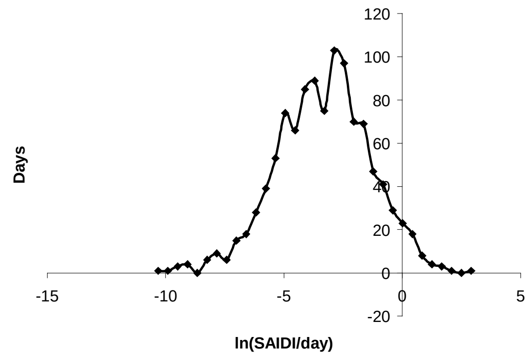


Fig. 3. Histogram of the natural logs of three years of daily SAIDI data from anonymous Utility 2 supplied by the Distribution System Design Working Group. The logs of the data are normally distributed, so the daily data is log-normally distributed.

The Three Beta method is identical in concept to the curve fitting method described in [1], although the description here is somewhat different.

The Three Beta method is only slightly more complicated than the Three Sigma method, and has the advantage that the expected number of MEDs per year from Table 1 is now correct, if *k* is the multiplier of the log-standard deviation $\beta$ rather than the standard deviation $\sigma$. With $k = 3$, approximately one day every two years should be classified as a major event day, and the expected number of MEDs per year should not vary with average $\mu$, standard deviation $\sigma$, log-average $\alpha$ or log-standard deviation $\beta$, and is thus an equitable method to apply to different utilities, or to different years of data from the same utility.

After examining the number of days classified as MEDs using their data, some utilities were concerned that not enough days were identified per year. One solution is to reduce the log-standard deviation multiplier *k* to 2.5, giving, on average, 2.3 days per year. Another solution is to set the expected number of MEDs per year and calculate the corresponding value of *k*. Even so, there are still cases where too many or too few MEDs to suit the expectations of distribution reliability engineers are identified. This suggests using a method that starts with the desired number of MEDs/year, which is the Bootstrap method described next.

## IV. BOOTSTRAP METHOD

The Bootstrap method was introduced in [1], which includes a discussion of the number of years of historical data to use for the analysis.

If the Three Beta method produces a number of MEDs which is too high or too low based on engineering judgement, this could be due to irregularities in the tail of the distribution that reflect physical phenomena. While there is no question that most of the distribution is log-normally distributed, the tail may not be, and the tail is where the thresholding process operates. The Bootstrap method therefore uses the tail of the actual distribution, rather than any theoretical distribution, to set the MED threshold.

The theory of the Bootstrap method is that the probability distribution of future events will be the same as the probability distribution of historical events, no matter what the shape of that distribution. The probability of a given number of days/year exceeding a given MED threshold will be the same as the probability of exceeding that MED threshold in the past. Thus the bootstrap method is as follows:

1. Assemble the preceding three to five years of daily SAIDI values.
2. Sort the daily reliability values into descending order, so the largest are first in the list.
3. Calculate the expected number of MEDs in the historical data. For example, if 3 MEDs/year are desired, and there are 5 years of historical data, the expected number of MEDs in the historical data is $3 \cdot 5 = 15$ days. The 15 largest SAIDI/day values are these expected MEDs.
4. The daily SAIDI of the lowest of the expected MEDs in the historical data (the 15$^{th}$ largest, in the example) is the threshold $R*$ to use in the following year.

Note that the expected MEDs in the historical data may or may not have been classified as MEDs at the time when they occurred, since the threshold used to classify them was based on data from previous years. To provide for consistency, days should probably not be retroactively reclassified.

The Bootstrap method is easy to implement, conceptually simple, equitable, and should in theory give the same results as the Three Beta method if the same expected number of MEDs/year is used for each process. In practice, the Bootstrap method may be more subject to saturation than the Three Beta method. Saturation occurs when one year has a large collection of severe events. Suppose, for example, that a particularly severe storm event causes 15 days of horrible reliability in the example system in a single year. That year will set the MED threshold for the next five years, probably at a high value that results in few or no days being classified as MEDs in the subsequent four years, until the bad year rolls out of the historical data set. In the Three Beta method, the normal reliability data of the non-horrible years will tend to mitigate the effect of the 15 bad days.

## V. EXAMPLE

The example data is from three example data sets provided to the Distribution Design Working Group membership by anonymous utilities. The data sets consist of three to five years of daily SAIDI values of minutes/day. They are designated Example 2, Example 6 and Example 7. (Other examples, e.g. Example 1, do not have dates for each value and thus cannot be sorted into historical and present years.) The three methods compared are as follows: The Three Sigma method (3σ) will be used with $k = 3$. This is the most commonly discussed value of $k$ used with σ, and there is little theoretical basis to set it to other values. The Three Beta method is used with $k = 2.4$, giving an expected number of MEDs/year of 3. Thus it is really the 2.4β method. The Bootstrap method is used with 3 expected MEDs/year (hence it is called the B3 method), allowing direct comparison to the 2.4β method. Thresholds (Table 2) and MEDs (Table 3) are calculated using the three most recent years of historical data, and applied to the last year in the data set to identify MEDs. If less than three years of historical data are available, then the available historical data is used.

TABLE 2 - EXAMPLE THRESHOLD VALUES

| | 3σ | 2.4β | B3 |
|---|---|---|---|
| Ex 2 | 1.33 | 3.25 | 1.91 |
| Ex 6 | 2.42 | 3.51 | 3.28 |
| Ex 7 | 20.64 | 4.37 | 14.93 |

TABLE 3 - EXAMPLE MED VALUES

| | 3σ | 2.4β | B3 |
|---|---|---|---|
| Ex 2 | 9 | 4 | 5 |
| Ex 6 | 4 | 2 | 3 |
| Ex 7 | 0 | 6 | 0 |

Thresholds should be compared for different methods for the same example, but not between different examples for the same method. MEDs can be compared both among examples and methods.

It's hard to say much from this data, since the "right" number of MEDs is not known, and there is insufficient data to make a reasonable assessment. Nevertheless an attempt at analysis will be made.

The most glaring discrepancy is with Example 7, where neither the Three Sigma method nor the Bootstrap method identified any MEDs for the present year, yet the 2.4β method identifies six. A look at the histogram of the logs of the data for Example 7 is instructive (Fig. 4).

The probability distribution is Gaussian, but exhibits a long tail to the right in comparison to, for example, the utility in Fig. 3. Example 7 probably is subject to severe weather phenomenon, like hurricanes, that Utility 2 does not see. In Example 7, the present year (lower curve) has not had events as severe as those in the previous three years (upper curve), but still has events that are severe in comparison to the normal portion of the curve. The 3σ threshold is pulled up by the effects of the large events in the historical data. The B3 threshold is similarly increased, although not as much. This is not exactly saturation, since the worst days in the historical data are distributed over the three preceding years. The 2.4β threshold is not pulled up as much by the tail because it includes the effects of all of the daily data, and the tail effects are reduced by using a logarithmic scale.

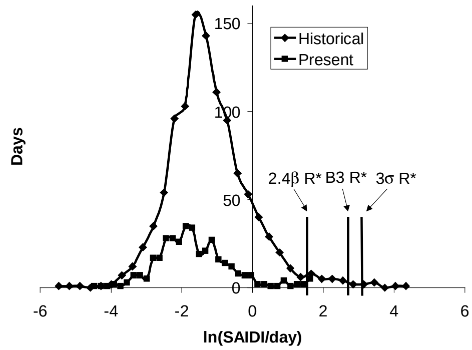


Fig. 4. Histogram of the natural logs of daily SAIDI data from Example 7 supplied by the Distribution System Design Working Group. The logs of the data are normally distributed, so the daily data is log-normally distributed.

Looking at the shape of the curves, the 2.4β threshold seems to be in the most appropriate place for Example 7. The six days in the present year that exceed the 2.4β threshold are represented by the last point in the present year data. They are not much above the threshold, but this proximity will occur with any thresholding process.

The real issue is whether days with SAIDI values between those of the 2.4β $R^*$ and the 3σ $R^*$ should be MEDs. Looking at the curves it appears that this region is well outside the normal distribution envelope and that points in this region - including the six days in the present year - are statistical major events. In the author's opinion 2.4β delivers the best threshold for this extreme case.

## VI. Conclusions

The paper has described three statistical methods of identifying major event days (MEDs) and given an example comparing them. The Three Sigma method is inequitable among different utilities. Example 7 shows its variation with mean and standard deviation and sensitivity to large daily SAIDI values. The Bootstrap method is equitable, less sensitive to large SAIDI values and theoretically identical to the Three Beta method, but in the limited example set presented here does not seem to perform as well in extreme cases like example 7. For this reason, the Three Beta method, or rather, *k* Beta with *k* set based on desired average number of MEDs, is the author's present preferred method for identifying MEDs.

## VII. Acknowledgments

The author is grateful to the members of the System Design Working Group of the Distribution Subcommittee for providing the example data, and specifically to Jim Bouford and Cheri Warren for compiling it.

## VIII. References

[1] R.D. Christie, "Statistical Classification of Major Event Days in Distribution System Reliability," *IEEE Transactions on Power Delivery*, to appear.

**Richard D. Christie** (M) is an Associate Professor of Electrical Engineering at the University of Washington in Seattle. He obtained the BS and MS in Electric Power Engineering from Rensselaer Polytechnic Institute and the PhD from Carnegie Mellon University. He has been active in developing predictive distribution reliability assessment methods, and is a member of the Working Group on System Design of the Distribution Subcommittee of the IEEE.