The Mighty 300Hz Clock

Gordon Bechtel, Symphony-Teleca Corp.

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1 The Mighty 300 Hz Clock

Everyone has their daily torment. You know what I'm talking about. It's that pesky, bothersome issue that is there every day and which one has to continually step around, or fix and re-fix, or put away again and again. It's that bothersome, niggling irritation that you have to deal with every day. For parents of teenage boys, it's having to pick up their room every day. For users of Windows, it's having to put up with another update just before that big meeting with your boss. For the video industry, it's interlaced video.

Digital audio is no different. The torment for digital audio is two numbers: 48,000 and 44,100. These two numbers are the two most prevalent clock rates for sampling audio, 48kHz and 44.1kHz. Almost all digital audio uses one or the other. The original compact disk used 44.1kHz and much of the digital audio produced and sold since then used 44.1kHz sampling. Most of the audio on iTunes is at 44.1kHz. In more recent times, much of the professional audio world has moved to 48kHz sampling.

2 Why Have Both?

Either should work, theoretically. As those who are familiar with Dr. Nyquist know, as long as you sample faster than twice the highest frequency of the signal you're discretizing, you can theoretically get the original signal back through filtering and reconstruction techniques. With audio, the highest frequency the human can hear is 20kHz (and old guys like me, don't even come close to that), so we don't bother with anything higher. This means a sampling frequency of anything higher than 40kHz should do the trick. 44.1kHz and 48kHz both qualify. And neither are too high that we're wasting a lot of bandwidth or storage or processing power by sampling too much.

So why have both of them? Good question. I think the answer lies more in the science of evolution rather than rational analysis and decision making. 44.1kHz worked well in the early days because it was just fast enough (that is, it satisfied Dr. Nyquist), but not so fast that an album's worth of music wouldn't fit on the fancy new CD. As technology evolved, storage grew bigger, networks sizzled faster, and CPUs cranked up their gigaflops, the industry decided it could afford a faster sampling rate. It would be nice to have a clock that's easily divisible, as this makes it easier to subdivide to lower rates for low-quality audio (like telephone audio). Divisibility also allows it to be created relatively easily from lower rate clocks running at an integral fraction of the base sampling rate. 48kHz obviously satisfies these conditions.

Regardless of how we got here, the world is left with two prevalent sampling frequencies: 44.1kHz and 48kHz.

3 It's More Than an Annoyance

For engineers who are designing systems that have multiple unknown sources of audio – such as a car audio system or a home theatre system – having two clocks is more than a simple annoyance.

Among many issues, this memo deals with one particularly troublesome issue: How to distribute a digital clock from the audio sources to the audio sinks? In an automobile and any professional audio systems, it is desired to distribute a common clock to all elements. A common clock eliminates a lot of problems and it allows audio elements to get away with only reproducing one clock, even though they may be creating many audio streams. Because the clock needs to have very high quality (low-jitter) the circuitry for reproducing the clock can be expensive. Having to only recreate a single clock therefore greatly reduces the cost of an audio element.

But, if there are two clocks being used by the audio systems – 44.1kHz and 48kHz – we now have to distribute two clocks. Ugh! It would be very nice if we could do it with just one.

That's where the mighty 300 Hz clock comes in.

4 Back to Middle School Algebra

Our basic question is this: What is the fastest clock on which both a 48kHz and a 44.1kHz clock share a common tick mark? To put this mathematically, we need to find two integers M and N such that:

$$\frac{N}{48,000} = \frac{M}{44,100}$$

Prime factorizations will help us figure this out. The prime factorization of 44,100 is:

$$44,100 = 100 \times 441 = 2^2 \times 5^2 \times 3^2 \times 7^2$$

And the prime factorization of 48,000 is:

$$48,000 = 1000 \times 48 = 2^3 \times 5^3 \times 2^4 \times 3 = 2^7 \times 3 \times 5^3$$

Substituting the prime factorizations and solving for the ratio $\frac{N}{M}$:

$$\frac{N}{M} = \frac{2^7 \times 3 \times 5^3}{2^2 \times 3^2 \times 5^2 \times 7^2} = \frac{2^5 \times 5}{3 \times 7^2} = \frac{160}{147}$$

This means that 160 ticks of a 48kHz clock spans the same duration in time as 147 ticks of a 44.1kHz clock. We can't reduce the span any shorter, either, because 160 and 147 are *coprime* or *mutually prime,* meaning that their greatest common divisor is 1. So, how long is this time span? It's $\frac{160}{48,000} = 3.333$ ms or, inverting the fraction, $\frac{48,000}{160} = 300$Hz .

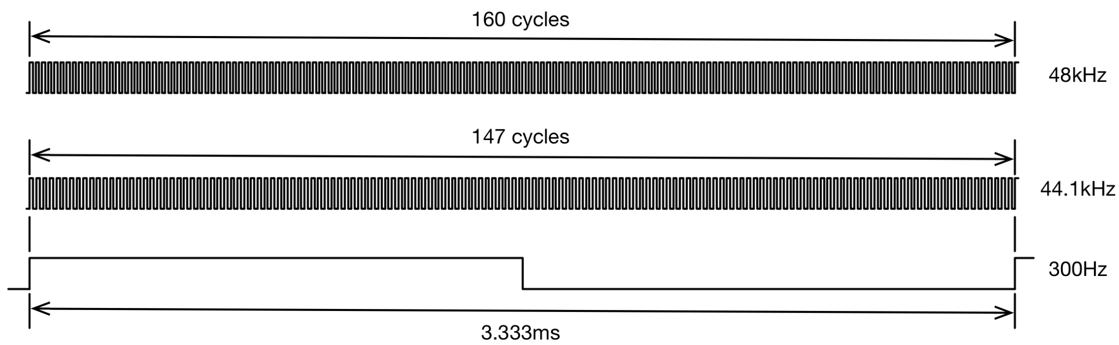

Figure 1: 300Hz, 44.1kHz and 48kHz Clocks

Magic! If we distribute a 300Hz clock, then the audio elements can re-create a 48kHz and a 44.1kHz clock by multiplying the 300Hz clock up by 160 and 147, respectively.

The figure diagrams the situation quite well.

5 Conclusion

After a long-winded and meandering narrative, the memo introduces and shows that a single 300Hz clock can be used to generate both 44.1kHz and 48kHz clocks. The author would like to thank the reader for their patience and perseverance.