1 Motion 7: NaI

P 1788 provides a unique NaI (Not an Interval)-datum for each abstract interval format. The datum indicates that the construction of an interval did not produce an $\mathbb{F}$ interval. It propagates through all interval operations. If at least one of the operands is NaI, so is the result. All comparisons with NaI are evaluated to false, with one exception, the result of isNaI(NaI) is true. NaI, however, is never returned by an operation on $\mathbb{F}$ intervals.

2 Rationale

2.1 Construction

In P1788 intervals are constructed and initialised by constructors or are returned from operations. There is at least one constructor `interval(l,u)` with 2 floating point datums as parameters. According to motion 3, if $l < u$ or if arguments are equal finite numbers, that constructor returns the $\mathbb{F}$ interval `[l,u]` otherwise NaI is returned.

There is a second constructor `empty()` producing the emptyset.

Constructor `interval(l,u)` is the only operation that is able to generate NaI without having NaI as an input parameter. The evaluation of a function outside its domain results in the empty set.

For extended interval arithmetic, say modal, the case where normally a NaI is generated, now constructs an extended interval.

Since there are many possible encodings of NaN, NaI and the empty set both can be represented as different pairs of NaNs. NaI as [sNaN,sNaN], a pair of signaling NaNs then have to trap on each occurence. True empty sets should be represented as a pair of quiet NaNs [qNaN,qNaN]. The propagation of NaIs can be guaranteed by handling of the invalid operation exception. Being an exceptional situation it does not matter that this may be expensive. The propagation of quiet NaNs on the other hand is efficient.

2.2 Arguments

Arguments against NaI may be

- "There is no such thing in mathematics, so stop computing, if it occurs."

  Stopping the computation of computing is only one option to handle this exception. Another option should be to continue with a defined value that propagates and indicates an illegal construction has occurred.

- "The empty set is enough to represent nothing."

  There is a semantic difference of an empty set resulting from an (intersection) operation and a dummy created by interval(4,3) or interval(NaN,5)

- There are other uses of NaI

  - missing data
  - division with gap
  - function not defined

  These can not or should not result in a NaI result, since some can be handled by sticky flags, others by proper choice of operation.

- NaI is used in fundamental motion 6 and proposed by the Vienna proposal 2.3