

Let's denote the arithmetic for closed real intervals by  $\mathbb{I}$  and that for connected real intervals by  $\mathbb{J}$ . Further let  $\mathbf{a} = [a_1, a_2]$  and  $\mathbf{b} = [b_1, b_2]$  be two floating-point intervals of  $\mathbb{I}$  and  $\circ$  an operation  $\circ \in \{+, -, \cdot, /\}$ . Then the lower bound of an interval operation  $\mathbf{a} \circ \mathbf{b}$  is computed by  $\nabla(a_i \circ b_j)$  and the upper bound by  $\Delta(a_\mu \circ b_\nu)$  where the  $i, j, \mu, \nu$  are to be selected by the usual formulas for interval operations.

In general the results  $a_i \circ b_j$  and  $a_\mu \circ b_\nu$  will not be floating-point numbers so that the roundings have to be applied. Then arithmetic in  $\mathbb{I}$  delivers the result  $I = [\nabla(a_i \circ b_j), \Delta(a_\mu \circ b_\nu)]$  while arithmetic in  $\mathbb{J}$  delivers  $J = (\nabla(a_i \circ b_j), \Delta(a_\mu \circ b_\nu))$  and we have  $J \subset I$ .

More drastic examples can be given in case of *reasonable* fused operations. Consider, for instance, two interval matrices. In  $\mathbb{J}$  the dot products are computed exactly with only one rounding at the end of the accumulation while in  $\mathbb{I}$  a rounding is applied after each addition and each multiplication in the dot products. In addition to the difference in accuracy there is a difference in computing speed. For details see section 8.6.2 in my book *Computer Arithmetic and Validity*. The unit discussed there was built in 1993/94. The book was on the market before IEEE P1788 was founded.