International Federation for Information Processing
Technical Committee 2: Software Theory and Practice
Working Group 2.5: Numerical Software
http://www.ifip.org/wg-2.5

11 July 2014

Prof. R. Baker Kearfott
Acting Chair, IEEE Project 1788
University of Louisiana at Lafayette
Box 4-1010
Lafayette, LA 70504-1010
U.S.A.

Dear Professor Kearfott,

In 2007, IFIP Working Group (WG) 2.5 on Numerical Software voted unanimously to urge IEEE Project 754 (Floating Point Arithmetic) to incorporate two facilities into the revision of the floating point standard under development at that time:

- for the data format double precision, provide interval arithmetic, at the speed of simple floating-point arithmetic;
- provide high speed arithmetic for dynamic precision for real and interval data types.

A letter to that effect was sent to Bob Davis, in his capacity as chair of the IEEE Microprocessor Standards Committee, on 4 September 2007. Copies were sent to D. Zuras, L. Tsai, W. Kahan, J. Darcy, P. Tang, D. Hough, and J. Demmel.

The request was considered to have been made too late in the revision process to be incorporated. Therefore, a new IEEE project, 1788, Interval Arithmetic, was formed. The first facility was the starting point for that project's work. The second facility was not initially included in the work plan.

On 9 September 2009, IFIP WG 2.5 sent a request to you, in your capacity as chair of the IEEE Interval Arithmetic Working Group, to urge inclusion of a requirement for an exact dot product, to provide the second facility. Copies were sent to Nathalie Revol, George Corliss, Jürgen Woff von Gudenberg, Dan Zuras, and John Pryce. An outcome of that request was Motion 9, which was approved, and which resulted in inclusion of that requirement in subsequent drafts, until draft 7.1, dated April 2013.

As described in [1], the exact dot product can be computed as rapidly as data can be made available: there cannot be a faster method. It always produces the same result, even if the order of the underlying computational steps varies. It is much simpler than an algorithm that uses floating-point operations in normal working precision, and extreme care, to compute a dot product as if using sufficient precision to avoid overflow or truncation, followed by rounding to working precision [as in, e.g., 2].

In June 2013, motion 45, which changed the status of the exact dot product from required to recommended, was adopted. This was based upon the premise that computing a correctly-rounded dot product is simpler than computing an exact dot product, but it would be difficult to construct a simpler method than the exact dot product described in [1].

An argument has been advanced that an exact dot product ought to have an independent standard. IFIP WG 2.5 believes this would be a mistake:

- The primary purpose of the exact dot product is to provide a method to produce close bounds of intervals. Standing on its own would bring into question the reason for its existence.
- Programs based upon an implementation of interval arithmetic that does not have facilities to provide close bounds can result in bounds so broadly separated as not to be interesting or useful. This would be counter-productive, serving only to re-enforce old objections to interval arithmetic, possibly resulting in failure to implement the standard produced by IEEE Project 1788.
- When an exact dot product is converted to an interval, the result is always either a point, or an interval with bounds that are consecutive floating-point numbers. This result cannot be achieved in general by the obvious naive floating-point dot product algorithm, and carefully contrived algorithms based upon floating point operations [as in, e.g., 2] can only achieve this result at higher cost in both complexity and execution time.
- A standard for an exact dot product would be extremely small.

IFIP WG 2.5 regrets that the unanimous request of leading numerical software experts has been incorporated as a recommendation, not a requirement, into the final draft of the standard produced by IEEE Project 1788, for which we understand that balloting was completed on 7 July 2014, just prior to our Working Group meeting of July 11 in Vienna. If final balloting on the standard has not been completed, IFIP WG 2.5 urges that Project 1788 incorporate a requirement for an exact dot product before final balloting. Otherwise, IFIP WG 2.5 urges that Project 1788 produce a revised standard, expeditiously in the normal course of IEEE standards maintenance, and that that revision will include a requirement for an exact dot product. This will increase the significance of the standard.

Sincerely,

Prof. dr. ir. Ronald Cools
Chair, IFIP Working Group 2.5, Numerical Software
Dept. of Computer Science, KU Leuven
Celestijnenlaan 200A bus 2402
B-3001 Heverlee, Belgium

References:

1. Ulrich Kulisch, **Computer Arithmetic and Validity — Theory, Implementation, and Applications**, de Gruyter, Berlin (2008), second edition (2013).
2. Takeshi Ogita, Siegfried M. Rump, Shin'ichi Oishi, *Accurate sum and dot product*, **SIAM Journal on Scientific Computing 26**, 5 (2005) pp 1955-1988.

Attachments:

- Letter of September 4, 2007, on IEEE 754R
- Letter of September 9, 2009, on IEEE P1788

Cc: Members of WG 2.5
Members of P1788

International Federation for Information Processing
Technical Committee 2: Software Theory and Practice
Working Group 2.5: Numerical Software
http://www.nsc.liu.se/wg25/

September 4, 2007

Bob Davis, Chair
IEEE Microprocessor Standards Committee
Summit Computer Systems, Inc.
22685 Summit Road
Los Gatos, CA 95033-9310

Dear Mr. Davis,

The IFIP Working Group 2.5 on Numerical Software very much appreciates the work of the Standards Committee of the IEEE Computer Society in revising the IEEE Floating-Point Arithmetic Standards 754 and 854.

We think that the tremendous progress in computer technology and the great increase in computer speed should be accompanied by extension of the mathematical capacity of the computer. Beyond what has already been done by IEEE754R, IFIP WG 2.5 expresses its desire that the following two requirements are included in the future computer arithmetic standard.

a) *For the data format double precision, interval arithmetic should be made available at the speed of simple floating-point arithmetic.*

   Most processors on the market are equipped with arithmetic for multimedia applications. On these processors we believe that it is likely that only about 0.1% more silicon in the arithmetic circuitry would suffice to realize this capability.

   See, for example, Reinhard Kirchner and Ulrich W. Kulisch, "Hardware Support for Interval Arithmetic," *Reliable Computing* **12**:3 (June 2006), pp.225-237.

b) *High speed arithmetic for dynamic precision should be made available for real and for interval data.*

The basic tool to achieve high speed dynamic precision arithmetic for real and interval data is an exact *multiply and accumulate* (i.e., continued addition) operation for the data format double precision. Pipelining gives it high speed and exactitude brings very high accuracy into computation.

See, for example, Ulrich W. Kulisch, "Advanced Arithmetic for the Digital Computer," Springer, 2002.

We believe that these requirements reflect the needs of a significant portion of the numerical computing community. With 30 members and 18 affiliates from 13 countries, WG 2.5 represents a wide cross-section of this community. These issues have been discussed at length by the Working Group, which voted unanimously at its meeting in August 2007 to make its views known to your committee.

Please alert us if there is to be further periods of public comment on the draft standard. We would be happy to provide the names of experts from the Working Group who could discuss these issues with you in more technical detail.

Thank you for your kind attention.

Sincerely,

Dr. Ronald F. Boisvert
Chair, IFIP Working Group 2.5

Cc: D. Zuras
L. Tsai
W. Kahan
J. Darcy
P. Tang
D. Hough
J. Demmel
Members, WG 2.5

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September 9, 2009

R. Baker Kearfott, Chair
IEEE Interval Arithmetic Working Group
Box 4-1010
University of Louisiana at Lafayette
Lafayette, LA 70504-1010

Dear Professor Kearfott,

The IFIP Working Group 2.5 on Numerical Software very much appreciates the important work of the IEEE Interval Arithmetic Working Group in the development of a standard for interval arithmetic to support scientific computing with guarantees.

In that regard, WG 2.5 strongly supports inclusion of an exact dot product in the IEEE Standard P1788. The exact dot product is essential for fast long real and long interval arithmetic, as well as for assessing and managing uncertainty in computer arithmetic. It is a fundamental tool for computing with guarantees and can be implemented with very high speed.

We believe that this capability would provide much needed support for verifiable numerical computing, whose easy availability would serve to increase the reliability of scientific computing for many critical applications. This issue was discussed at length by the Working Group, which voted unanimously at its meeting in August 2009 to make its views known to your committee. With 30 members and 18 affiliates from 13 countries,

WG 2.5 represents a wide cross-section of the numerical computing community, and we believe our views are representative.

Thank you for your kind attention.

Sincerely,

Dr. Ronald F. Boisvert
Chair, IFIP Working Group 2.5

Cc: Members, WG 2.5
Nathalie Revol
George Corliss
Juergen Wolff von Gudenberg
Dan Zuras
John Pryce