Motion 10v2: Elementary Functions

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2 Rationale

We think that an interval standard should provide interval extensions for the most commonly used elementary functions. A trade-off will always exist between providing many functions and keeping the standard small and simple. To cope with this trade-off we introduced 2 sets of functions, a normative list of required functions and a non-normative list of recommended functions. Mandatory functions comprise the usual set of math standard functions provided in languages and libraries for scientific applications. Namely power, exponential, and logarithmic functions as well as the trig and hyperbolic functions together with their inverses.

The selection of functions was taken from IEEE 754-2008 standard, the Vienna proposal, and from the C++ interval extension approach.

We ask for a vote concerning the required functions as a whole list. People who want to exchange some of the functions of the required list are welcome to argue during the discussion.

Separate power functions are usually provided with integer, and real exponent. Whereas the latter is defined by an exponentiation with the log function and thus is restricted to non-negative bases, the former may be called with negative bases.

Even more options open up, if we consider rational exponents. $pown(-2, 3) = -8$, $powr(-32, 1, 5) = -2$, $rootn(-16, 4) = \emptyset$, e.g. Instead of giving the user the choice of the proper variation, we suggest to provide an "all inclusive" power function $\mathbf{x}^{\mathbf{y}}$ for arbitrary intervals with an appropriate semantics. This function has been proposed by Dan Zuras, see also [1]

Since this function may be a bit controversary, we ask for a separate vote.

Table 1 shows a possible implementation of the interval square root function for bare intervals using a correctly rounded implementaion of the floating point sqrt function.

The decoration trits for definedness and boundedness are touched by the calculation of square root. Just as an example we show how the definedness propagates in the last 3 colums of Table 2. Please note that details concerning the decoration are not topic of this motion, but of motion 8.

[1] W. Krämer, J. Wolff v. Gudenberg,Extended Interval Power Function,Proceedings of

X	sqrt(X)
$x_1 < x_2 < 0$	$\emptyset$
$x_1 < 0 \leq x_2$	$[0, \Delta sqrt(x_2)]$
$x_1 < 0, x_2 = \infty$	$[0, \infty]$
$0 <= x_1 \leq x_2$	$[\nabla sqrt(x_1), \Delta sqrt(x_2)]$
$0 <= x_1, x_2 = \infty$	$[\nabla sqrt(x_1), \infty]$

Table 1: sqrt for bare intervals

X	sqrt(X)	X.deco,def		
		no	maybe	yes
$x_1 < x_2 < 0$	$\emptyset$	no	no	no
$x_1 < 0 \leq x_2$	$[0, \Delta sqrt(x_2)]$	no	maybe	maybe
$x_1 < 0, x_2 = \infty$	$[0, \infty]$	no	maybe	maybe
$0 <= x_1 \leq x_2$	$[\nabla sqrt(x_1), \Delta sqrt(x_2)]$	no	maybe	yes
$0 <= x_1, x_2 = \infty$	$[\nabla sqrt(x_1), \infty]$	no	maybe	yes

Table 2: sqrt for decorated intervals

Validated Computing, Reliable Computing, 2003