

MOTION 24.02

ROUNDED FLOATING-POINT ARITHMETIC

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ABSTRACT. The IEEE standard 754 provides floating-point operations with rounding downwards and upwards. It is highly desirable making these easily accessible for the user. This proposal deals with the question how this can or should be done.

KEY WORDS: computer arithmetic, floating-point arithmetic, interval arithmetic, arithmetic standards.

In interval arithmetic the lower bound of the result of an interval operation is computed with rounding downwards and the upper bound with rounding upwards. It is desirable that with an interval arithmetic standard notations for arithmetic operations with the directed roundings be introduced. They could be:

$+\rangle$, $-\rangle$, $*\rangle$, $/\rangle$ for operations with rounding upwards, and
 $+\langle$, $-\langle$, $*\langle$, $/\langle$ for operations with rounding downwards.

This is much simpler and leads to much more easily readable expressions than for instance the use of operators like `.addup.` or `.adddown.` in Fortran. In languages like C++ which just provide operator overloading and do not allow user defined operators these operations could be written as functions:

addp, subp, mulp, divp, and *addn, subn, muln, divn.*

Here *p* stands for rounding toward positive and *n* for rounding toward negative.

Every IEEE 1788 compliant system **shall** provide the 8 operations with the directed roundings by distinct operation codes. Each of the 8 operations **shall** be callable as a single instruction. The rounding **shall** be an integral part of the arithmetic operation. Employing an operation with a directed rounding must be as simple as employing the corresponding operation with rounding to nearest.

The operations with the directed roundings **should** be implemented directly in hardware for efficiency reasons.

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