

11.11.1. *Bare and decorated interval literals.*

This subclause defines an **interval literal**: a (text) string that denotes a bare or decorated interval. This entails defining a **number literal**: a string, occurring within an interval literal, that denotes an extended-real number.

The interval denoted by an interval literal is its *value* (as an interval literal). Any other string has no value in that sense. For convenience a string is called *valid* or *invalid* (as an interval literal) according as it does or does not have such a value. The usage of value, valid and invalid for number literals is similar.

[*Note. The notion of number literal is only used in this subclause. Interval literals are used as input to text2interval, see §11.11.8. The definition of interval literal is not intended to constrain the syntax and semantics that a language might use to denote intervals in other contexts.*]

This definition of an interval literal  $s$  is placed in Level 2 because its use is not mandatory at Level 1, see §9.6.1. However the value of  $s$  is a Level 1 interval  $x$ . Within  $s$ , conversion of a number literal to its value shall be done as if in infinite precision, ignoring any default or explicit language-defined precision specification. Conversion of  $x$  to a Level 2 datum  $y$  is a separate operation. In all cases  $y$  shall contain  $x$ ; typically  $y$  is the  $\mathbb{T}$ -hull of  $x$  for some interval type  $\mathbb{T}$ .

[*Examples.*

- The interval denoted by the literal `[1.2345]` is the Level 1 single-point interval  $x = [1.2345, 1.2345]$ . However the result of  $\mathbb{T}\text{-text2interval}(\text{"[1.2345]"})$ , where  $\mathbb{T}$  is the 754 *infsup* binary64 type, is the interval, approximately  $[1.2344999999999999, 1.2345000000000002]$ , whose endpoints are the nearest binary64 numbers either side of 1.2345.
- Suppose the host language is C/C++. There, conversion of a number literal to a floating point format is considered part of its value, and a “floating-suffix” can be added to change this. E.g., `1.2345f` denotes 1.2345 rounded to the nearest float (single-precision)—instead of to the nearest double, which is the default. This is ignored within an interval literal: `1.2345` and `1.2345f` both denote the mathematical number 1.2345.

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The following forms of number literal shall be supported:

- (1) A number literal<sup>3</sup> in the syntax provided by the host language of the implementation, in decimal or any other supported radix.
- (2) Either of the strings `inf` or `infinity`, ignoring case, optionally preceded by `+`, with value  $+\infty$ ; or preceded by `-`, with value  $-\infty$ .
- (3) A string in the hexadecimal-significand form of IEEE 754-2008, §5.12.3, with the value specified there.
- (4) A string  $p/q$ , that is  $p$  and  $q$  separated by the `/` character, where  $p, q$  are decimal integer strings, with  $p$  optionally signed and  $q > 0$ . Its value is the exact rational number  $p/q$ .

[*Note. These categories need not be mutually exclusive, e.g., in C/C++, form (3) is included in (1).*]

**Bare intervals.** The forms of bare interval literal that shall be supported are listed below. To simplify stating the needed constraints, e.g.  $l \leq u$ , the number literals  $l, u, m, r$  are identified with their values. Space shown between elements of a literal, below, denotes zero or more characters that count as whitespace in the host language. [*Note. Some characters, e.g., newline, might be whitespace in some contexts but not others.*]

**Constants:** The string `[empty]`, ignoring case, whose value is the empty set  $\emptyset$ ; and the string `[entire]`, ignoring case, whose value is the whole line  $\mathbb{R}$ .

**Inf-sup form:** A string `[ l , u ]` where  $l$  and  $u$  are number literals with  $l \leq u$ ,  $l < +\infty$  and  $u > -\infty$ , see §9.2. Its value is the mathematical interval  $[l, u]$ . A string `[ x ]` is equivalent to `[ x , x ]`.

**Uncertain form:** a string  $m?rue$  where:  $m$  is a decimal number literal of form (1) above, without exponent;  $r$  is empty or is a non-negative decimal integer *ulp-count*;  $u$  is empty or is a *direction character*, either `u` (up) or `d` (down); and  $e$  is empty or is a decimal *exponent field* for the number  $m$ . No whitespace is permitted within the string.

One *ulp* equals one unit in the last place of the number  $m$  as written. The literal  $m?$  by itself denotes  $m$  with a symmetrical uncertainty of  $\frac{1}{2}\text{ulp}$ , that is the interval

<sup>3</sup>“floating constant” in C, “real literal constant” in Fortran, “real numeric literal” in Ada, etc.

$[m - \frac{1}{2}\text{ulp}, m + \frac{1}{2}\text{ulp}]$ . The literal  $m?r$  denotes  $m$  with a symmetrical uncertainty of  $r$  ulps, that is  $[m - r \times \text{ulp}, m + r \times \text{ulp}]$ . Adding **d** (down) or **u** (up) converts this to uncertainty in one direction only, e.g.  $m?\text{d}$  denotes  $[m - \frac{1}{2}\text{ulp}, m]$  and  $m?\text{ru}$  denotes  $[m, m + r \times \text{ulp}]$ . Finally the exponent field if present multiplies the whole interval by the appropriate power of 10, e.g.  $m?\text{rue}$  denotes  $10^e \times [m, m + r \times \text{ulp}]$ .

[Examples. Assuming the common types of decimal number literals are provided:

Form	Literal	Value
Inf-sup	[1.e-3, 1.1e-3]	[0.001, 0.0011]
	[-Inf, 2/3]	$[-\infty, 2/3]$
Uncertain	3.56?	[3.555, 3.565]
	3.56?1	[3.55, 3.57]
	3.560?2	[3.558, 3.562]
	3.560?2u	[3.560, 3.562]
	3.56?1e+1	[35.5, 35.7]

A formal description of bare interval literals is by the following grammar (using the notation of 754§5.12.3) which defines a `bareIntvlLiteral`, subject to the constraints on  $l, u, m, r$  stated above. `anycase(s)` matches all upper/lower case variants of the string  $s$ , e.g. `anycase("ab")` matches any of "ab", "Ab", "aB", "AB".

 **Meta-characters are temporarily colored red, to help spot any meta-syntax-errors in this grammar.**

```

sign          [+ -]
digit         [0123456789]
natural       {digit} +
langNumLit    {number literal of host language}
langNumLitMant {mantissa of number literal of host language}
langNumLitExp {exponent of number literal of host language}
sp            {whitespace character of host language} *
infLit        {sign} ? ( anycase("inf") | anycase("infinity") )
hexNumLit     {see 754§5.12.3}
ratNumLit     {sign} ? {digit} + "/" {digit} +
numberLiteral ( {langNumLit} | {infLit} | {hexNumLit} | {ratNumLit} | )
pointIntvl    "[" {sp} {numberLiteral} {sp} "]"
infSupIntvl   "[" {sp} {numberLiteral} {sp} "," {sp} {numberLiteral} {sp} "]"
uncertIntvl   {langNumLitMant} "?" {natural} ? [du]? {langNumLitExp} ?
constIntvl    ( anycase("[empty]") | anycase("[entire]") )
bareIntvlLiteral ( {pointIntvl} | {infSupIntvl} | {uncertIntvl} | {constIntvl} )

```

*Decorated intervals.* The syntax of decorated interval literals is given by extending the above grammar as follows, to define a `decoratedIntvlLiteral`.

```

connectChar   {language-defined character}
decorationLit ( "ill" | "trv" | "def" | "dac" | "com" )
decoratedIntvlLiteral {bareIntvlLiteral} {connectChar} {decorationLit}

```

That is,  $s$  is a bare interval literal  $sx$ , followed by a language-defined single character  $c$  followed by a 3-character decoration string  $sd$ . It is recommended that  $c$  be the underscore ' $_$ ', which is assumed here. The string  $sd$  is one of `ill`, `emp`, `trv`, `def`, `dac` or `com` representing the corresponding decoration  $dx$ . If  $sx$  has the value  $x$ , and if  $x_{dx}$  is a permitted combination according to §10.4, then  $s$  has the value  $x_{dx}$ . Otherwise  $s$  has no value as a decorated interval literal.