

Dear all,

There are some discrepancies in the notations of Drafts 4.02 and 4.04 and I think we should straighten these out before less suited denotations spread. Let me briefly comment on the history of these notations.

The real numbers $\mathbb{R}$ are defined as conditionally complete, linearly ordered field. Conditionally complete means: Every bounded subset has an infimum and a supremum.

Every conditionally completely ordered set can be completed by joining a least and a greatest element. In case of the real numbers $\mathbb{R}$ these are $-\infty$ and $+\infty$. However, these new elements are not real numbers. For instance $\infty - \infty \neq 0$, or $\infty/\infty \neq 1$. I think there was general agreement that the completion should be expressed by overlining the $\mathbb{R}$. So $\overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, +\infty\}$.

Since the early days of interval arithmetic the set of nonempty, closed and bounded real intervals has been denoted by $\mathbb{IR}$. The ordering of the set $\{\mathbb{IR}, \subseteq\}$ also is only conditionally complete. For every bounded subset the infimum is the intersection and the supremum is the interval hull. Completion of $\{\mathbb{IR}, \subseteq\}$ brings the empty set and unbounded intervals into the game. In my book (2008) and in the paper I prepared for the proceedings of the Dagstuhl meeting (January 2008) I denoted the completed set by $(\mathbb{IR})$. This was criticized within P1788. Then I suggested writing $\mathbb{JR}$ for the completed set. After some discussion I think we all agreed indicating the completion again by overlining the set $\mathbb{IR}$. In $\overline{\mathbb{IR}}$ the empty set is the least element. However, the empty set is not an interval. As $-\infty$ and $+\infty$ are not real numbers the empty set does not follow conventional rules of interval arithmetic, for instance, $\emptyset \cdot 0 \neq 0$.

For consistency the same scheme of denotations should be kept for the subsets representable on computers. This leads to the following denotations:

$\mathbb{R}$ the set of real numbers.

$\overline{\mathbb{R}}$ $\overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, +\infty\}$.

$\mathbb{IR}$ the set of nonempty, closed and bounded real intervals.

$\overline{\mathbb{IR}}$ the set of closed real intervals, including unbounded intervals and the empty set.

$\mathbb{F}$ the set of (finite) floating-point numbers representable in some floating-point format.

$\overline{\mathbb{F}}$ $\overline{\mathbb{F}} := \mathbb{F} \cup \{-\infty, +\infty\}$.

$\mathbb{IF}$ the intervals of $\mathbb{IR}$ whose bounds are in $\mathbb{F}$.

$\overline{\mathbb{IF}}$ the intervals of $\overline{\mathbb{IR}}$ whose bounds are in $\overline{\mathbb{F}}$ and the empty set.

Best regards
Ulrich