MOTION 24.03
ROUNDING CONTROL

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Introduction: On the computer the lower bound of the result of an interval operation is computed with rounding downwards and the upper bound with rounding upwards. It is desirable that with an interval arithmetic standard a more transparent use of rounding be introduced. Special operators with directed roundings are needed for arithmetic operations and for type conversions (from complete to floating-point, between different floating-point precisions, or between different radixes).

The Motion: Every IEEE 1788 compliant system shall provide the four basic arithmetic operations addition, subtraction, multiplication, and division with rounding downwards and upwards. Type conversions with directed roundings shall also be provided.

Sample realizations: In [4, 6] the following notations are used for the required operations:

`+>`, `->`, `*>`, `/>` for operations with rounding upwards, and

`+<`, `-<`, `*<`, `/<` for operations with rounding downwards.

`#*` for rounding to the nearest floating-point number,

`#>` for rounding upwards to the next floating-point number,

`#<` for rounding downwards to the next floating-point number,

`##` for rounding to the smallest floating-point interval.

References

[1] American National Standards Institute / Institute of Electrical and Electronics Engineers: *A Standard for Binary Floating-Point Arithmetic.* ANSI/IEEE Std. 754-1987, New York, 1985. (reprinted in SIGPLAN **22**, 2, pp. 9-25, 1987). Also adopted as IEC Standard 559:1989.

[2] IEEE Floating-Point Arithmetic Standard 754, 2008.

[3] Pryce, J. D. (Ed.): *P1788, IEEE Standard for Interval Arithmetic*, http://grouper.ieee.org/groups/1788/email/pdfOWdtH2mOd9.pdf.

[4] IBM, *ACRITH–XSC: IBM High Accuracy Arithmetic – Extended Scientific Computation. Version 1, Release 1*, IBM Deutschland GmbH (Department 3282, Schönaicher Strasse 220, D-71032 Böblingen), 1990.
1. General Information, GC33-6461-01.
2. Reference, SC33-6462-00.
3. Sample Programs, SC33-6463-00.
4. How To Use, SC33-6464-00.
5. Syntax Diagrams, SC33-6466-00.

[5] Kirchner, R., Kulisch, U.: *Hardware support for interval arithmetic.* Reliable Computing 12:3, 225–237, 2006.

[6] Klatte, R.; Kulisch, U.; Neaga, M.; Ratz, D.; Ullrich, Ch.: *PASCAL-XSC – Sprachbeschreibung mit Beispielen*, Springer, Berlin Heidelberg New York, 1991. English translation Springer 1992, Russian translation MIR, Moskau, 1995, third edition 2006.

[7] Kulisch, U.,W.: *Computer Arithmetic and Validity – Theory, Implementation and Applications.* De Gruyter, Berlin, New York, 2008.

[8] Neumaier, A.: *Vienna Proposal for Interval Standardization.*

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