Improved Analysis of Component of dTER for Synchronization Transport over an IEC/IEEE 60802 Network due to GM Time Error

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Outline

- Introduction
- Analysis and results
- Conclusion and next steps

Introduction - 1

- Reference [2] contains the latest simulation results for relative dynamic time error (dTE_R) for the transport of time synchronization over an IEC/IEEE 60802 network
  - This presentation describes 11 simulation cases
  - Cases 1 – 8 are revisions of simulations presented in [1]; cases 9 – 11 are new
- In all the simulation cases in [2], the effect of GM dTE on dTE_R is ignored
  - As explained in [1], the GM dTE is a component of the total time error input to the endpoint filter, and results in additional relative dynamic time error
  - The filtering of the GM dTE results in a phase shift, which results in additional dTE_R at the output of the endpoint filter, relative to the GM output
  - It is also explained in [1] that, if the effect of GM dTE is included in the simulations, interpolation is needed because the times at which dTE is computed at the GM output and the endpoint output are not the same
    - dTE_R must be computed by taking the difference between the two time errors at the same instant of time
  - Finally, and most importantly, sufficient precision is needed when computing dTE_R , because total dTE at the GM and endpoint filter outputs is much greater than the difference between these quantities (i.e., dTE_R)

Introduction - 2

- However, an approximate computation of the component of dTE_R due to GM dTE was done in [1] by considering the effect of the endpoint filter on the GM dTE, and computing the difference between the output of the endpoint filter for GM dTE as the input, and GM dTE itself
  - This is equivalent to filtering GM dTE with a high-pass filter whose corner frequency and gain peaking are the same as those of the endpoint filter
  - The result, for the endpoint filter parameters, and GM maximum frequency offset and frequency drift rate, assumed in [1], was approximately 46 ns (and was rounded to 50 ns)
  - Based on this, 50 ns was added to the simulation results obtained assuming GM dTE is zero
- However, in the discussion of [1] it was pointed out that there can be additional phase shift, and therefore additional error, due to the time delay between the GM and the endpoint
  - This time delay is mainly due to the residence times in the successive PTP Relay Instances
  - There also is component due to link delay, but this is much smaller than residence time and can be neglected
- The present presentation extends the analysis of [1] to include the effect of time delay due to residence times in the successive PTP Relay Instances

Analysis and Results - 1

- Some of the material on this and succeeding slides is taken from [1]
- First, the GM phase and frequency variations waveforms are needed; this computation was done in [1], and is copied below

$$y(t) = A\sin(2\pi f_0 t)$$

$$\dot{y}(t) = 2\pi f_0 A\cos(2\pi f_0 t)$$

where

$y(t) =$ instantaneous frequency offset in ppm

$A =$ frequency offset amplitude in ppm

Setting A to 50 ppm and the maximum frequency drift rate $(2\pi f_0 A)$ to 3 ppm/s produces

$$2\pi f_0 (50 \text{ ppm}) = 3 \text{ ppm/s}$$

$$f_0 = \frac{3}{100\pi} \text{ Hz} = 9.549\times 10^{-3} \text{ Hz} = 9.549 \text{ mHz}$$

Note that the maximum time offset of the GM is

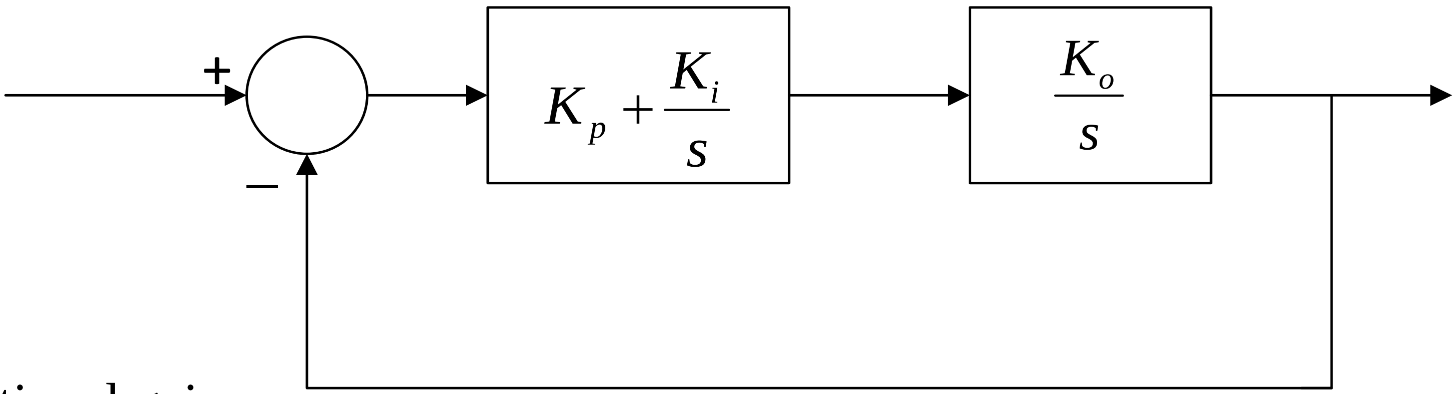
$$\frac{50}{2\pi(9.549\times 10^{-3} \text{ Hz})} = 8.334\times 10^{-4} \text{ s} = 0.8334 \text{ ms}$$

As was seen in previous presentations ([4], [5]), the GM error waveform is of relatively low frequency and large amplitude

Analysis and Results - 2

- Next, the properties of the endpoint filter are needed, in order to compute its transfer function
- These are given in [2]; the following 3 slides are copied from [2]

Analysis and Results - 3



$K_p =$ proportional gain

$K_i =$ integral gain

$K_o =$ VCO/DCO gain

Transfer function:

$$H(s) = \frac{K_p K_o s + K_i K_o}{s^2 + K_p K_o s + K_i K_o} = \frac{2\varsigma\omega_n s + \omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

with

$$\omega_n = \sqrt{K_i K_o} \qquad \varsigma = \frac{K_p}{2}\sqrt{\frac{K_o}{K_i}}$$

Analysis and Results - 4

- Often the filter parameters (and requirements) are expressed in terms of 3 dB bandwidth ($f_{3\text{dB}}$) and gain peaking (H_p)
  - These are related to damping ration (ζ) and undamped natural frequency (ω_n) by (see [6] and [7] of reference [2] here):

$$f_{3\text{dB}} = \frac{\omega_n}{2\pi}\left[1 + 2\varsigma^2 + \sqrt{\left(1 + 2\varsigma^2\right)^2 + 1}\right]^{1/2}$$

$$H_p(\text{dB}) = 20\log_{10}\left\{\left[1 - 2\alpha - 2\alpha^2 + 2\alpha\sqrt{2\alpha + \alpha^2}\right]^{-1/2}\right\}$$

where

$$\alpha = \frac{1}{4\varsigma^2} = \frac{K_i}{K_p^2 K_o}$$

Analysis and Results - 5

- As in previous simulation models, the VCO gain was folded into the proportional gain and integral gain (this is equivalent to setting the VCO gain to 1)
- Filter assumption:
  - $K_p K_o = 11$, $K_i K_o = 65$
  - Using the equations on the previous slides, we obtain
    - $\zeta = 0.68219$
    - $\omega_n = 8.06226$ rad/s ≈ 8.06 rad/s
    - H_p (gain peaking) = 1.28803 dB = (approx) 1.3 dB
    - $f_{3dB} = 2.5998$ Hz ≈ 2.6 Hz
- Note that this filter is underdamped, and has appreciable gain peaking
  - However, the damping ratio (ζ) is close to $1/\sqrt{2}$ = (approx) 0.707); this is often used to obtain a fast response with small overshoot, in cases where the filters are not cascaded (the endpoint filters are not cascaded)

Analysis and Results - 6

- The transfer function of the endpoint filter, and the corresponding error transfer function (i.e., the transfer function between the input and the error, which is the difference between the input and output) were given in [1] as:

The transfer function for the 2nd order endpoint filter, $H(s)$, is

$$H(s) = \frac{2\varsigma\omega_n s + \omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

where ω_n is the undamped natural frequency in rad/s and ς is the damping ratio (see slides 7 - 9).

The difference between the GM waveform, which to first approximation is the input to the input to the endpoint filter (ignoring the effect of the network, which is much smaller) and the output of the endpoint filter, is the high-pass error transfer function $H_e(s) = 1 - H(s)$, i.e.

$$H_e(s) = \frac{s^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}.$$

Setting $s = j\omega = 2\pi jf$, $\omega_n = 2\pi f_n$ produces

$$H_e(j\omega) = \frac{-\omega^2}{-\omega^2 + 2\varsigma\omega_n\omega j + \omega_n^2}.$$

- However, as indicated in the introduction, the present analysis needs to also take into account the time delay between the GM and the endpoint filter
- This analysis is shown in the following slides

Analysis and Results - 7

- Let the time delay between the GM output and the endpoint filter output be T
- The transfer function of a pure time delay is e^{-sT} (refer to any reference on Laplace Transforms for this result)

Including the effect of the time delay, the transfer function between the GM output and the endpoint filter output, $H(s)$, is

$$H(s) = \frac{2\varsigma\omega_n s + \omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} e^{-sT}$$

where ω_n is the undamped natural frequency in rad/s and ς is the damping ratio (see slides 7 - 9).

The difference between the GM waveform, which to first approximation is the output of the GM, and the output of the endpoint filter (neglecting the PTP transport as that has been simulated separately), is the error transfer function $H_e(s) = 1 - H(s)$, i.e.

$$H_e(s) = 1 - H(s) = 1 - \frac{2\varsigma\omega_n s + \omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} e^{-sT}.$$

Setting $s = j\omega = 2\pi jf$, $\omega_n = 2\pi f_n$ produces

$$H_e(j\omega) = 1 - \frac{-\omega^2 +}{-\omega^2 + 2\varsigma\omega_n\omega j + \omega_n^2} e^{-j\omega T} = \frac{-\omega^2 + 2\varsigma\omega_n\omega j(1 - e^{-j\omega T}) + \omega_n^2(1 - e^{-j\omega T})}{-\omega^2 + 2\varsigma\omega_n\omega j + \omega_n^2}.$$

Dividing the numerator and denominator of the final equation by ω_n^2, and setting $x = \omega / \omega_n$, produces

$$H_e(j\omega) = \frac{-x^2 + 2\varsigma xj(1 - e^{-j\omega T}) + (1 - e^{-j\omega T})}{-x^2 + 2\varsigma xj + 1}$$

Analysis and Results - 8

- The component of max|dTE_R| due to the GM time error is equal to the GM time error amplitude, i.e., 0.8334 ms (see slide 5), multiplied by the magnitude (i.e., absolute value) of the error transfer function evaluated at the GM time error waveform frequency and the time delay
- In cases 9 – 11 of [2], the residence time is either 4 ms or 10 ms
  - For the GM, followed by 99 PTP Relay Instances, followed by a PTP End Instance, the maximum total delay due to residence time (as indicated previously, it is assumed that link delay is much smaller and therefore negligible) is 99(10 ms) = 990 ms $\approx$ 1 s
- For convenience, the component of max|dTE_R| due to the GM time error was evaluated for time delay ranging from 0 to 1 s, in increments of 1 ms (0.001 s)
- The evaluation was done using the application Mathcad® 8 [3]
- The evaluation worksheets are shown on the following slides

Analysis and Results - 9

$i := 0, 1 .. 1000$

$T_i := 0.001 \cdot i$

$wn := (65)^{0.5}$

$wn = 8.06226$

$f := 0.009549$

$w := 2 \cdot \pi \cdot f$

$w = 0.06$

$x := \frac{w}{wn}$

$x = 7.44185 \cdot 10^{-3}$

$zeta := 0.68219$

$E_i := 1 - \exp\left(w \cdot T_i \cdot j\right)$

$N_i := -x^2 + 2 \cdot zeta \cdot x \cdot E_i \cdot j + E_i$

$D_i := 1 - x^2 + 2 \cdot zeta \cdot x \cdot j$

$He_i := \frac{N_i}{D_i}$

$He\_mag_i := \left| He_i \right|$

$He\_mag_0 = 5.53814 \cdot 10^{-5}$

$maxdter_i := He\_mag_i \cdot 0.8334$

Note that He_mag_0 is the transfer function magnitude for zero time delay; the result (5.538×10^{-5}) agrees with the result obtained in [1] (see slide 61 of [1])

Analysis and Results - 10

$He\_mag_i =$	$maxdter_i =$
$5.53814\cdot10^{-5}$	$4.61548\cdot10^{-5}$
$8.12379\cdot10^{-5}$	$6.77037\cdot10^{-5}$
$1.31651\cdot10^{-4}$	$1.09718\cdot10^{-4}$
$1.87788\cdot10^{-4}$	$1.56503\cdot10^{-4}$
$2.45758\cdot10^{-4}$	$2.04814\cdot10^{-4}$
$3.04515\cdot10^{-4}$	$2.53782\cdot10^{-4}$
$3.63678\cdot10^{-4}$	$3.03089\cdot10^{-4}$
$4.23076\cdot10^{-4}$	$3.52592\cdot10^{-4}$
$4.82624\cdot10^{-4}$	$4.02219\cdot10^{-4}$
$5.42271\cdot10^{-4}$	$4.51929\cdot10^{-4}$
$6.01988\cdot10^{-4}$	$5.01697\cdot10^{-4}$
$6.61757\cdot10^{-4}$	$5.51508\cdot10^{-4}$
$7.21563\cdot10^{-4}$	$6.01351\cdot10^{-4}$
$7.814\cdot10^{-4}$	$6.51219\cdot10^{-4}$
$8.4126\cdot10^{-4}$	$7.01106\cdot10^{-4}$
$9.01138\cdot10^{-4}$	$7.51008\cdot10^{-4}$
$9.61031\cdot10^{-4}$	$8.00924\cdot10^{-4}$
$1.02094\cdot10^{-3}$	$8.50849\cdot10^{-4}$
$1.08085\cdot10^{-3}$	$9.00783\cdot10^{-4}$
$1.14078\cdot10^{-3}$	$9.50725\cdot10^{-4}$

Below are values for total delay of 400 ms (case 9) and 1 s (case 10)

$$He\_mag_{400} = 0.024$$

$$maxdter_{400} = 0.02$$

$$He\_mag_{1000} = 0.05999$$

$$maxdter_{1000} = 0.05$$

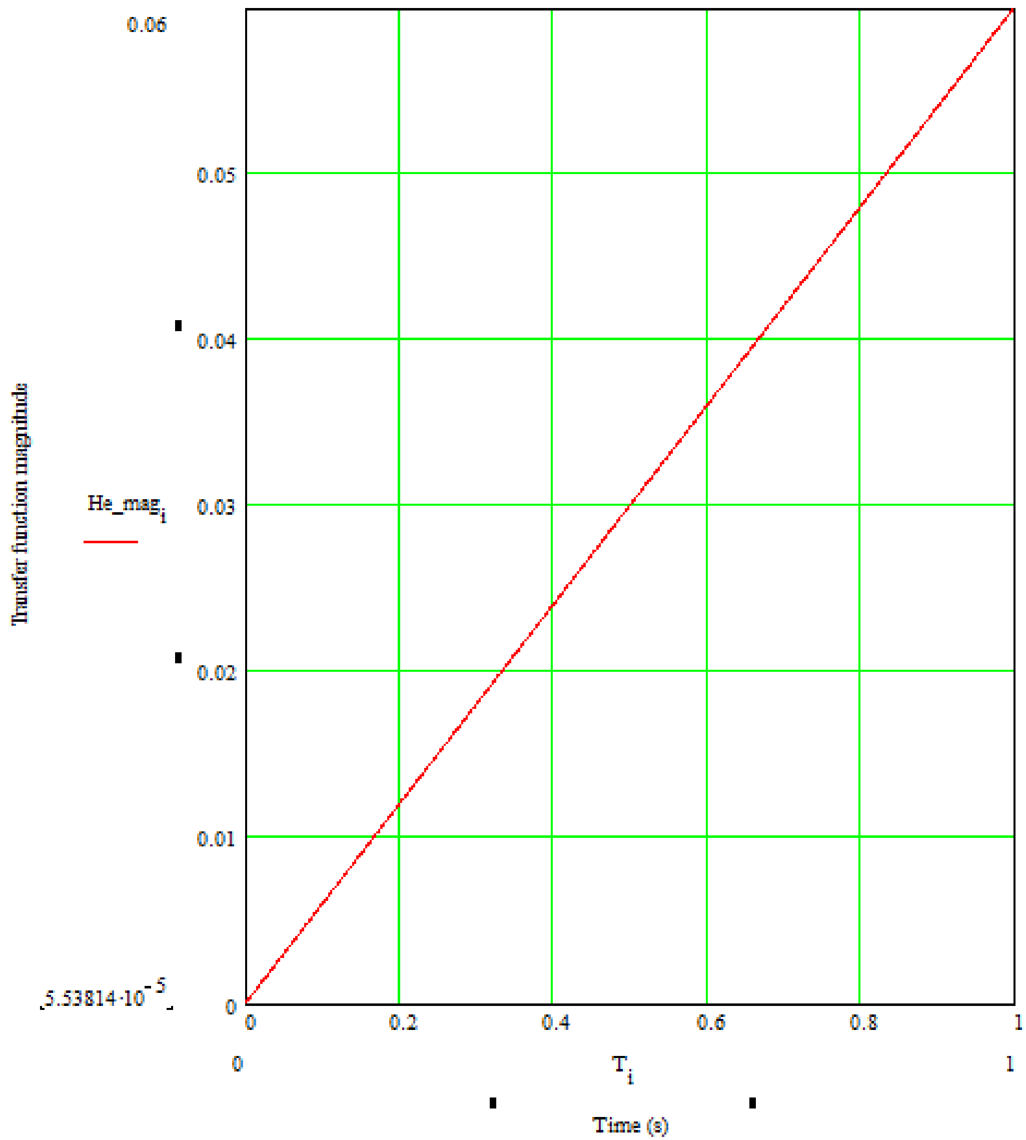
$$He\_mag_{256} = 0.01536$$

$$maxdter_{256} = 0.0128$$

$$He\_mag_{640} = 0.0384$$

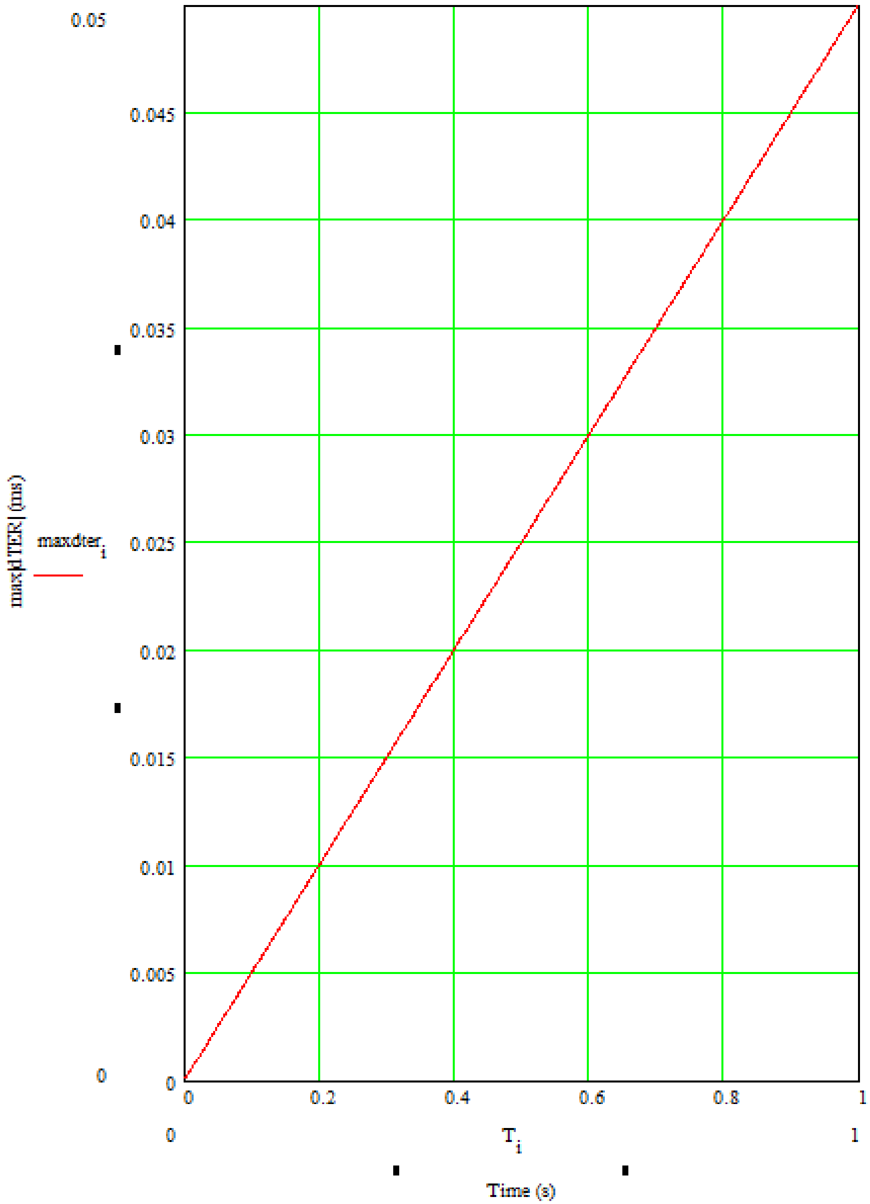
$$maxdter_{640} = 0.032$$

Analysis and Results - 11



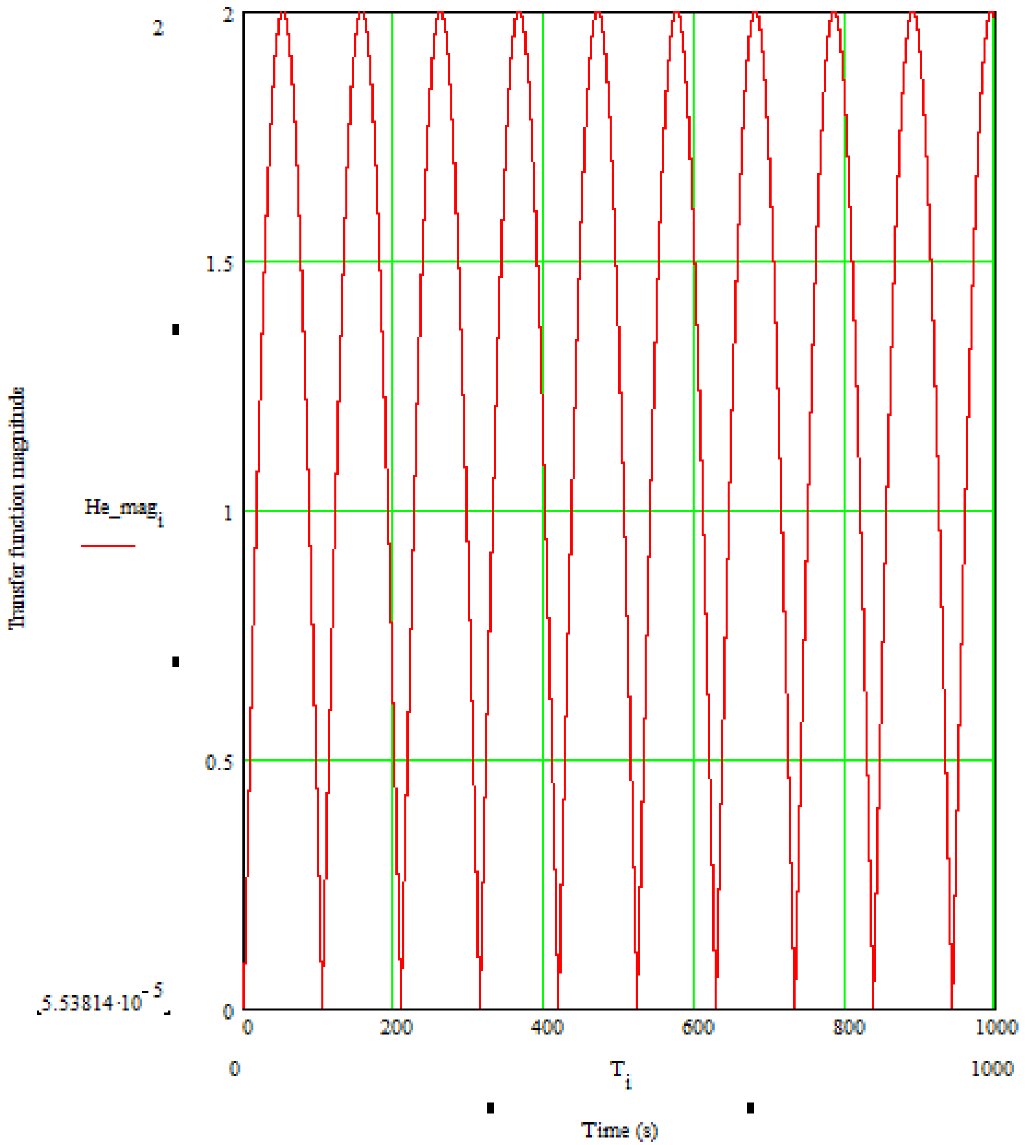
Transfer function magnitude

Analysis and Results - 12



Max|dTE$_R$ (ms)

Analysis and Results - 13



Transfer function for 0 – 1000 s, to show its periodicity

Period = $1/f_0$ (see slide 5)

= $\mathbf{1/(0.009549\ Hz)}$

= 104.72 s

Analysis and Results - 14

- For case 9, the total delay for 100 nodes is 99(4 ms) = 396 ms ≈ 400 ms = 0.4 s
- For case 10, the total delay for 100 nodes is 99(10 ms) = 990 ms ≈ 1 s
- The approximate error component due to GM dTE is, from the above results for maxdter, are 0.02 ms = 20 μs and 0.05 ms = 50 μs, respectively
- For 65 nodes (64 hops), the delays are 64(4 ms) = 256 ms = 0.256 s, and 64(10 ms) = 640 ms = 0.64 s, respectively
- The components are $\max|dTE_R|$ due to GM dTE, at node 64, is approximately 0.0128 ms = 12.8 μs and 0.032 ms = 32 μs, respectively
- It is seen that the component of $\max|dTE_R|$ due to the GM dTE exceeds the 1 μs objective for $\max|TE_R|$ by more than a factor of 10 (for both 64 hops and 100 hops)

Conclusion and Next Steps - 1

- The results for the component of max|dTE_R| due to the GM dTE exceeds the 1 μs objective for max|TE_R| by more than a factor of 10 (for both 64 hops and 100 hops)
  - It also exceeds the results for cases 9 – 11 in [2] by more than a factor of 10 (see slide 43 of [2])
- The analysis was approximate, in that only the effect of the endpoint filter and of delay through the network due to residence time was considered
- It is possible to do a more accurate analysis via simulation
  - It would be necessary to ensure that the computations are done with sufficient precision
  - It would also be necessary to perform the necessary interpolation
  - **However, given how much the error due to GM dTE exceeds the 1 μs objective for max|TE_R| and also the results of [2] for cases 9 – 11, it is unlikely that simulation results would be within the limit**

Conclusion and Next Steps - 2

- The reason the component of max|dTE_R| due to GM dTE is so large is that this error waveform has extremely large amplitude compared to the 1 μs objective (i.e., 0.8334 ms = 833.4 μs, see slide 5)
- The large amplitude of the GM dTE waveform is due to the assumptions:
  - 50 ppm maximum frequency offset
  - 3 ppm/s maximum frequency drift rate
  - Sinusoidal phase and frequency variation
- Results would very likely be different if the GM time and frequency error model were different
  - e.g., max|dTE_R| due to the GM dTE would very likely be much smaller if the GM error model were a random noise (e.g., power-law noise model
- If the 50 ppm maximum frequency offset and 3 ppm/s maximum frequency drift rate assumptions are needed for the GM (i.e., here we are considering only GM PTP Instances), the sinusoidal (i.e., deterministic and periodic) error model should be re-examined)

Thank you

References - 1

[1] Geoffrey M. Garner, *New Simulation Results for Time Error Performance for Transport over an IEC/IEEE 60802 Network Based on Updated Assumptions, Revision 3*, IEEE 802.1 presentation, September 2020 (available at https://www.ieee802.org/1/files/public/docs2020/60802-garner-new-simulation-results-dte-updated-assumptions-60802-network-0920-v03.pdf)

[2] Geoffrey M. Garner, Further Simulation Results for Dynamic Time Error Performance for Transport over an IEC/IEEE 60802 Network Based on Updated Assumptions, Revision 1, IEEE 802.1 presentation, September 2020 (available at https://www.ieee802.org/1/files/public/docs2020/60802-garner-further-simulation-results-time-sync-transport-1120-v01.pdf)

[3] Mathcad® 8 was a product of MathSoft, Inc. Information on the latest version of Mathcad is available at http://www.Mathcad.com.