Phase and Frequency Offset, and Frequency Drift Rate Time History Plots Based on New Frequency Stability Data

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Introduction - 1

- Reference [2] presented initial results for the time history of frequency offset and frequency drift rate for a crystal oscillator (XO), based on:
  - A simple, quadratic, model for the temperature dependence of frequency offset with temperature, given in [1] (i.e., frequency offset versus temperature was a quadratic polynomial in this model)
  - A proposed temperature profile (temperature as a function of time) given in [1]
- In the discussion of [1] and [2], there were questions on whether the quadratic dependence of frequency on temperature (i.e., frequency stability) is representative of crystal oscillators currently available
- In response to this discussion, reference [3] was presented
- Reference [3] distinguished between a crystal unit, which is a passive device, and a crystal oscillator, which contains the crystal unit and associated electronics
  - Reference [3] presented frequency stability data for a typical AT-cut crystal (i.e., crystal unit), which showed (at least approximately) a cubic (i.e., 3^{rd}-order) dependence on temperature in the temperature range of interest (see slide 6 of [3])

Introduction - 2

- In addition, reference [4] was cited during the discussions of [1], [2], and [3]
  - This shows data on the dependence of frequency offset on temperature for AT-cut crystals that, at least for some cut angles, is consistent (though not identical) with the data shown in [3]
  - Note that [4] is a very old paper (i.e., from 1956)
- Based on the above, it was suggested (by the author of the current presentation) that the frequency offset dependence on temperature provided in [3], along with the temperature versus time profile of [1], could be used to obtain frequency offset versus time
  - This could then be integrated to obtain phase/time offset versus time (i.e. time history), to be used in subsequent simulations of time error performance in an IEC/IEEE 60802 network
  - It was indicated that appropriate margin should be applied to the frequency versus temperature dependence of [3] for use in modeling or budgeting, as this data is for a single crystal unit

Introduction - 3

- The author of [3] kindly provided the Excel file attached to this presentation, which contains the numerical data for the frequency versus temperature characteristics on slide 6 of [3]
  - Again, note that some margin should be applied if this data is used for modeling or budgeting, because this data is for a single crystal unit
  - In the work presented in the current presentation, a margin of 10% is used

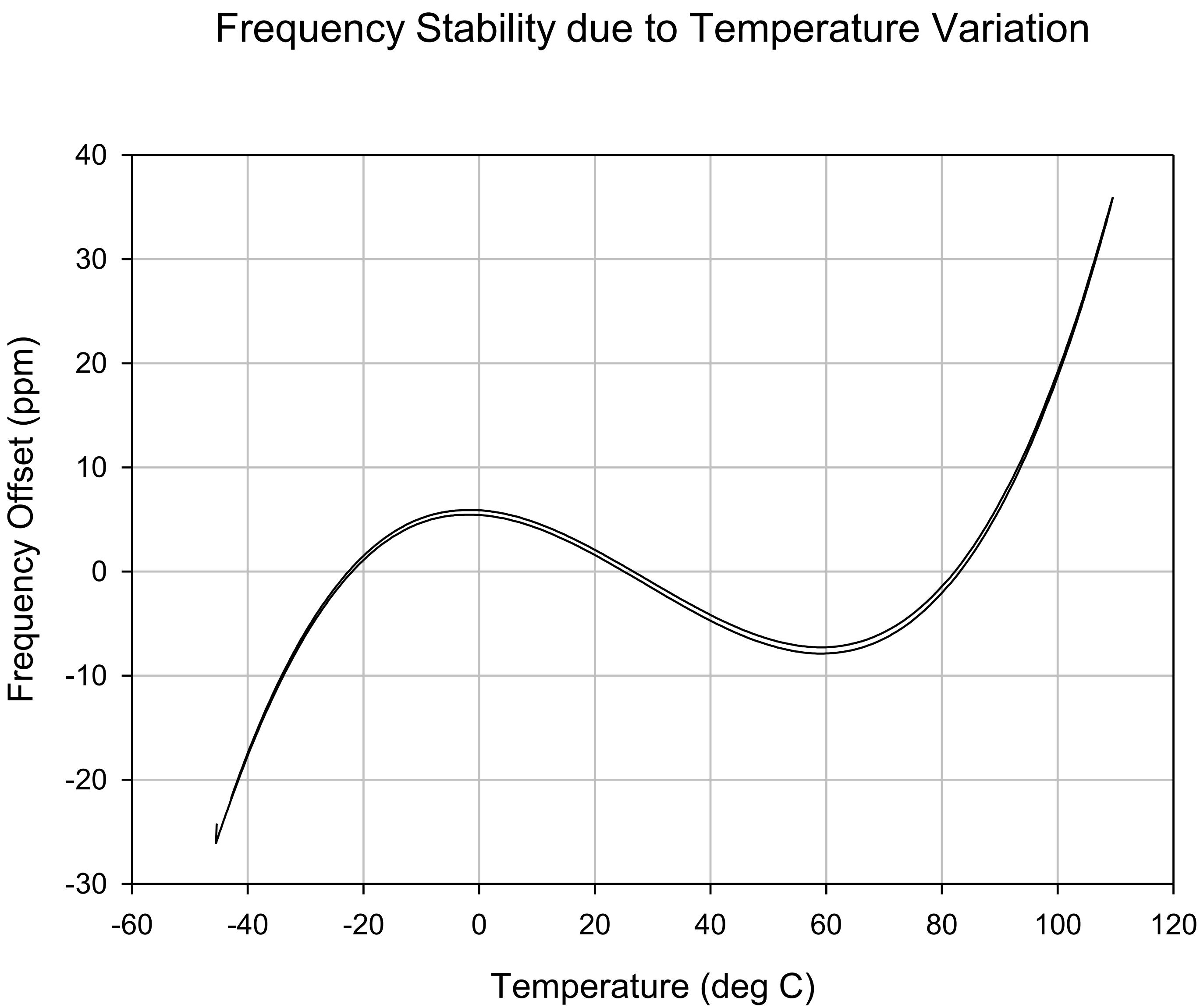
Introduction - 4

- The current presentation obtains frequency drift rate, frequency offset, and phase offset time histories for the above assumptions
  - The results will be shown for both the frequency versus temperature data as given and with a 10% margin applied, so that the effect of the margin can be seen

Temperature Dependence of Frequency - 1

- The temperature dependence of frequency given the data provided by the author of [3] (see the attached Excel file) is shown on the next slide
  - This data corresponds to the blue plot on slide 6 of [3]
- Note that the dependence shows a small amount of hysteresis
- The plot has the characteristic of a third-order polynomial
- Note that temperature dependence of frequency is often referred to as frequency stability due to temperature variation

Temperature Dependence of Frequency - 2



Temperature Dependence of Frequency - 3

- To facilitate the use of this data in the subsequent calculations of this presentation and in future simulations, the LINEST function of Excel was used to obtain a third-order polynomial least-squares fit
- The polynomial is of the form

$$y(T) = a_3 T^3 + a_2 T^2 + a_1 T + a_0$$

where

T = temperature in °C

y = frequency offset in ppm

$a_3, a_2, a_1,$ and a_0 are coefficients to be determined from a least-squares fit

- The least-squares fit of LINEST produces:

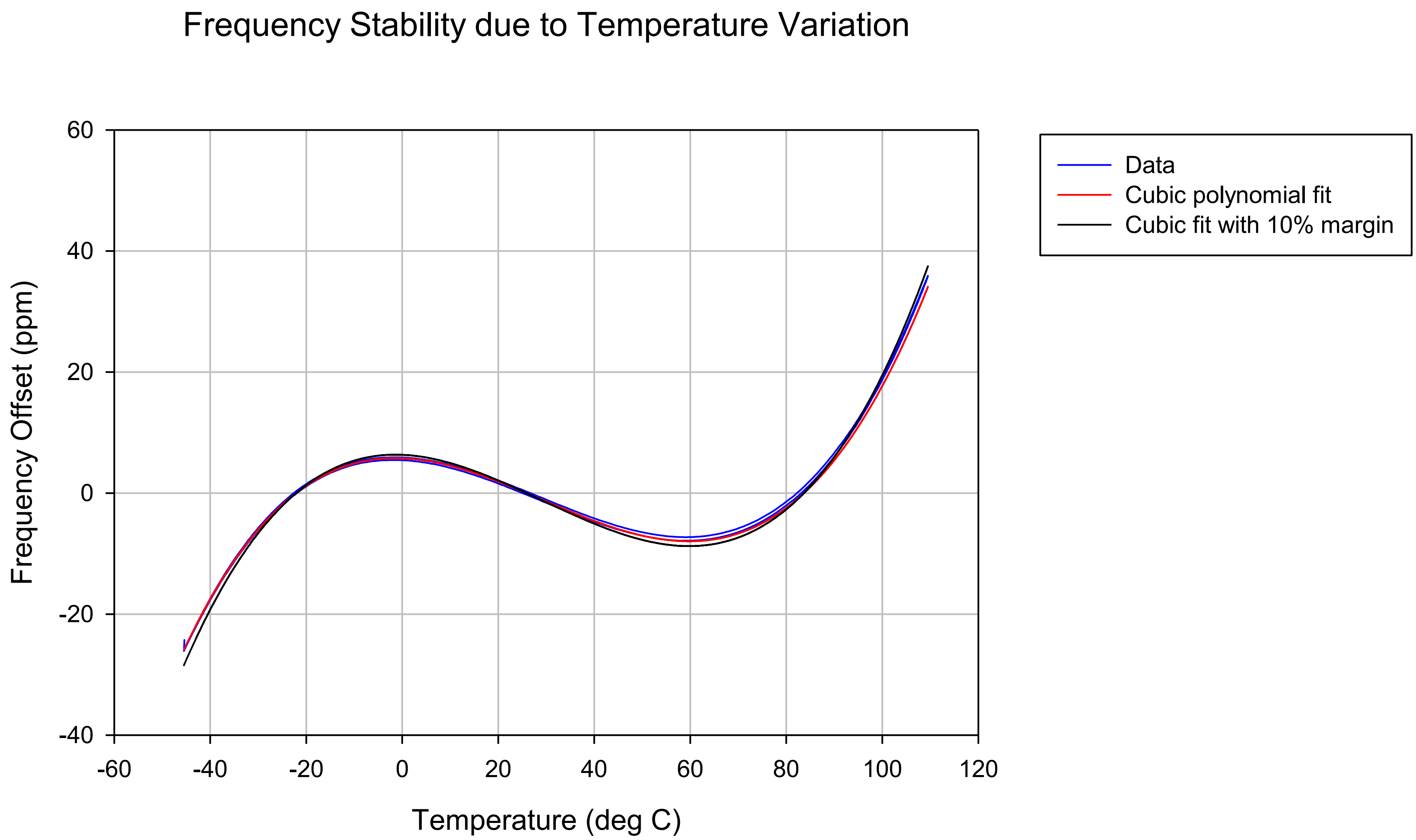
$$a_3 = 0.00012$$
$$a_2 = -0.0105$$
$$a_1 = -0.0305$$
$$a_0 = 5.73845$$

Temperature Dependence of Frequency - 4

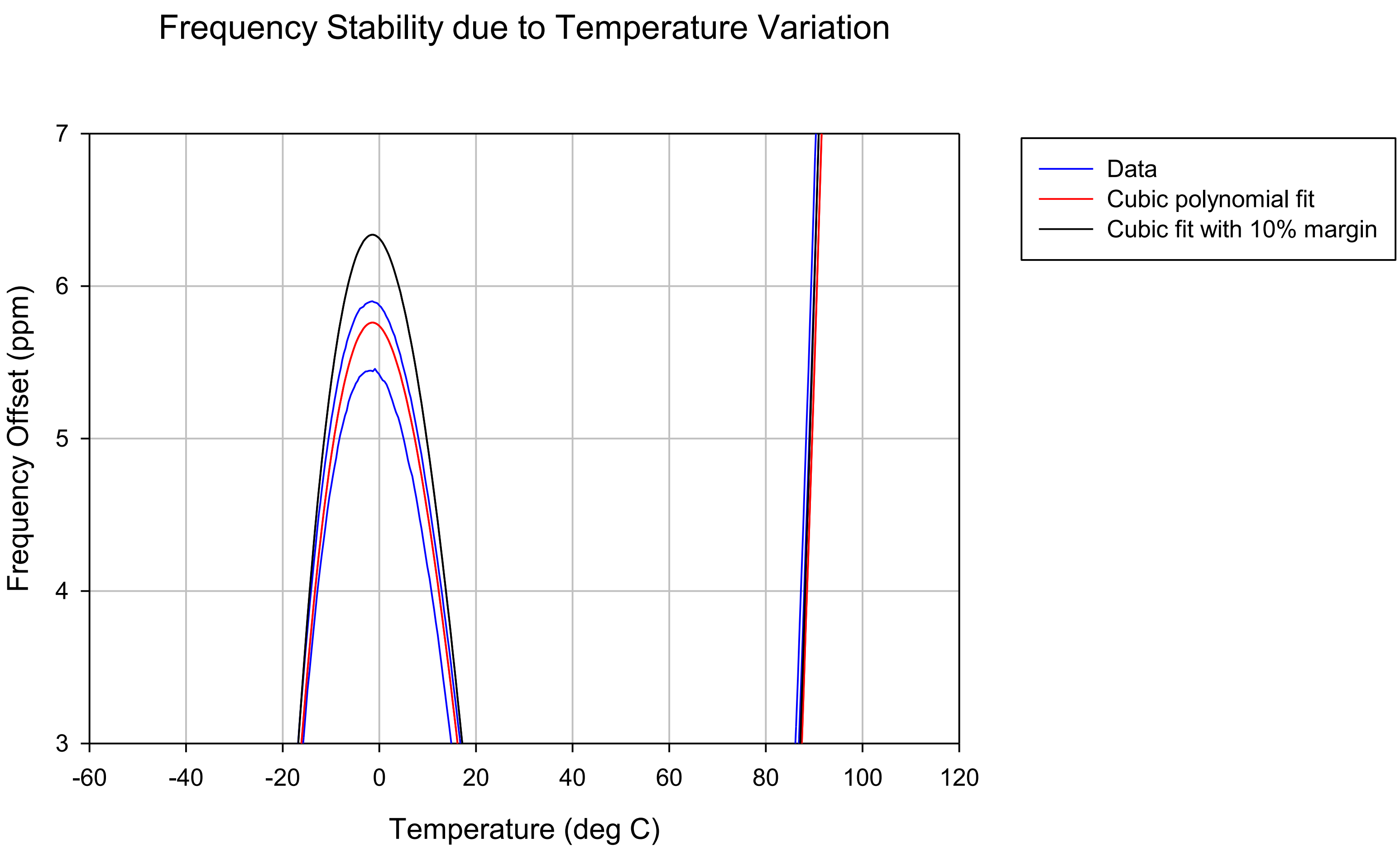
- The 3rd-order polynomial with these coefficients, along with the Excel file data, is shown plotted on the next slide
- At least visually, the results confirm that the cubic polynomial is a good fit
- In addition, a second polynomial, with 10% margin, is obtained by multiplying each of the coefficients a_i on the previous slide by 1.1
  - This second polynomial, with 10% margin, also is plotted
- The differences between the plots is not easily visible on the scale of the plots
- To see the differences more easily, a second plot is presented (on the slide after the next one) showing the detail of -18°C to +18 -18°C, and 87°C to 92 °C
  - On this scale, the factor of 1.1 that relates the curve with margin (black curve) and curve without margin (red curve) is easily seen
- As indicated earlier, the results for the time dependence of frequency drift rate, frequency offset, and phase offset will be shown for cases both with and without the 10% margin
- Finally, the rate of change of frequency with respect to temperature is plotted

$$\frac{dy}{dt} = 3a_3[T(t)]^2 + 2a_2[T(t)] + a_1$$

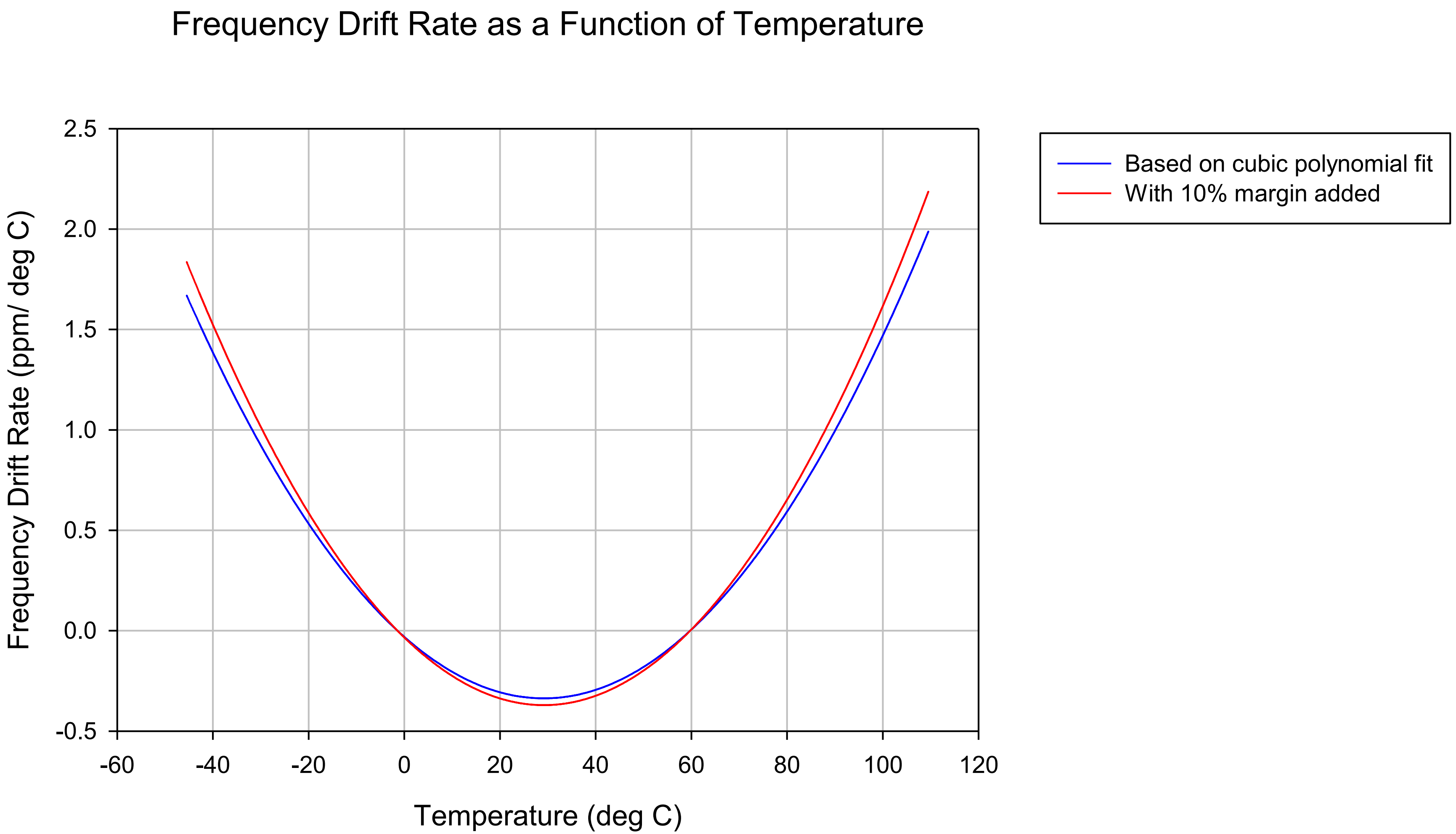
Temperature Dependence of Frequency - 5



Temperature Dependence of Frequency - 5



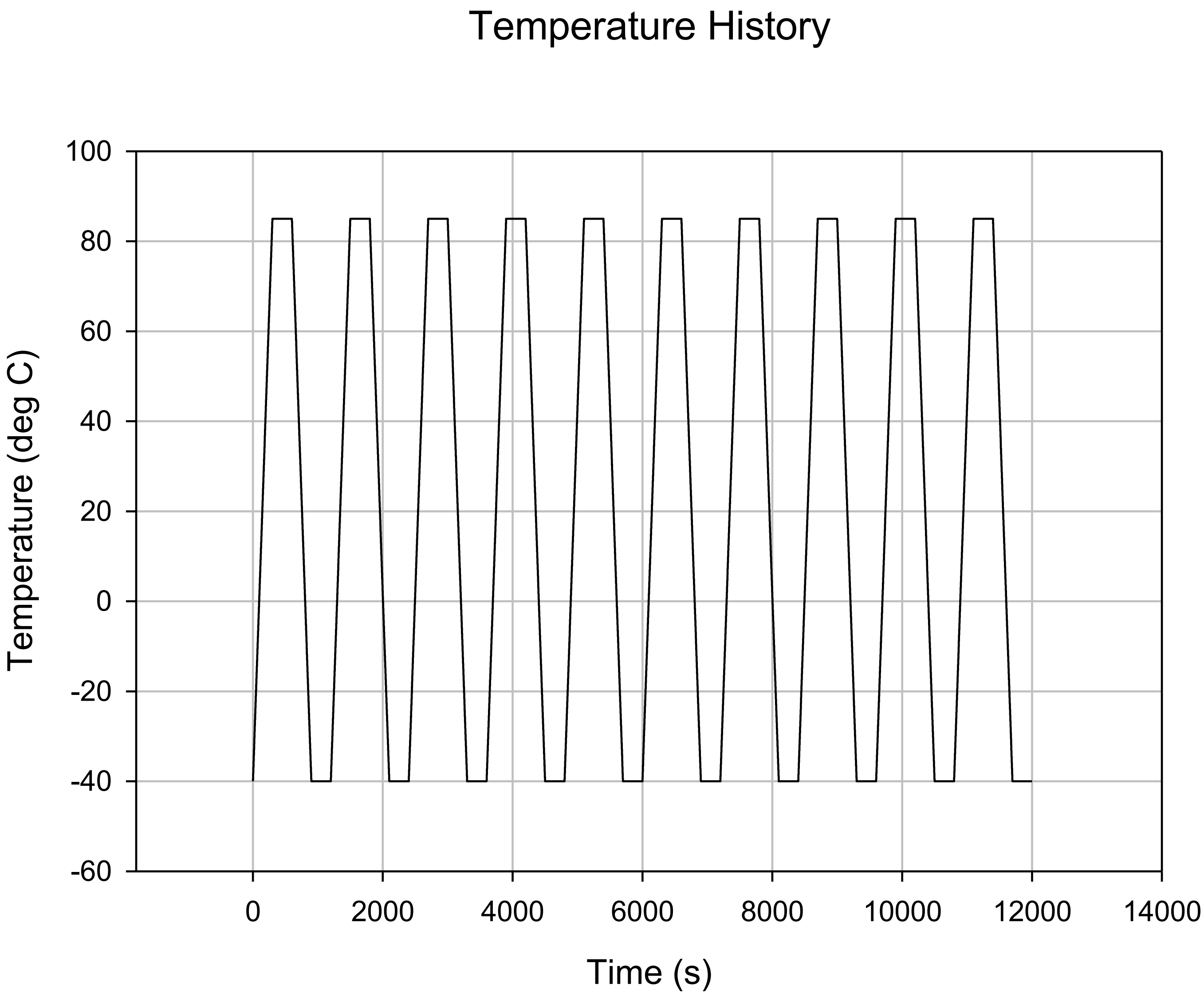
Temperature Dependence of Frequency - 5



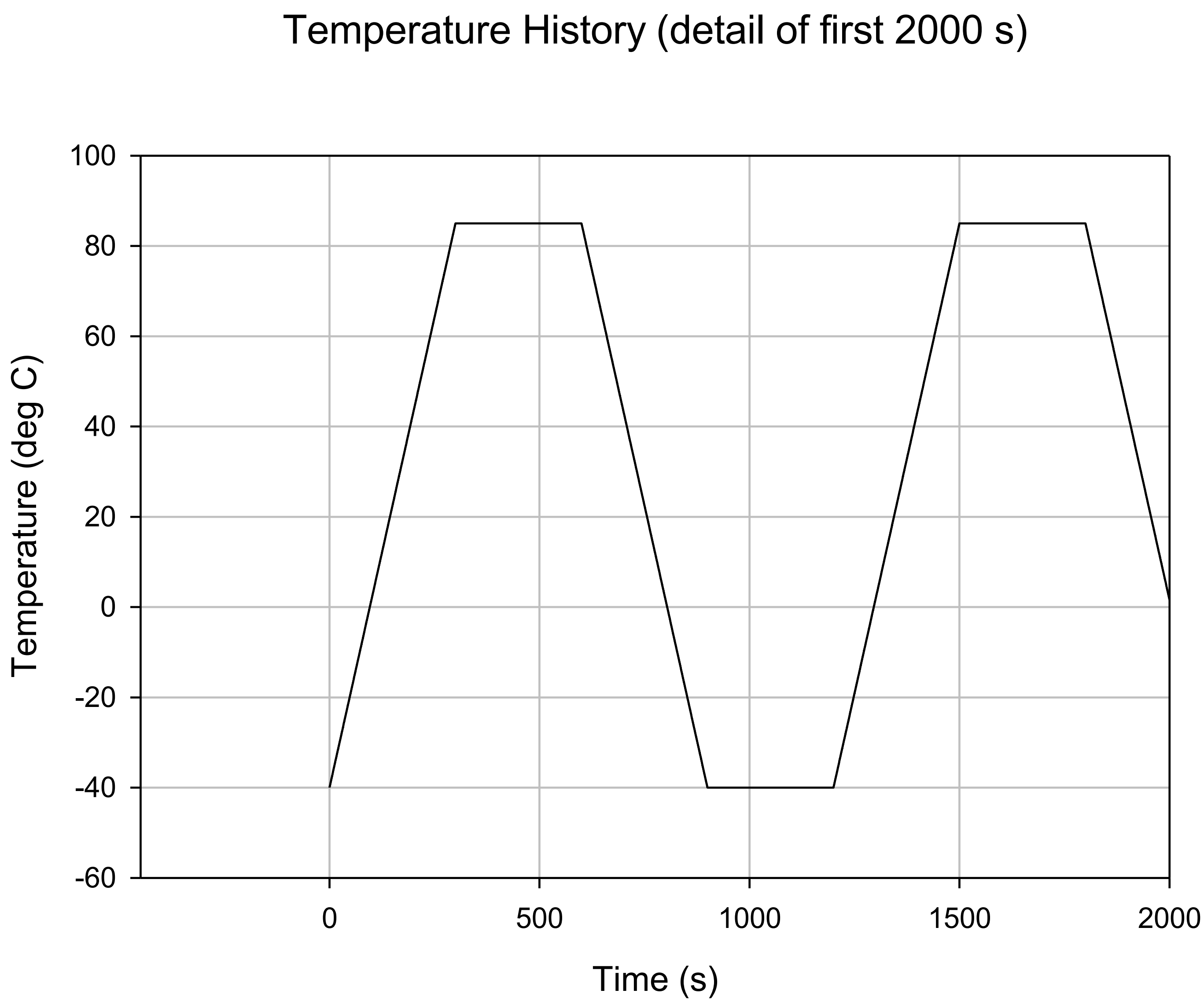
Model of [1] - Temperature Variation

- The temperature history of [1] is assumed to vary between – 40°C and +85°C, at a rate of 25°C /minute, or 0.41666667°C /s
- When the temperature is increasing and reaches +85°C, it remains at +85°C for 5 minutes (300 s)
- The temperature then decreases from +85°C to – 40°C at a rate of 25°C /minute; this takes 5 minutes (300 s)
- The temperature then remains at – 40°C for 5 minutes
- The temperature then increases to +85°C at a rate of 25°C /minute; this takes 5 minutes
- The duration of the entire cycle (i.e., the period) is therefore 20 minutes (1200 s)
- The temperature dependence is plotted over 12000 s (10 periods) on the next slide, and 2000 s (1.6667 periods) on the slide after that

Temperature, 0 - 12000 s



Temperature, 0 - 2000 s



Resulting Dependence of Frequency Offset on Time - 1

- As indicated previously, the frequency offset, y, as a function of temperature, T, is modeled as a cubic polynomial

$$y(T) = A[a_3T^3 + a_2T^2 + a_1T + a_0]$$

where

T = temperature in °C

y = frequency offset in ppm

$a_3, a_2, a_1,$ and a_0 are coefficients to be determined from a least-squares fit

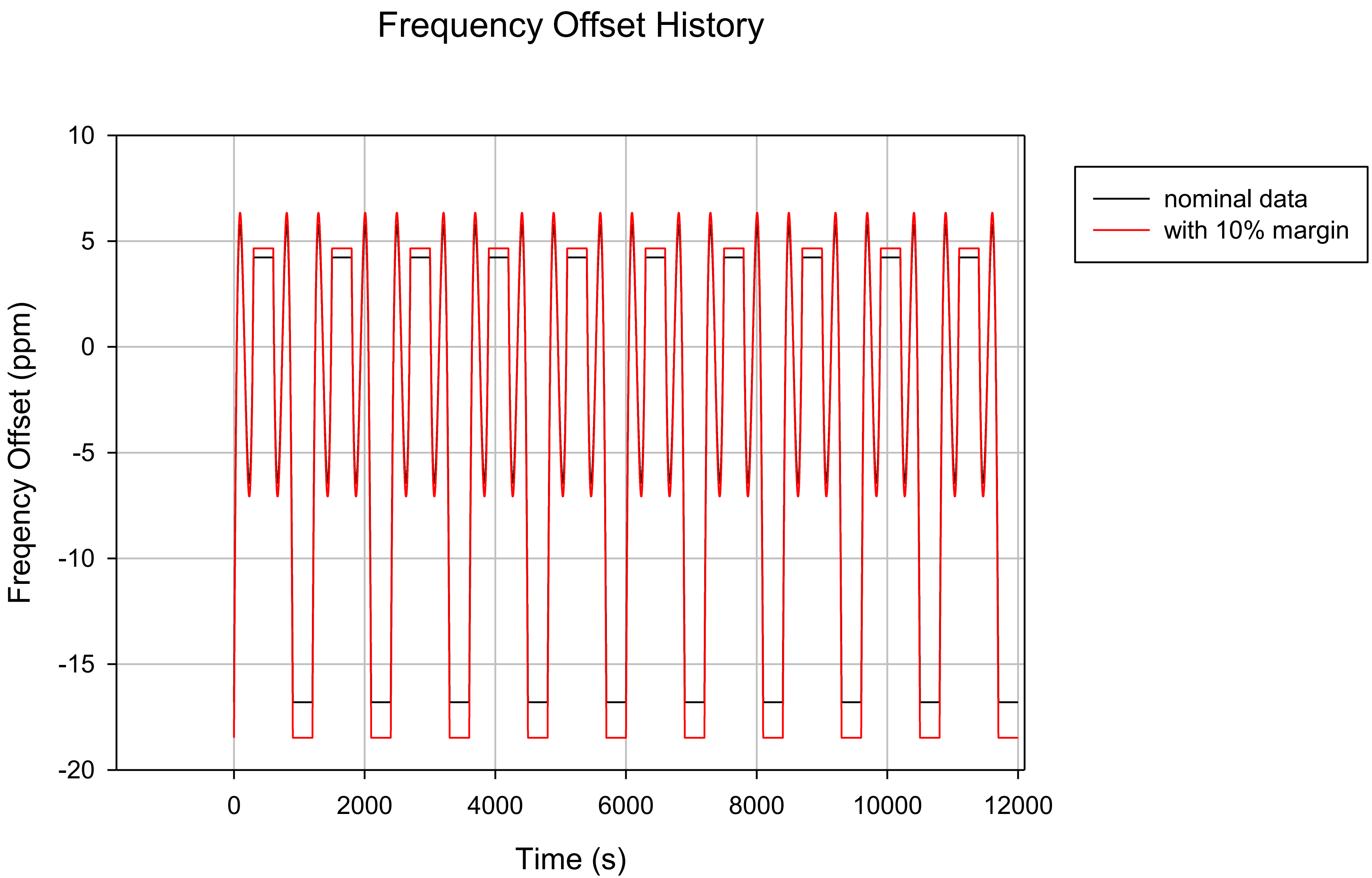
A = factor applied corresponding to desired margin (e.g., $A = 1.1$ for 10% margin)

- Let the time dependence of temperature (on the previous three slides) be represented by $T(t)$; then the time-dependence of frequency offset is given by

$$y(t) = A\{a_3[T(t)]^3 + a_2[T(t)]^2 + a_1T(t) + a_0\}$$

- The time history of frequency offset, with and without 10% margin for the frequency stability data, is plotted on the next two slides for 12000 s and 2000 s, respectively

Resulting Dependence of Frequency Offset on Time - 2



Resulting Dependence of Frequency Offset on Time - 3

Resulting Dependence of Frequency Offset on Time - 4

- The flat portions of the frequency offset curve, between 4.5 and 5 ppm and between -15 and -20 ppm, correspond to the periods when the temperature is constant at +85°C and -40°C, respectively
- The 10% margin results in a 2 ppm difference in frequency offset during the quarter cycle when the temperature is constant at -40°C, and less than 1 ppm difference during other portions of the cycle

Resulting Dependence of Frequency Drift Rate on Time - 1

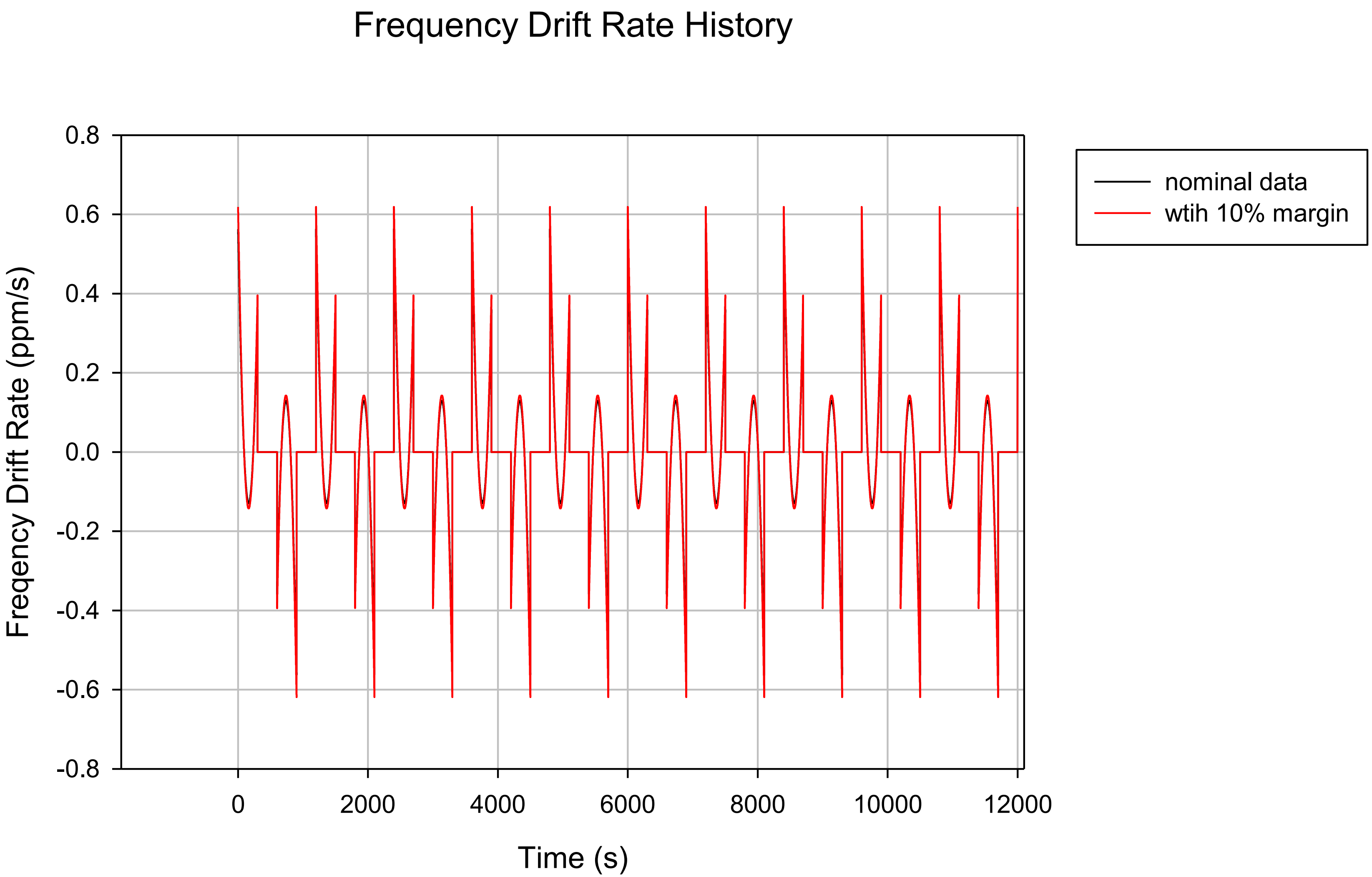
- The frequency drift rate is obtained by differentiating $y(t)$ with respect to time

$$y(t) = A\{a_3[T(t)]^3 + a_2[T(t)]^2 + a_1T(t) + a_0\}$$

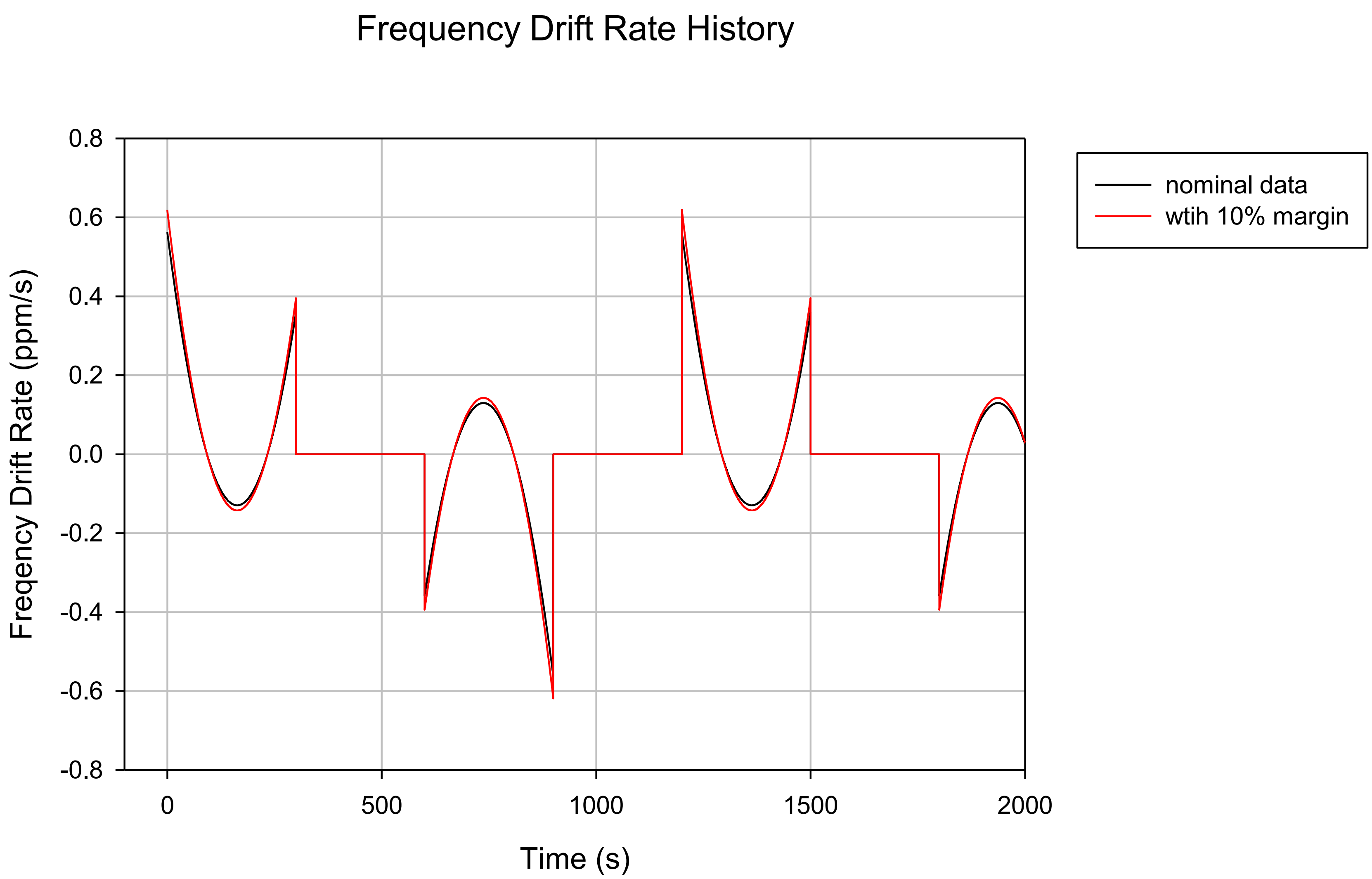
$$\frac{dy}{dt} = \frac{dy}{dT}\cdot\frac{dT}{dt} = A\{3a_3[T(t)]^2 + 2a_2[T(t)] + a_1\}\cdot\frac{dT}{dt}$$

- For the temperature profile above, dT/dt is 0 during the periods when temperature is constant at -40°C or +85°C, +0.41666667°C /s during the periods when temperature is increasing, and -0.41666667°C /s during the periods when temperature is decreasing
- The time history of frequency drift rate, with and without 10% margin for the frequency stability data, is plotted on the next two slides for 12000 s and 2000 s, respectively

Resulting Dependence of Frequency Drift Rate on Time - 2



Resulting Dependence of Frequency Drift Rate on Time - 3



Resulting Dependence of Frequency Drift Rate on Time - 4

- The maximum absolute value of frequency drift rate is approximately 0.56 ppm/s without the 10% margin, and 0.62 ppm/s with the 10% margin
- However, for half the period the frequency drift rate is zero, because the temperature is constant for half the period
- In addition, the absolute value of the frequency drift rate exceeds 0.2 ppm/s for approximately 13.3% of each cycle (approximately 160 s out of 1200 s)

Resulting Dependence of Phase Offset on Time - 1

- The phase offset as a function of time, $x(t)$, is obtained by integrating the frequency offset time history, $y(t)$, and expressing $x(t)$ in appropriate units
  - Assuming phase offset is zero at time zero:

$$x(t) = \int_0^t y(u)\ du$$

  - If $y(t)$ is in ppm, then $x(t)$ will be in μs
- The integration is done numerically, using the trapezoidal rule
  - A higher-order scheme (e.g., Simpson's rule) was not used because this would require values of in between time steps (or, alternatively, would not produce a result at every time step)
  - The iteration is:

$$x_{k+1} = x_k + \frac{\Delta t}{2}\left(y_{k+1} + y_k\right)$$

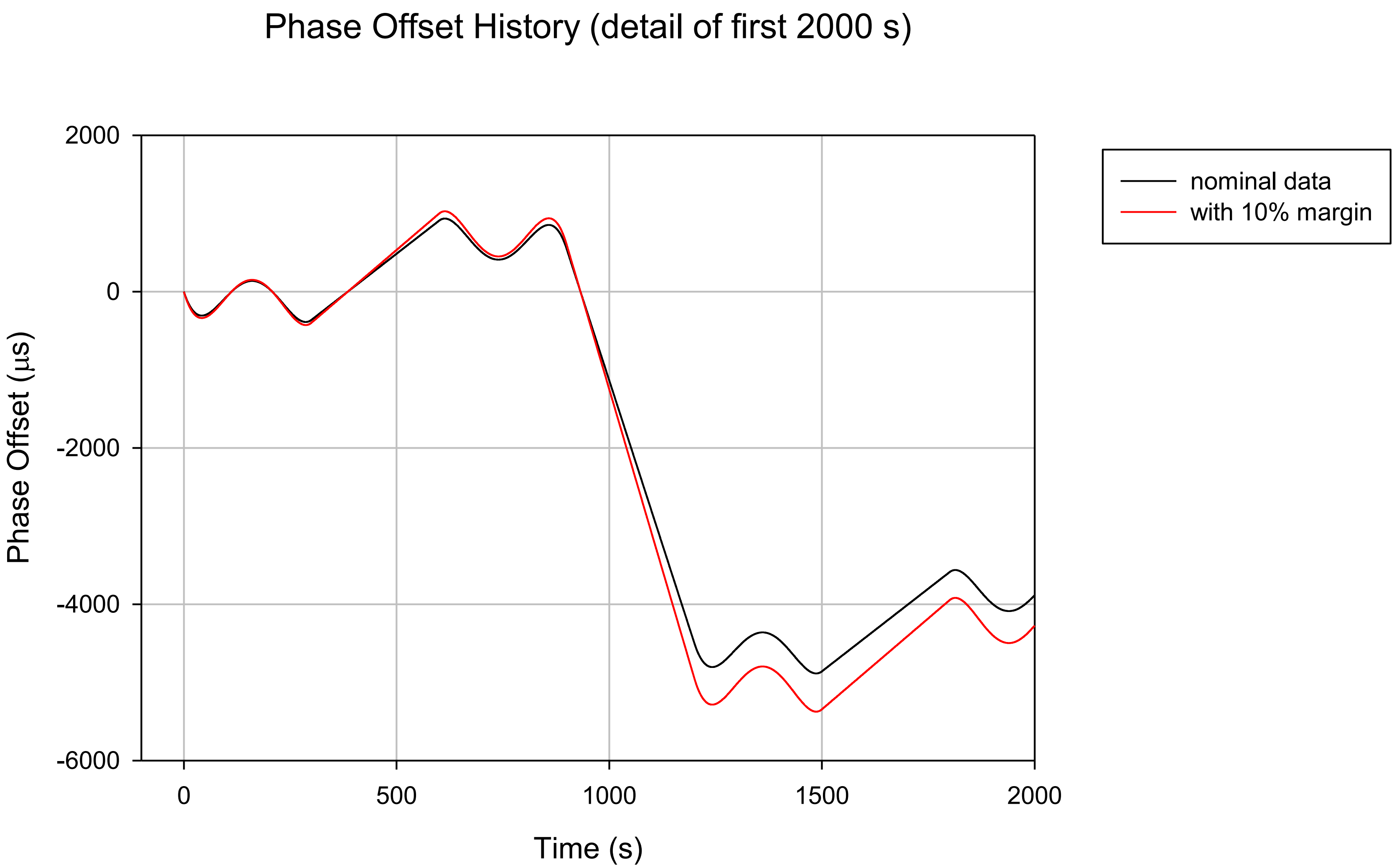
where

k = timestep index (i.e., $t = k\Delta t$)

Δt = timestep size in s

Resulting Dependence of Phase Offset on Time - 2

Resulting Dependence of Phase Offset on Time - 3



Resulting Dependence of Phase Offset on Time - 4

- The phase time history shows a phase drift of approximately – 50 ms over 12000 s
- This is equivalent to a frequency offset of 50000/12000 μs/s = – 4.2 ppm
- This frequency offset arises because, with the frequency offset as a function of temperature (slide 8) and temperature profile (slide 15) assumptions, the frequency offset averaged over one cycle of temperature variation is not zero (i.e., it is approximately – 4.2 ppm)
- Actually, the average frequency offset of the crystal unit does not matter, because this is the frequency offset of the free-running LocalClock entity in 802.1AS
  - The neighborRateRatio is measured using the Pdelay messages and the overall rateRatio relative to the GM is accumulated in a TLV
  - What matters is how much the frequency of the free-running LocalClock entity changes between Pdelay messages
- Finally, as described in [3], the actual long-term frequency offset can be different from this value because the frequency versus temperature curve of slide 8 can be shifted vertically due to the frequency tolerance of the crystal unit

Relative Time Offsets of Phase Offsets at Each Node - 1

- If the above model for frequency and phase offset for the LocalClock entity, as a function of time, is still used, assumptions are still needed for the relative offsets (i.e., relative phases) of the phase waveforms (slides 26 and 27) at each node
- These assumptions are important; we cannot simply assume that the waveforms are exactly in phase at all the nodes because, if we did this and assumed negligible propagation delays for the PTP timing messages, the resulting transported time error relative to the GM would be negligible
- In previous simulations, two types of assumptions were used
  - The periodic phase error waveforms had the same period but completely random phases at each node (with phase at each node chosen randomly at initialization)
  - The phase error waveforms at the different nodes had slightly different frequencies, where frequency was chosen randomly, at initialization, within a range (see [5] for details)

Relative Time Offsets of Phase Offsets at Each Node - 2

- Initial phases were also chosen randomly at initialization; however, this was not important because, with different frequencies, the waveforms would be out of phase over time anyway
- In any case, it was suggested in discussion that those assumptions might be overly conservative because they imply that the temperature history at each node could be at an arbitrary point in its cycle relative to other nodes
  - This would mean, for example, that temperature could be increasing at one node and decreasing at the next node
- An alternative, less conservative, assumption would be to use the first assumption but allow the time offsets of the phase error waveforms to be random within a certain fraction β of the cycle
  - For example, if $\beta = 0.1$ (10%), with the above period (1200 s) any two waveforms would be offset from each other by at most 120 s
  - Note that, with this notation, the 0.1 is a peak-to-peak offset; on initialization the random phases would be chosen randomly within ± 0.05 cycle (i.e., ± 60 s)
- Finally, note that if initial simulation runs are for single replications, it will be possible to look at cases where the random phase is chosen over the entire period versus only a fraction of the period, to see what the impact is on the results

Other Assumptions - 1

- Some other assumptions were briefly suggested in email discussion
  - Mean Sync interval: 125 ms
  - Mean Pdelay interval: 31.25 ms
  - Timestamp granularity: 8 ns, 4 ns (both cases)
  - Residence times: 1 ms, 4 ms, 10 ms (all 3 cases)
  - Timestamp error (±8 ns, each with 0.5 probability)
- The above implies at least 6 simulation cases
  - But, single-replication cases are run with different assumptions on initial random phases for phase error waveforms (see previous two slides), the above (6 cases) would be multiplied by the number of different assumptions on initial random phases considered
- Other assumptions can be taken from the most recent simulations [5], and are summarized on the following slides
  - Note that initial simulations will assume GM error of zero; GM error will be added after other assumptions are settled on

Other Assumptions - 2

Assumption/Parameter	Description/Value
Hypothetical Reference Model (HRM), see note following the tables	101 PTP Instances (100 hops; GM, followed by 99 PTP Relay Instances, followed by PTP End Instance
Computed performance results	$\max\lvert dTE_{R(k,\,0)}\rvert$ (i.e., maximum absolute relative time error between node k ($k > 0$) and GM; here, GM time error is 0, so $\max\lvert dTE_{R(k,\,0)}\rvert = \max\lvert dTE\rvert$)
Use syncLocked mode for PTP Instances downstream of GM	Yes
Window size for successive Sync messages method, when used	7 (take difference between respective timestamps of current Sync message and 7th previous message)
Compute median for successive Sync messages method, when used	Yes
Endpoint filter parameters	$K_pK_o = 11$, $K_iK_o = 65$ ($f_{3dB} = 2.5998$ Hz, 1.288 dB gain peaking, $\zeta = 0.68219$)
Simulation time	(a) For single replication cases:12050 s; discard first 50 s to eliminate any startup transient before computing $\max\lvert dTE_{R(k,\,0)}\rvert$ (b) For multiple replication cases, may need to be shorter than 12050 s depending on run times
Number of independent replications, for each simulation case	(a) Single replication cases (i.e. 1) (b) 300

Other Assumptions - 3

Assumption/Parameter	Description/Value
GM rateRatio and neighborRateRatio computation granularity	0
Mean link delay	500 ns
Link asymmetry	0
Dynamic timestamp error for event messages (Sync, Pdelay-Req, Pdelay_Resp) due to variable delays within the PHY	±8 ns; for each timestamp taken, a random error is generated. The error is + 8 ns with probability 0.5, and – 8 ns with probability 0.5. The errors are independent for different timestamps and different PTP Instances.
Window Size for mean link delay averaging (i.e., how many mean link delay samples are averaged over, assuming a sliding window)	16

Thank you

References - 1

[1] Jordan Woods, *Concerns regarding the clock model used in 60802 time synchronization simulations*, Revision 1, IEC/IEEE 60802 presentation, December 21, 2020 call (available at https://www.ieee802.org/1/files/public/docs2020/60802-woods-ClockModel-1220-v02.pdf)

[2] Geoffrey M. Garner, *Temperature, Frequency Offset, and Frequency Drift Rate Time History Plots for Model of [1]*, IEC/IEEE 60802 presentation, December 21, 2021 call (available at https://www.ieee802.org/1/files/public/docs2020/60802-garner-temp-freqoffset-freqdrift-plots-1220-v00.pdf)

[3] Chris McCormick, *Crystal Fundamentals & State of the Industry*, IEC/IEEE 60802 presentation, February 22, 2021 call (available at https://www.ieee802.org/1/files/public/docs2021/60802-McCormick-Osc-Stability-0221-v01.pdf)

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[4] Rudolph Bechmann, *Frequency-Temperature-Angle Characteristics of AT-Type Resonators Made of Natural and Synthetic Quartz*, Proceedings of the IRE, vol. 44, no. 11, November 1956, pp. 1600 – 1607 (available at https://ieeexplore.ieee.org/document/4051918/authors#authors)

[5] Geoffrey M. Garner, *Further Simulation Results for Dynamic Time Error Performance for Transport over an IEC/IEEE 60802 Network Based on Updated Assumptions*, Revision 2, IEC/IEEE 60802 presentation, December 14, 2021 call (available at https://www.ieee802.org/1/files/public/docs2020/60802-garner-further-simulation-results-time-sync-transport-1120-v02.pdf)