

Further 60802 Time Sync Simulation Results using Offset Compensation Factor Revision 1

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IEEE 802.1 TSN TG

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Introduction - 1

- In IEEE Std 802.1AS-2020, syntonization is performed by measuring neighborRateRatio using the T_3 and T_4 timestamps of successive Pdelay_Resp and Pdelay_Resp_Follow_Up messages
 - neighborRateRatio is then accumulated in a TLV attached to Sync or Follow_Up to obtain the measured rateRatio (i.e., rateRatio relative to the grandmaster (GM) frequency)
- In [1], an alternative syntonization scheme is proposed
- The current presentation contains initial analysis of this scheme
 - The transfer functions for a single PTP Instance and a chain of PTP Instances are computed
 - Initial simulation results are presented and compared with previous results obtained in [2] based on the standard 802.1AS syntonization scheme (measurement and accumulation of neighborRateRatio)
- This revision fixes missing mathematical operations in the block diagram of slide 6

Introduction - 2

- ❑ The analysis here is based on [1] (mainly slide 15 of [1]) and subsequent email discussion on the 802.1 reflector
- ❑ Actually, the simulation results presented here for $\max|dTE_R|$ (i.e., maximum absolute value of relative dynamic time error, relative to the GM) are significantly worse than previous results obtained using the standard syntonization scheme of 802.1AS-2020
- ❑ However, after the work of the current presentation was completed, additional details on the scheme of [1] were provided in a White Paper [4]
 - This details might result in modification of the models described in the current presentation
 - If so, subsequent analysis will be provided in a future presentation

Model of [1] - 1

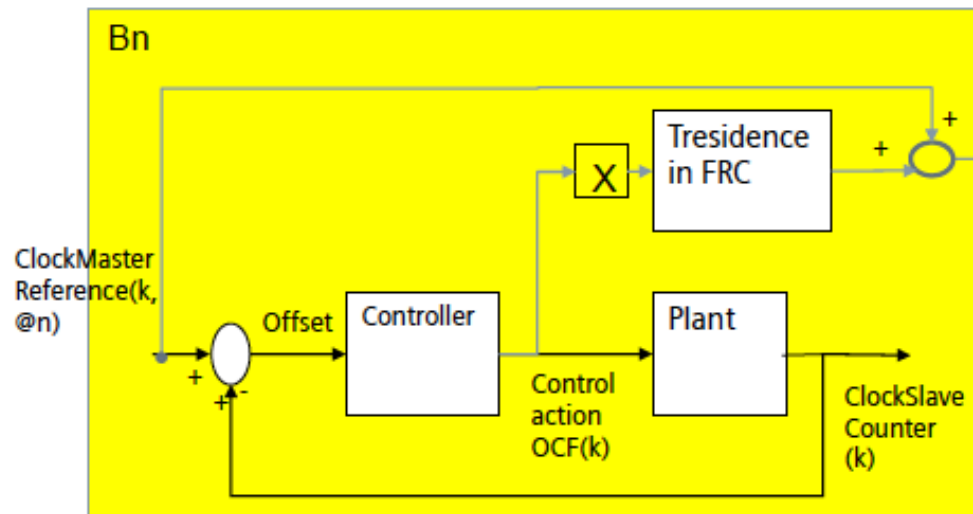
Below is a reproduction of slide 15 of [1]

Using controlled clock to update Sync Message

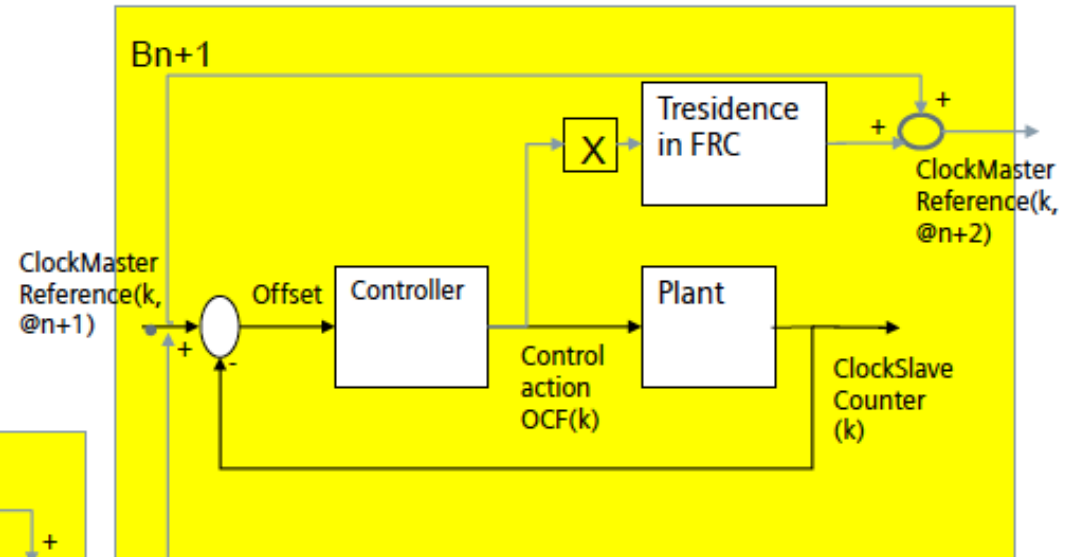
The transfer function relevant for the residence delay compensation with the ClockSlave "time" is between the ClockMaster_Reference and the control signal OCF

$$TF = \frac{C}{1 + PC}$$

- The gain of this TF is $\ll 1$ at all frequencies (due to the large gain in P)
- There is no amplification over consecutive elements if $\max(|TF(j\omega)|) * \text{Residence} < 1!$ (but other factors play a role too)



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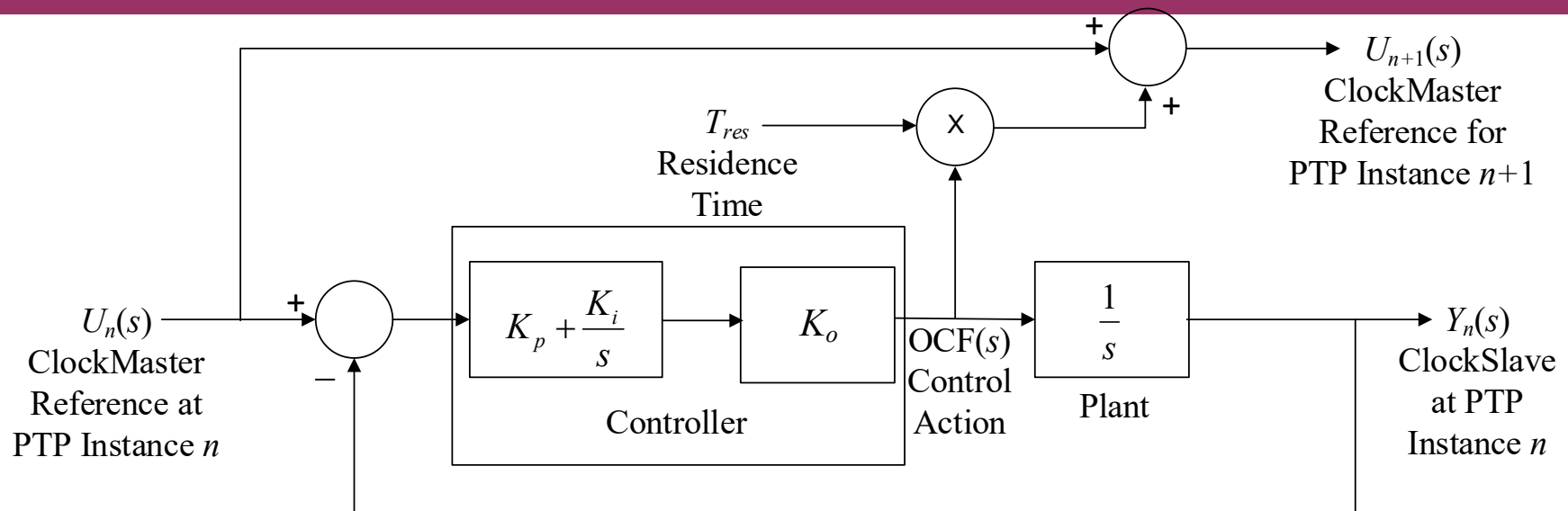


We also have a limit on the OCF, hence it can not grow indefinitely!

Model of [1] - 2

- In this model, the controller is a proportional plus integral (PI) controller, and the plant is either an oscillator whose frequency is controlled by the controller output (e.g., the plant could be a voltage-controlled oscillator (VCO), a digital-controlled oscillator (DCO), etc.)
- Actually, based on earlier slides in [1], the controller output is the frequency offset between the controlled oscillator and the ClockMaster Reference input, and the plant is the controller oscillator, i.e., it is an integrator
- The following slide shows the model of [1], but with the details of the controller and plant shown using the typical continuous-time model for a second order phase-locked loop (PLL) with 20 dB/decade roll-off; the nomenclature is:
 - C = Controller
 - P = Plant
 - FRC = free-running counter
 - TF = transfer function between ClockMaster input and Control Action OCF
 - OCF = offset compensation factor

Model of [1] - 3



- ❑ In this model, the controlled oscillator gain, K_o , is combined with the controller, and the plant is an integrator that converts the oscillator frequency to phase
- ❑ The OCF is the frequency offset between the ClockMaster input $U_n(s)$ and the ClockSlave output $Y_n(s)$
- ❑ The ClockMaster input includes the uncorrected residence time (T_{res}), i.e., measured relative to the timestamping clock, plus the measured upstream meanLinkDelay

Model of [1] - 4

- ❑ The product $OCF \times T_{res}$ is the correction to the residence time due to the difference between the timebase of the residence time measurement, i.e., the FRC, and the timebase of the ClockSlave
- ❑ $OCF \times T_{res}$ is computed in continuous time; however, its value is added to the ClockMaster reference for PTP Instance n (which includes the measured T_{res}) only when a Sync message is sent to PTP Instance $n+1$
- ❑ OCF is intended to be analogous to the measured frequency offset (i.e., the measured GM rateRatio minus 1) in the standard 802.1AS-2020 specifications

Transfer Functions - 1

- The transfer function between the ClockMaster input $U_n(s)$ and the output of the multiplier of OCF(s) and T_{res} , represented by $TF(s)$, is given by (note that this is a slight change of notation compared to [1]; in [1], TF does not include the multiplication by T_{res})

$$\begin{aligned} TF(s) &= \frac{\left(K_p + \frac{K_i}{s}\right) K_o T_{res}}{1 + \left(K_p + \frac{K_i}{s}\right) \frac{K_o}{s}} = \frac{s T_{res} (K_p K_o s + K_i K_o)}{s^2 + K_p K_o s + K_i K_o} \\ &= \frac{s T_{res} (2\zeta \omega_n s + \omega_n^2)}{s^2 + 2\zeta \omega_n s + \omega_n^2} \end{aligned}$$

where

ω_n = undamped natural frequency in rad/s = $\sqrt{K_i K_o}$

ζ = damping ratio (dimensionless) = $\frac{K_p K_o}{2\sqrt{K_i K_o}}$

Transfer Functions - 2

- As expected, the denominator of the above transfer function is the same as that of the standard second-order PLL with 20 dB/decade roll-off, but the numerator is different
- The 3dB bandwidth and gain peaking of the PLL are related to the undamped natural frequency and damping ratio as given in previous presentations (for example, see [3]). The parameter values are also given in [3]
 - The following 2 slides contain this information, and are copied from [3] (except for the slide titles and slide numbers at the lower right, they are copied as is)

Transfer Functions - 3

- Often the filter parameters (and requirements) are expressed in terms of 3 dB bandwidth ($f_{3\text{dB}}$) and gain peaking (H_p)
 - These are related to damping ratio (ζ) and undamped natural frequency (ω_n) by (see [6] and [7] of reference [2] here):

$$f_{3\text{dB}} = \frac{\omega_n}{2\pi} \left[1 + 2\zeta^2 + \sqrt{(1 + 2\zeta^2)^2 + 1} \right]^{1/2}$$

$$H_p \text{ (dB)} = 20 \log_{10} \left\{ \left[1 - 2\alpha - 2\alpha^2 + 2\alpha\sqrt{2\alpha + \alpha^2} \right]^{-1/2} \right\}$$

where

$$\alpha = \frac{1}{4\zeta^2} = \frac{K_i}{K_p^2 K_o}$$

Transfer Functions - 4

- As in previous simulation models, the VCO gain was folded into the proportional gain and integral gain (this is equivalent to setting the VCO gain to 1)
- Filter assumption:
 - $K_p K_o = 11$, $K_i K_o = 65$
 - Using the equations on the previous slides, we obtain
 - $\zeta = 0.68219$
 - $\omega_n = 8.06226 \text{ rad/s} \approx 8.06 \text{ rad/s}$
 - H_p (gain peaking) = 1.28803 dB = (approx) 1.3 dB
 - $f_{3\text{dB}} = 2.5998 \text{ Hz} \approx 2.6 \text{ Hz}$
- Note that this filter is underdamped, and has appreciable gain peaking
 - However, the damping ratio (ζ) is close to $1/\sqrt{2} =$ (approx) 0.707; this is often used to obtain a fast response with small overshoot, in cases where the filters are not cascaded (the endpoint filters are not cascaded)

Transfer Functions - 5

- The transfer function between the ClockMaster Reference input at PTP Instance n and PTP Instance $n+1$, represented by $H(s)$, is given by

$$\begin{aligned} H(s) &= 1 + TF(s) = 1 + \frac{\left(K_p + \frac{K_i}{s}\right) K_o T_{res}}{1 + \left(K_p + \frac{K_i}{s}\right) \frac{K_o}{s}} = 1 + \frac{s T_{res} (K_p K_o s + K_i K_o)}{s^2 + K_p K_o s + K_i K_o} \\ &= 1 + \frac{s T_{res} (2\zeta \omega_n s + \omega_n^2)}{s^2 + 2\zeta \omega_n s + \omega_n^2} \\ &= \frac{s^2 (1 + 2\zeta \omega_n T_{res}) + (\omega_n^2 T_{res} + 2\zeta \omega_n) s}{s^2 + 2\zeta \omega_n s + \omega_n^2} \end{aligned}$$

Transfer Functions

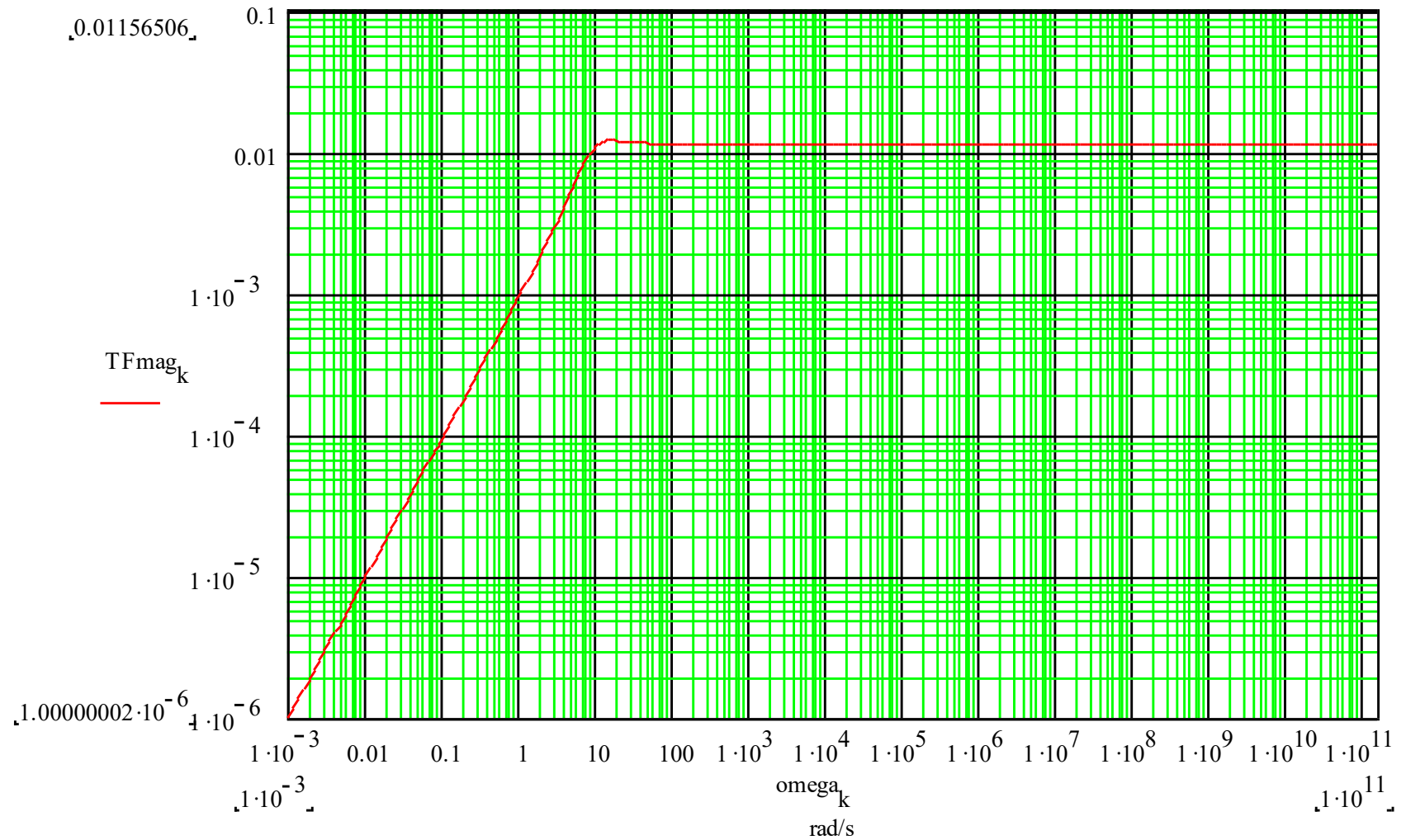
- ❑ The units of s and ω_n are rad/s. T_{res} has units of time. Therefore, both $TF(s)$ and $H(s)$ are dimensionless
- ❑ This means that the ClockMaster references at PTP Instances n and $n+1$, the ClockSlave, and T_{res} all have the same units, e.g., all have units of ns (or all have units of s, etc.)
- ❑ Since the integral of OCF has units of time, the units of OCF are [Time]/s, where [Time] represents the units of T_{res} (which must be the same as the units of ClockMaster and ClockSlave). For example, if [Time] is expressed in seconds, OCF is a dimensionless frequency offset. If [Time] is expressed in ns, OCF has units ns/s.
- ❑ The reason the denominator of the OCF units is seconds is that we have expressed the undamped natural frequency, ω_n , in rad/s, and s has units of rad/s

Frequency Responses - 1

- The frequency response is given by the absolute value of the transfer function, evaluated at $s = j\omega$, plotted as a function of ω
- For N hops (i.e., the GM, followed by N PTP Instances), the transfer function and frequency response are equal to the transfer function and frequency response, respectively, for a single PTP Instance, raised to the N^{th} power
- The frequency responses for OCF, for a single hop (i.e., from the ClockMaster at PTP Instance n to the ClockMaster at PTP Instance $n+1$) and for the GM followed by 64 and by 100 PTP Instances are shown on the following slides (the computations and plots were made using Mathcad)
 - Frequency responses are shown for residence times of 1 ms (0.001 s) and 10 ms (0.01 s)
 - In the plots, the horizontal axis is the angular frequency ω ; the subscript k is a discrete index of the points plotted

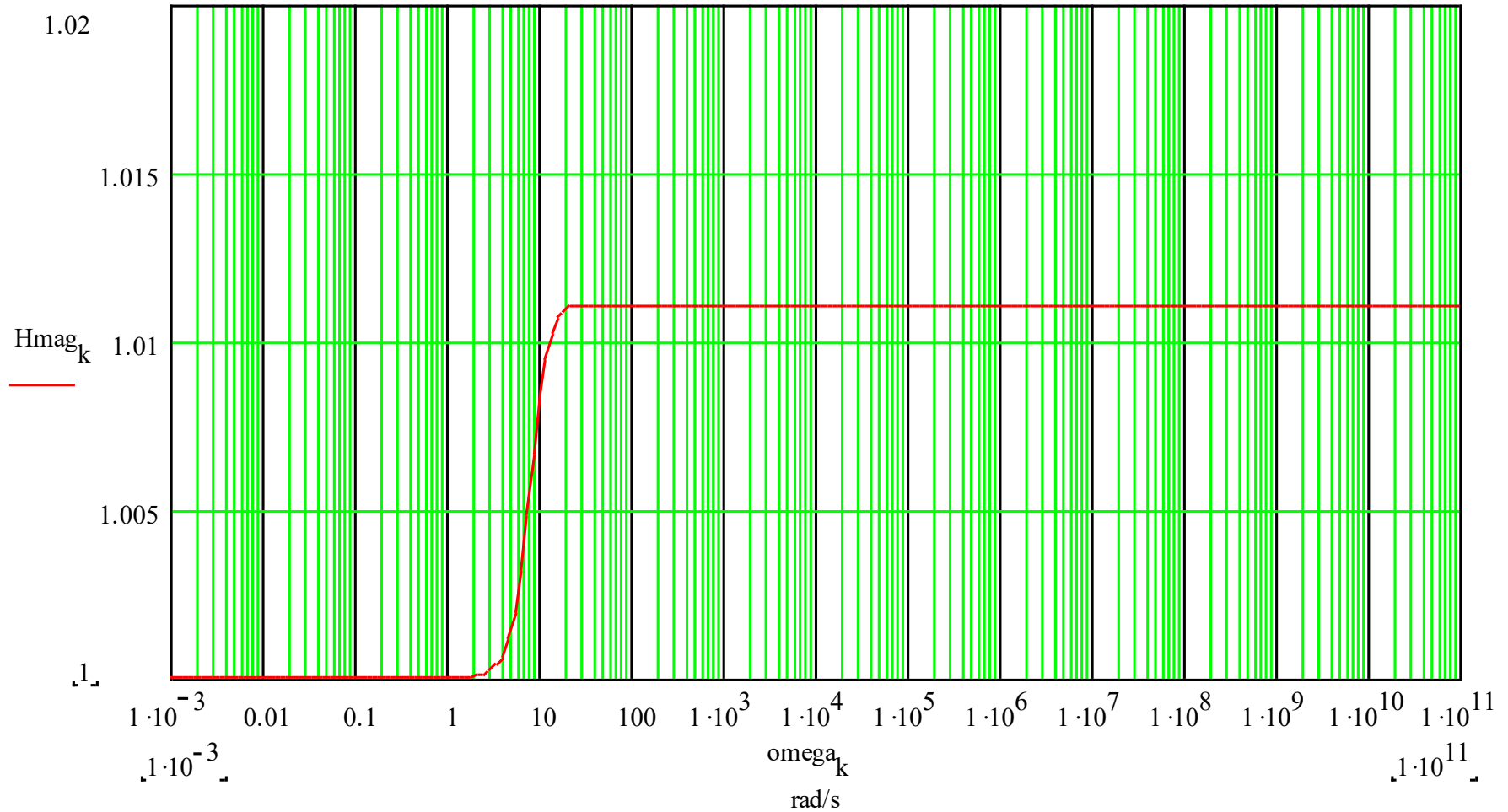
Frequency Responses - 2

OCF frequency response – 1 ms residence time



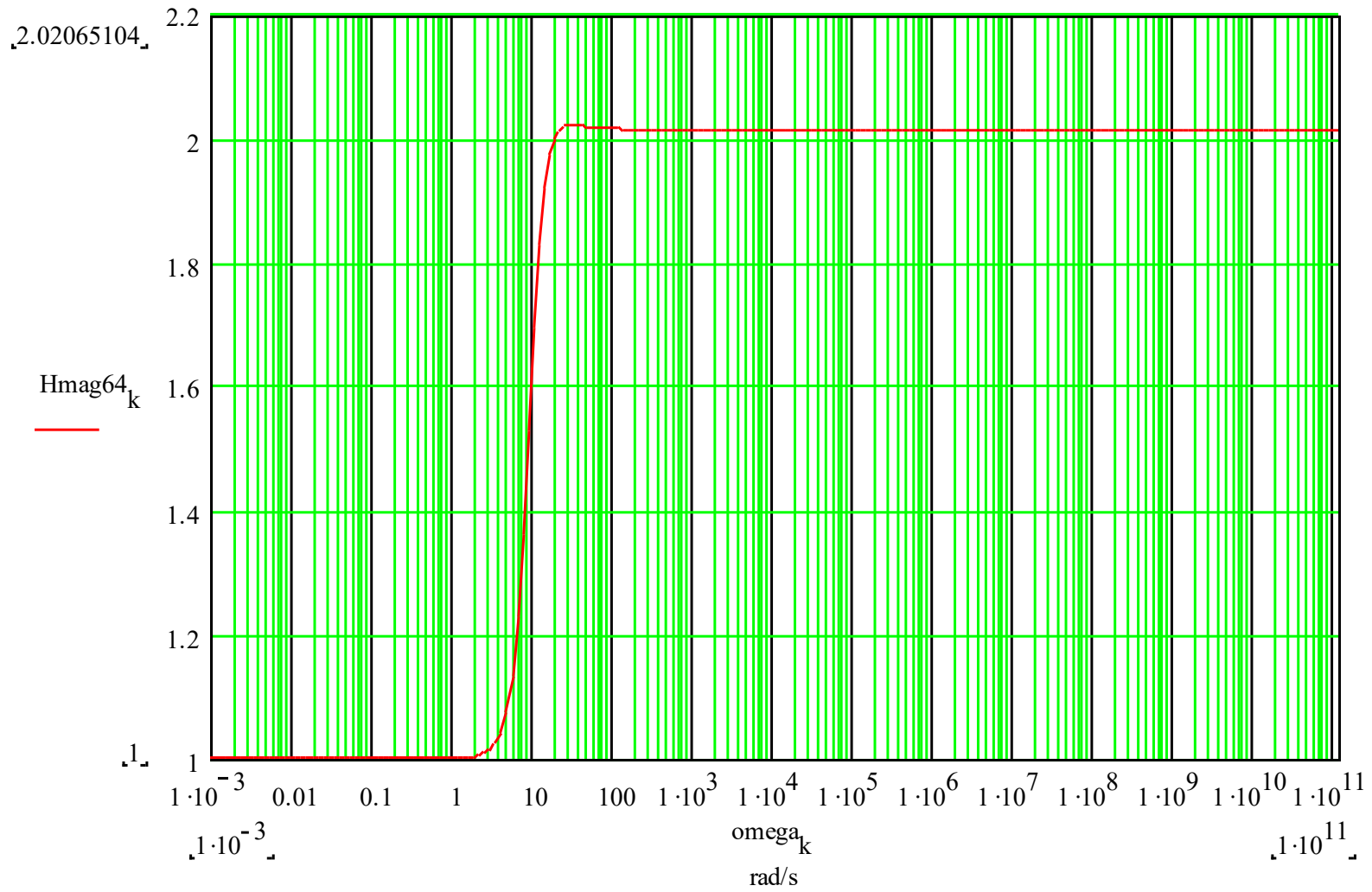
Frequency Response - 3

- ClockMaster n to ClockMaster $n+1$ frequency response ($|H(j\omega)|$) – 1 ms residence time



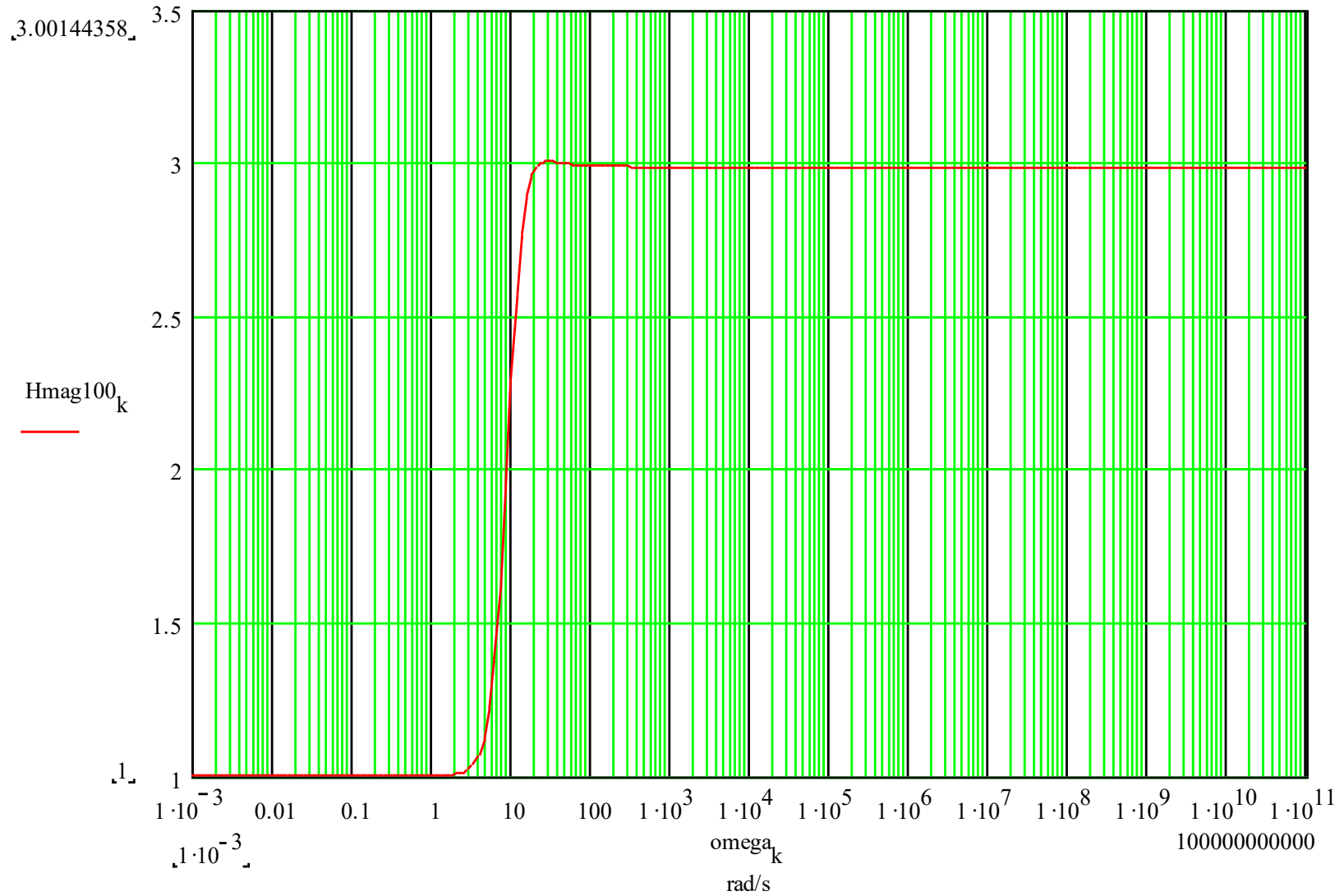
Frequency Response - 4

□ Frequency response for 64 hops ($|H(j\omega)|^{64}$) – 1 ms residence time



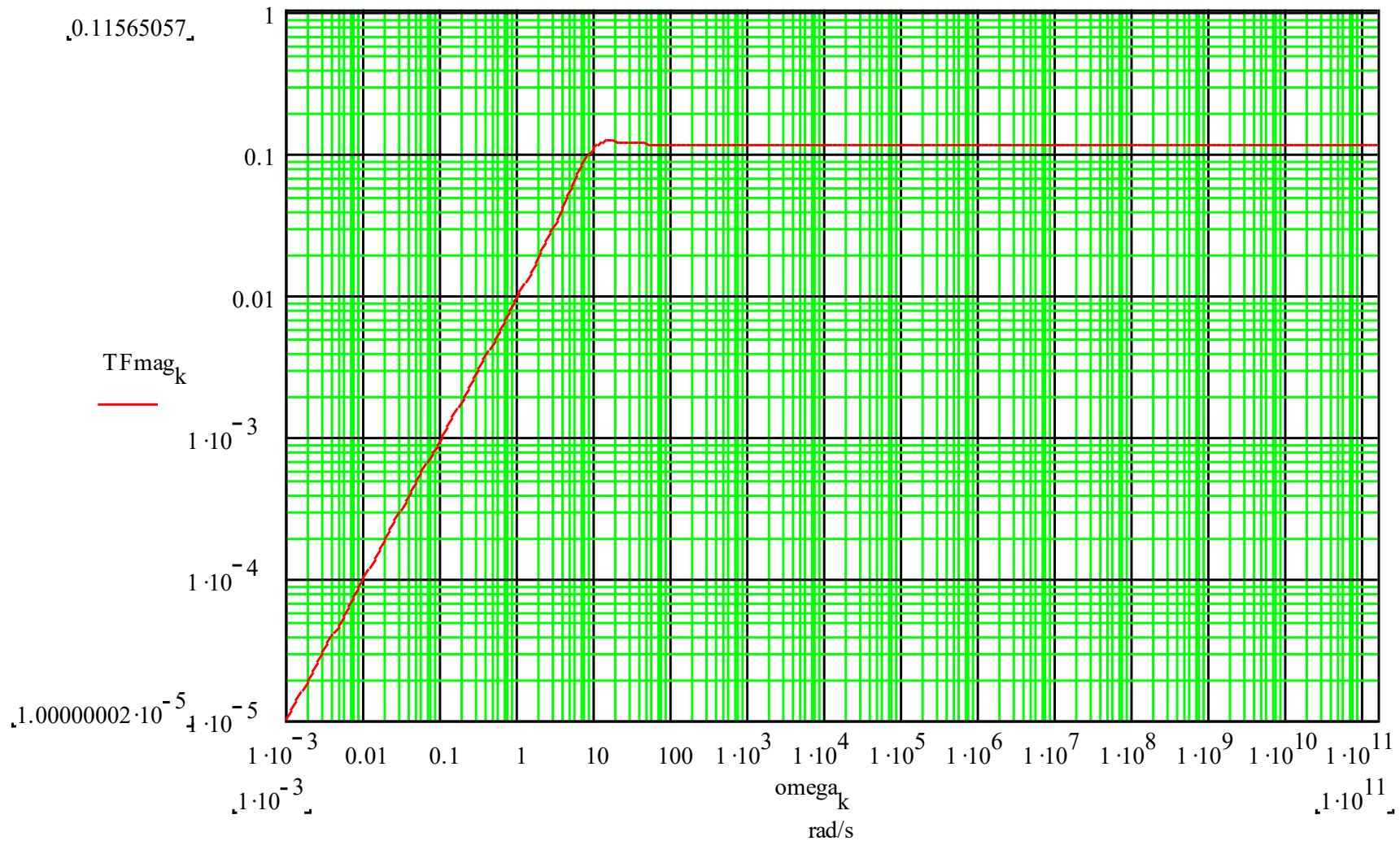
Frequency Response - 5

□ Frequency response for 100 hops ($|H(j\omega)|^{100}$) – 1 ms residence time



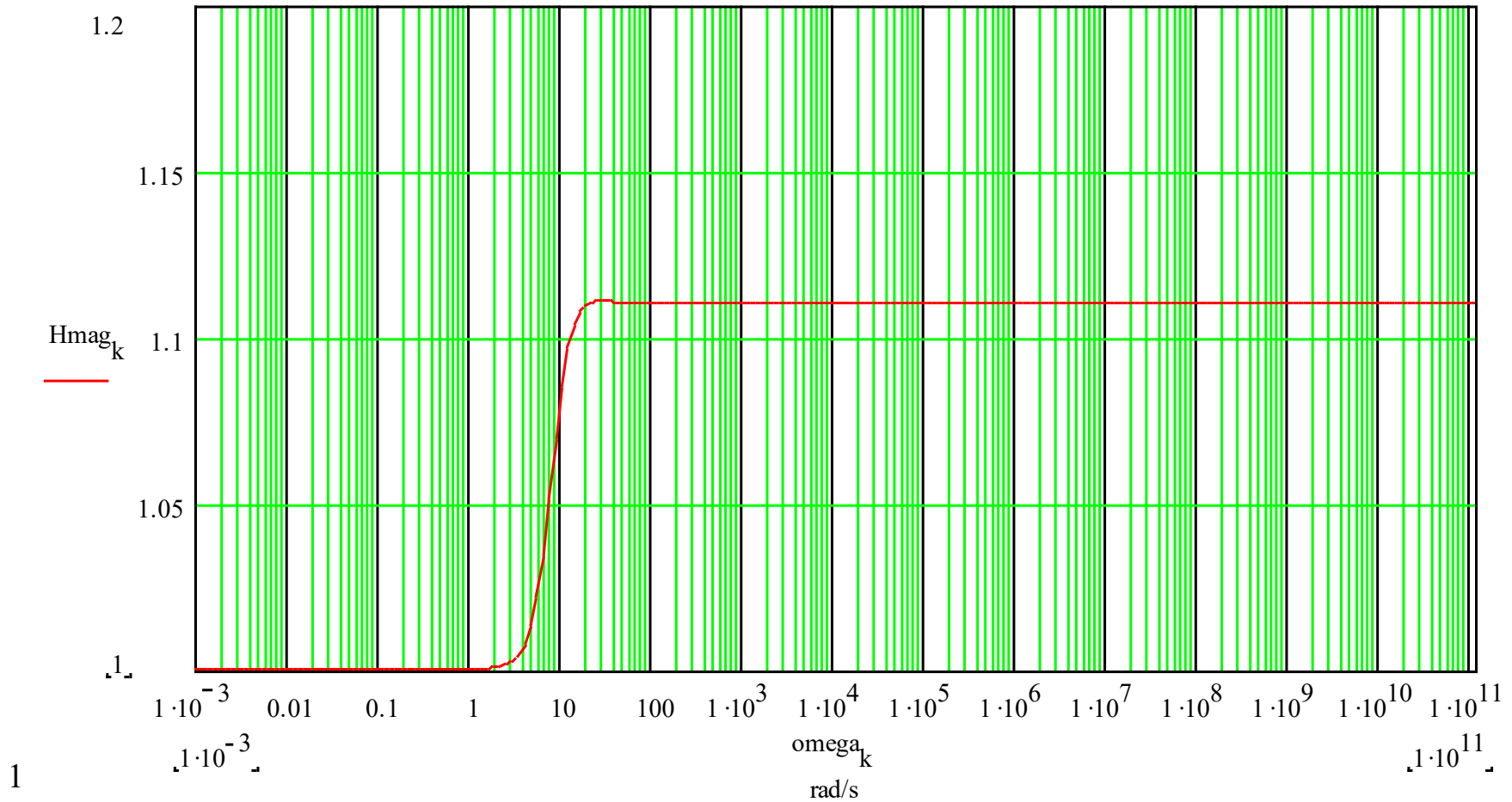
Frequency Responses - 6

□ OCF frequency response – 10 ms residence time



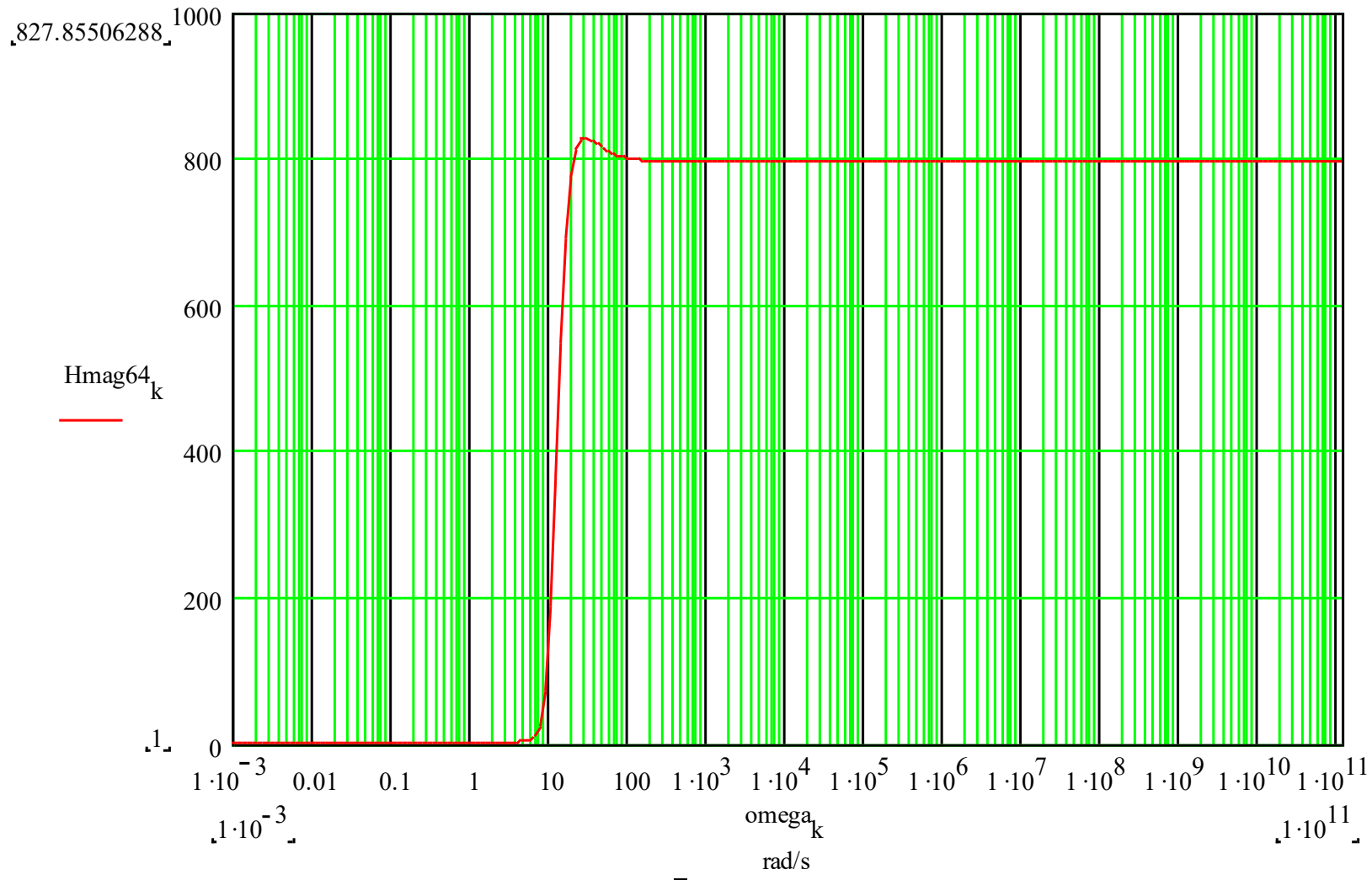
Frequency Response - 7

- ClockMaster n to ClockMaster $n+1$ frequency response ($|H(j\omega)|$) – 10 ms residence time



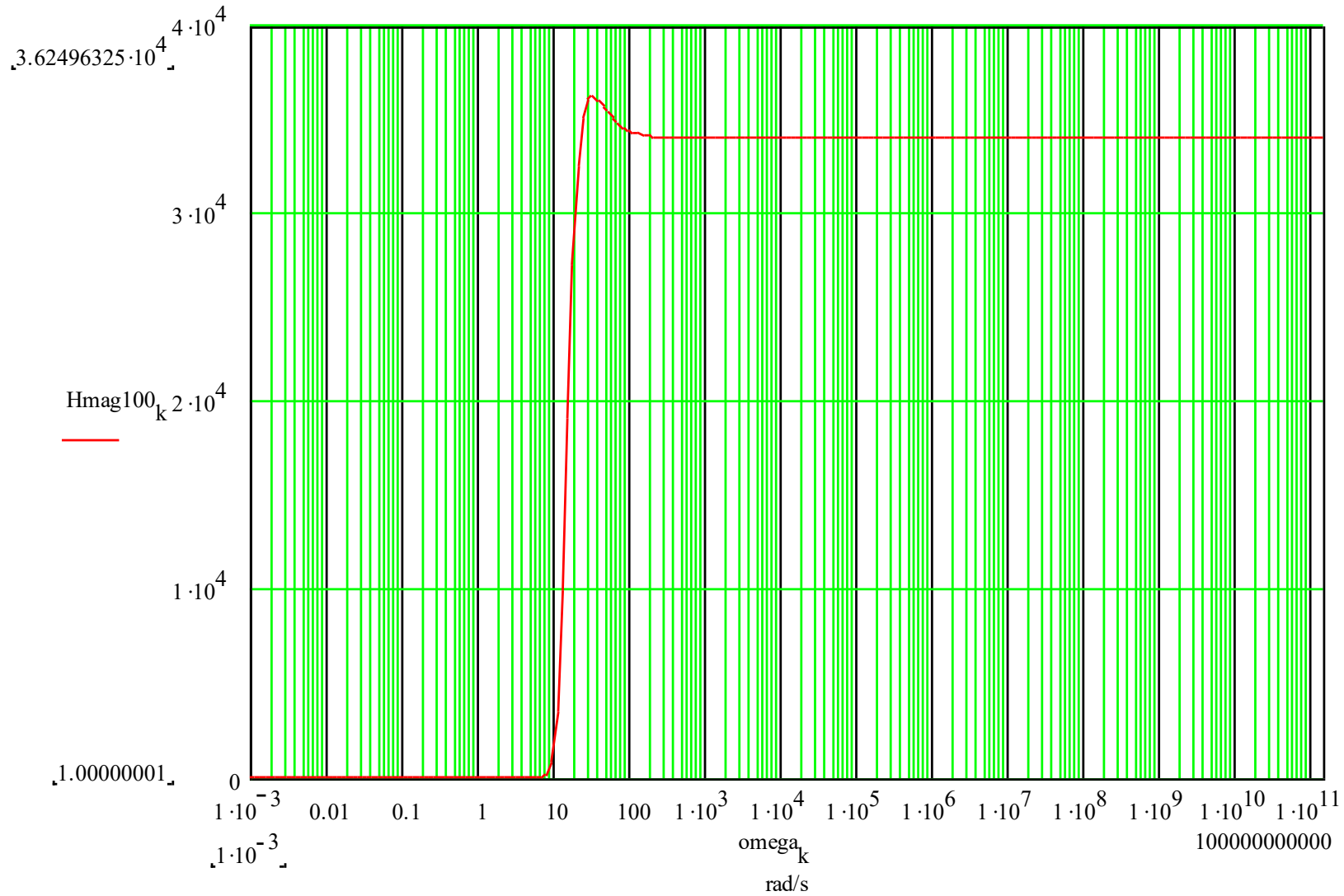
Frequency Response - 8

□ Frequency response for 64 hops ($|H(j\omega)|^{64}$) – 10 ms residence time



Frequency Response - 9

□ Frequency response for 100 hops ($|H(j\omega)|^{100}$) – 10 ms residence time



Frequency Responses - 10

- The OCF frequency response shows a high-pass characteristic
 - This is seen from $TF(s)$, which approaches zero for $s \rightarrow 0$ and $2\zeta\omega_n T_{res}$ for $s \rightarrow \infty$
- The corner frequency for $TF(s)$ is approximately 10 rad/s (close to ω_n of 8.06 rad/s)
- The frequency response for a single hop (PTP Instance n to PTP Instance $n+1$) is approximately 1 for frequencies less than the corner frequency, and increases to $1 + 2\zeta\omega_n T_{res}$ for frequencies above the corner frequency
 - The high-frequency gain is approximately 1.011 for 1 ms residence time and 1.11 for 10 ms residence time
- As a result, the gain over many hops can be large, and becomes larger as residence time increases (it grows exponentially with the number of hops)
 - The high-frequency gain for 1 ms residence time is approximately 2.02 for 64 hops and 3 for 100 hops
 - The high-frequency gain for 10 ms residence time is approximately 800 for 64 hops and 3.4×10^4 for 100 hops
 - The high-frequency gain goes like $(1 + 2\zeta\omega_n T_{res})^N$ for N hops

Assumptions for Simulation Cases - 1

- The assumptions of simulation case 1 of [2] are used as the basis for the new simulations in the current presentation
- Details of the assumptions are given in [2] and the presentations referenced there; in summary, the assumptions are
 - Temperature profile that includes sinusoidal variation between $-40\text{ }^{\circ}\text{C}$ and $+85\text{ }^{\circ}\text{C}$ over a period of 125 s
 - Dwell time of 30 s between each period where temperature increases or decreases, resulting in a period for the complete temperature cycle of 310 s
 - Frequency stability versus temperature that is a cubic polynomial, with coefficients as given in [2]
 - Dynamic timestamp error having uniform probability distribution over $[-8\text{ ns}, +8\text{ ns}]$
 - The GM is assumed to have the same frequency stability (i.e., time error) as the subsequent PTP Instances
 - dTE_R and $\max|dTE_R|$, relative to the GM, are the results computed
 - Since the dTE samples at the GM and at subsequent PTP Instances are not necessarily computed at the same time, interpolation is used to compute dTE_R

Assumptions for Simulation Cases - 2

- ❑ Variation in Sync and Pdelay intervals as described in slides 20 – 23 of [2]
- ❑ rateRatio relative to the GM is computed using OCR model, as described in previous slides in the current presentation
- ❑ Other assumptions as given in the tables in slides 24 and 25 of [2] (copied to the next two slides for convenience)
- ❑ As indicated on the next slide, two cases are considered here for residence time; in addition, simulations for both (a) the GM having the same frequency stability as the PTP Relay and PTP End Instances, and (b) the GM having zero error are considered
 - Case 1a – 1 ms (corresponds to case 1 of [2])
 - Case 2 – 10 ms
 - Case 3 – 1 ms, but with zero GM error
 - Case 4 – 10 ms, but with zero GM error

Assumptions for Simulation Cases - 3

Assumption/Parameter	Description/Value
Hypothetical Reference Model (HRM), see note following the tables	101 PTP Instances (100 hops; GM, followed by 99 PTP Relay Instances, followed by PTP End Instance)
Computed performance results	(a) $\max dTE_{R(k, 0)} $ (i.e., maximum absolute relative time error between node k ($k > 0$) and GM, both filtered (PLL filter output at each node) and unfiltered (input to PLL filter at each node))
Use syncLocked mode for PTP Instances downstream of GM	Yes
Endpoint filter parameters	$K_p K_o = 11$, $K_i K_o = 65$ ($f_{3dB} = 2.5998$ Hz, 1.288 dB gain peaking, $\zeta = 0.68219$)
Timestamp granularity	8 ns
Dynamic timestamp error	Base case: +8 ns with probability 0.5, -8 ns with probability 0.5 Case 1: Uniformly distributed over [-8 ns, +8 ns]
Simulation time	3150 s; discard first 50 s to eliminate any startup transient before computing $\max dTE_{R(k, 0)} $ (i.e., 10 cycles of frequency variation after discard)
Residence time	1 ms, 10 ms
Pdelay turnaround time	10 ms

Assumptions for Simulation Cases - 4

Assumption/Parameter	Description/Value
Number of independent replications, for each simulation case	300
GM rateRatio and neighborRateRatio computation granularity	0 (i.e., we do not truncate when computing timestamp differences and ratios of differences, but use floating point arithmetic)
Mean link delay	500 ns
Link asymmetry	0
Any variable PHY delay in addition to the dynamic timestamp error described above is assumed to be zero	0

Simulation Results - 1

- ❑ Results for $\max|dTE_R|$ for Case 1 of [2] and Cases 1a and 3 here (1 ms residence time) are shown on the next two slides (the first slide shows results after the endpoint PLL filter; the second shows results before the endpoint PLL filter; all the case 1a and case 3 results include the OCF function)
- ❑ Case 1a, which includes GM frequency instability, shows very large variability over successive PTP Instance
- ❑ Comparing Case 1a and Case 1, it is seen that replacing neighborRateRatio accumulation to obtain rateRatio by the OCF function causes the largest filtered $\max|dTE_R|$ to increase from approximately 400 ns to 2400 ns, i.e., by a factor of 6
- ❑ The Case 2 results are slightly less than the Case 1a results; for Case 2 the largest $\max|dTE_R|$ is approximately 2200 ns
- ❑ The large variability in the Case 1a results is likely due the successive PTP Instances having difficulty tracking the GM phase variation

Simulation Results - 2

Cases 1, 1a, and 3; assumptions of [2], except as noted in previous slides

Single replication of simulations

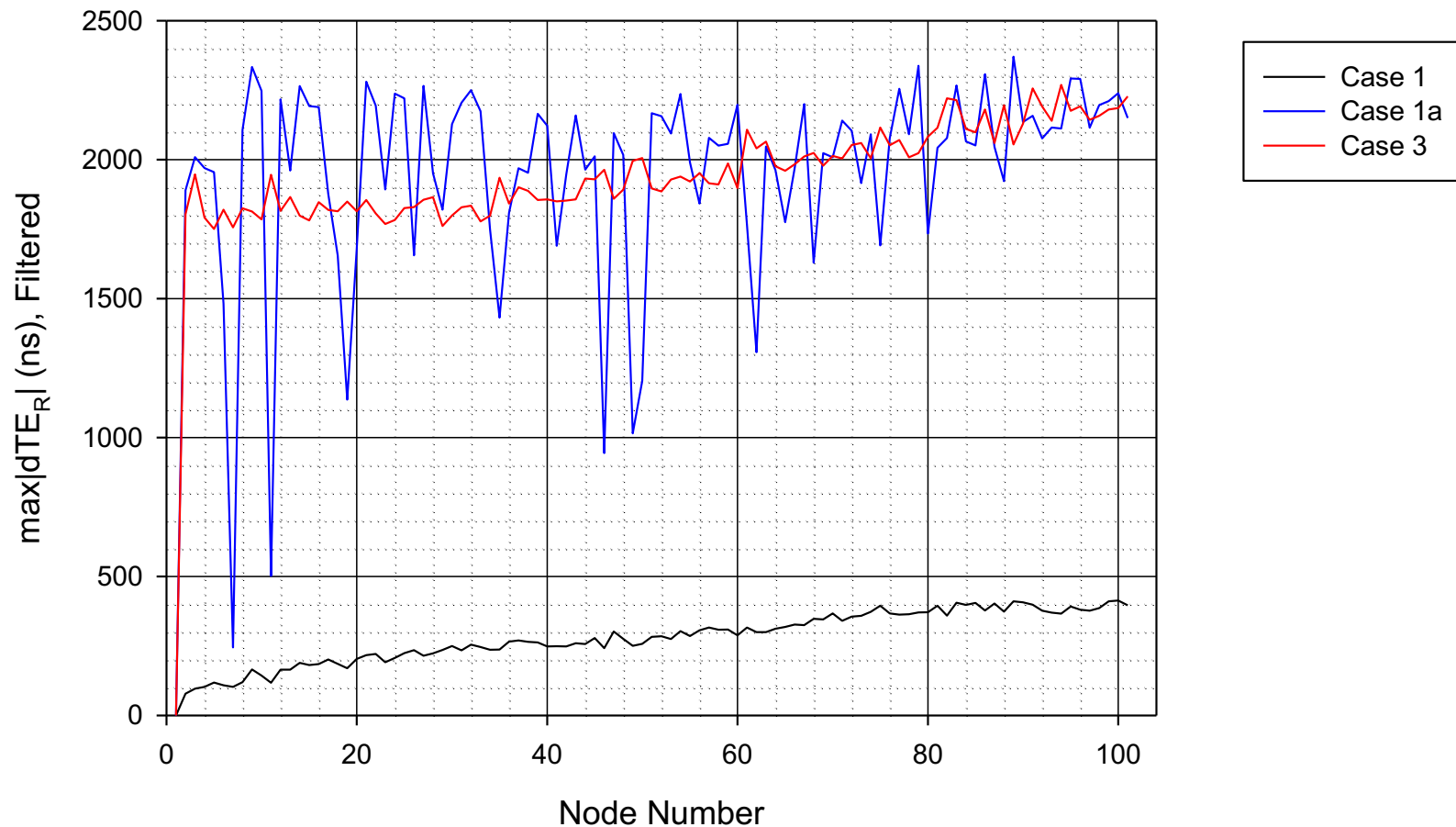
Residence time = 1 ms

+/-10% Sync Interval variation, 0-30% Pdelay Interval variation

GM time error modeled in Cases 1 and 1a; GM time error is zero in Case 3

GM labeled node 1

KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



Simulation Results - 3

Cases 1, 1a, and 3; assumptions of [2], except as noted in previous slides

Single replication of simulations

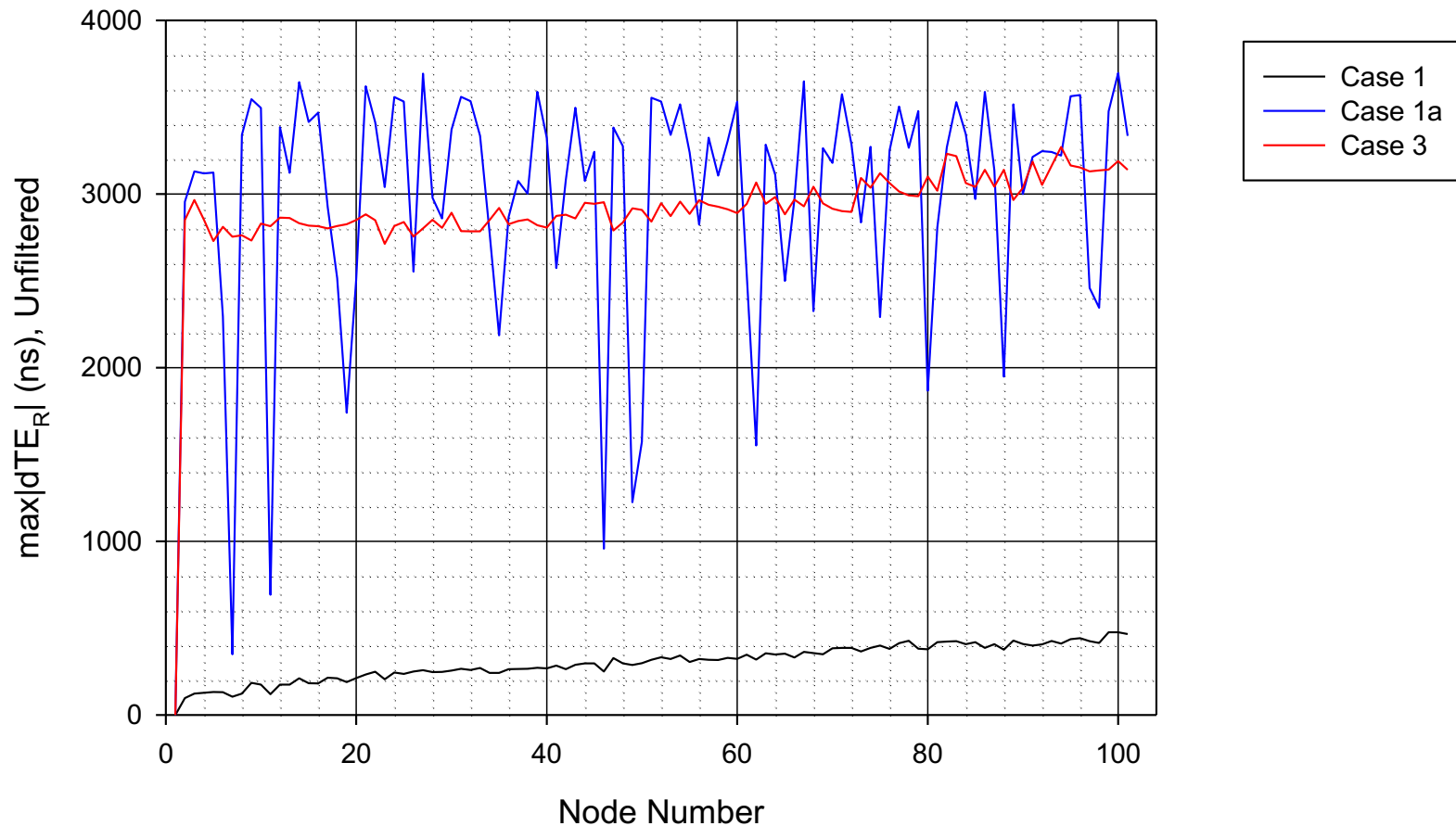
Residence time = 1 ms

+/-10% Sync Interval variation, 0-30% Pdelay Interval variation

GM time error modeled in Cases 1 and 1a; GM time error is zero in Case 3

GM labeled node 1

KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



Simulation Results - 4

- ❑ Cases 2 and 4 have residence time increased to 10 ms
- ❑ In these cases, the large gain over many hops (see the frequency response plots on slides 21 and 22) results in extremely large $\max|dTE_R|$ as the number of hops increases
- ❑ After 100 hops, filtered and unfiltered $\max|dTE_R|$ has increased to approximately 30×10^6 ns (i.e., 30 ms) and 40×10^6 ns (i.e., 40 ms), respectively
- ❑ Both linear and semi-log plots are provided for $\max|dTE_R|$ as a function of the number of hops; the results indicate that after approximately 30 – 40 hops, the increase in $\max|dTE_R|$ with hop number is exponential
- ❑ Recall (slide 23) that the high-frequency gain goes like $(1 + 2\zeta\omega_n T_{res})^N$ for N hops
- ❑ The high-frequency gain peaking for a single hop is $1 + 2\zeta\omega_n T_{res}$, which is larger for larger residence time
- ❑ The high-frequency gain for 10 ms residence time is approximately 1.11 for 1 hop, 800 for 64 hops, and 3.4×10^4 for 100 hops

Simulation Results - 5

- The large growth in time error, jitter, or wander that can occur over a chain of clocks, PLLs, or filters when gain peaking is excessive has been known for at least 40 years
 - Some examples are given in Appendix IV of [5]
 - Early work on this is contained in [6]

Simulation Results - 3

Cases 2 and 4; assumptions of [2], except as noted in previous slides

Single replication of simulations

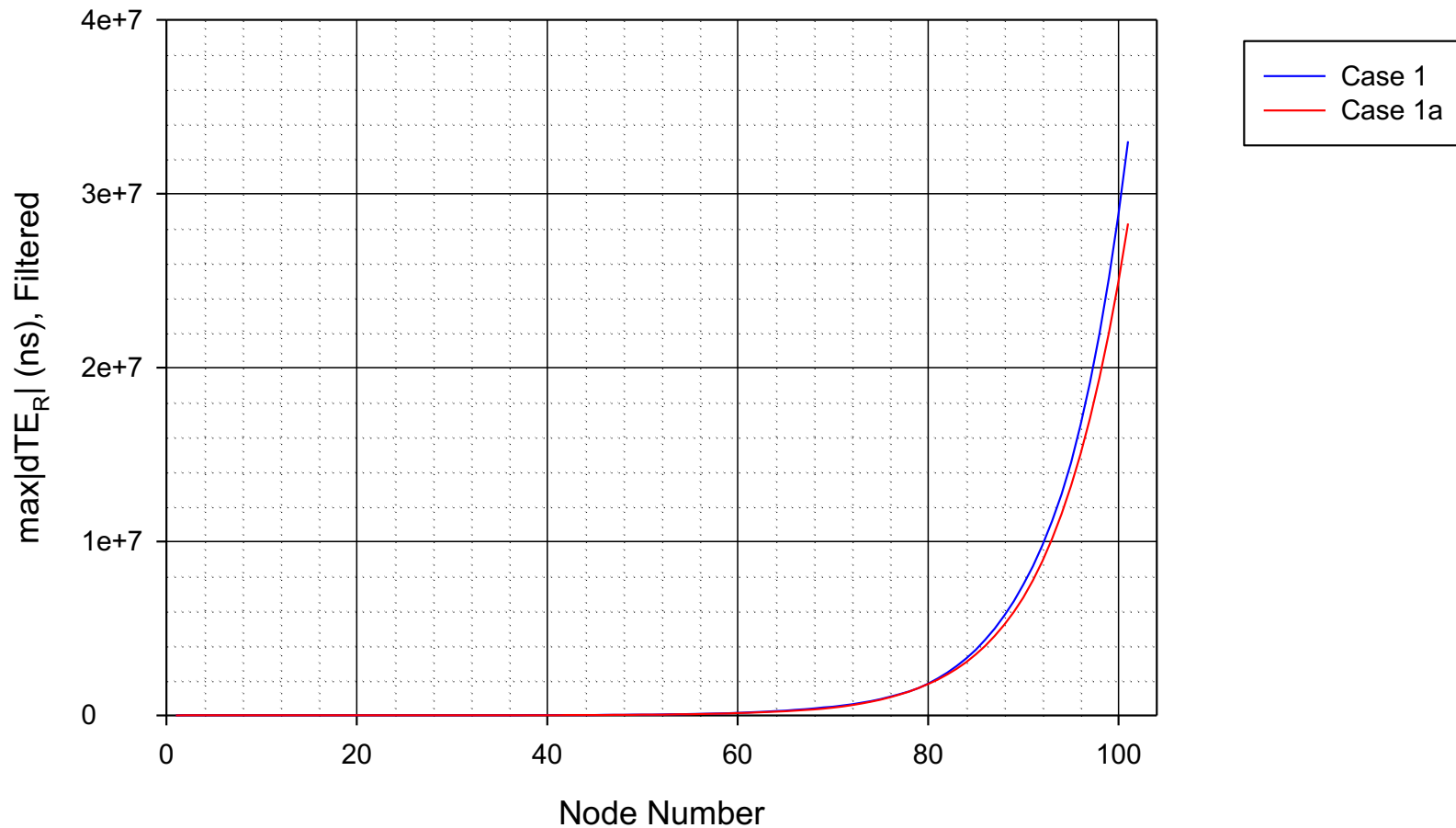
Residence time = 10 ms

+/-10% Sync Interval variation, 0-30% Pdelay Interval variation

GM time error modeled in Cases 1 and 1a; GM time error is zero in Case 3

GM labeled node 1

KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



Simulation Results - 4

Cases 2 and 4; assumptions of [2], except as noted in previous slides

Single replication of simulations

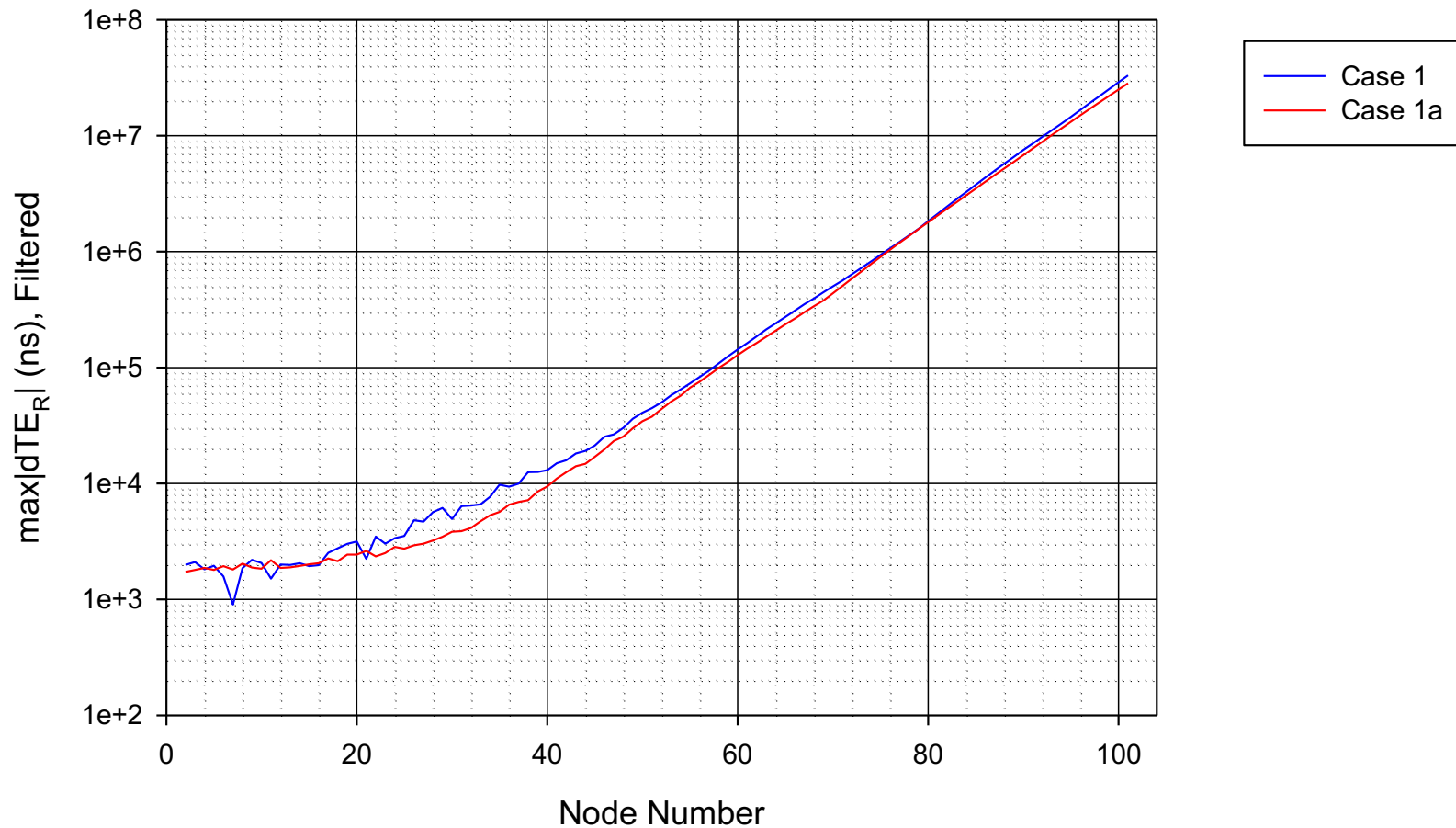
Residence time = 10 ms

+/-10% Sync Interval variation, 0-30% Pdelay Interval variation

GM time error modeled in Cases 1 and 1a; GM time error is zero in Case 3

GM labeled node 1

KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



Simulation Results - 5

Cases 2 and 4; assumptions of [2], except as noted in previous slides

Single replication of simulations

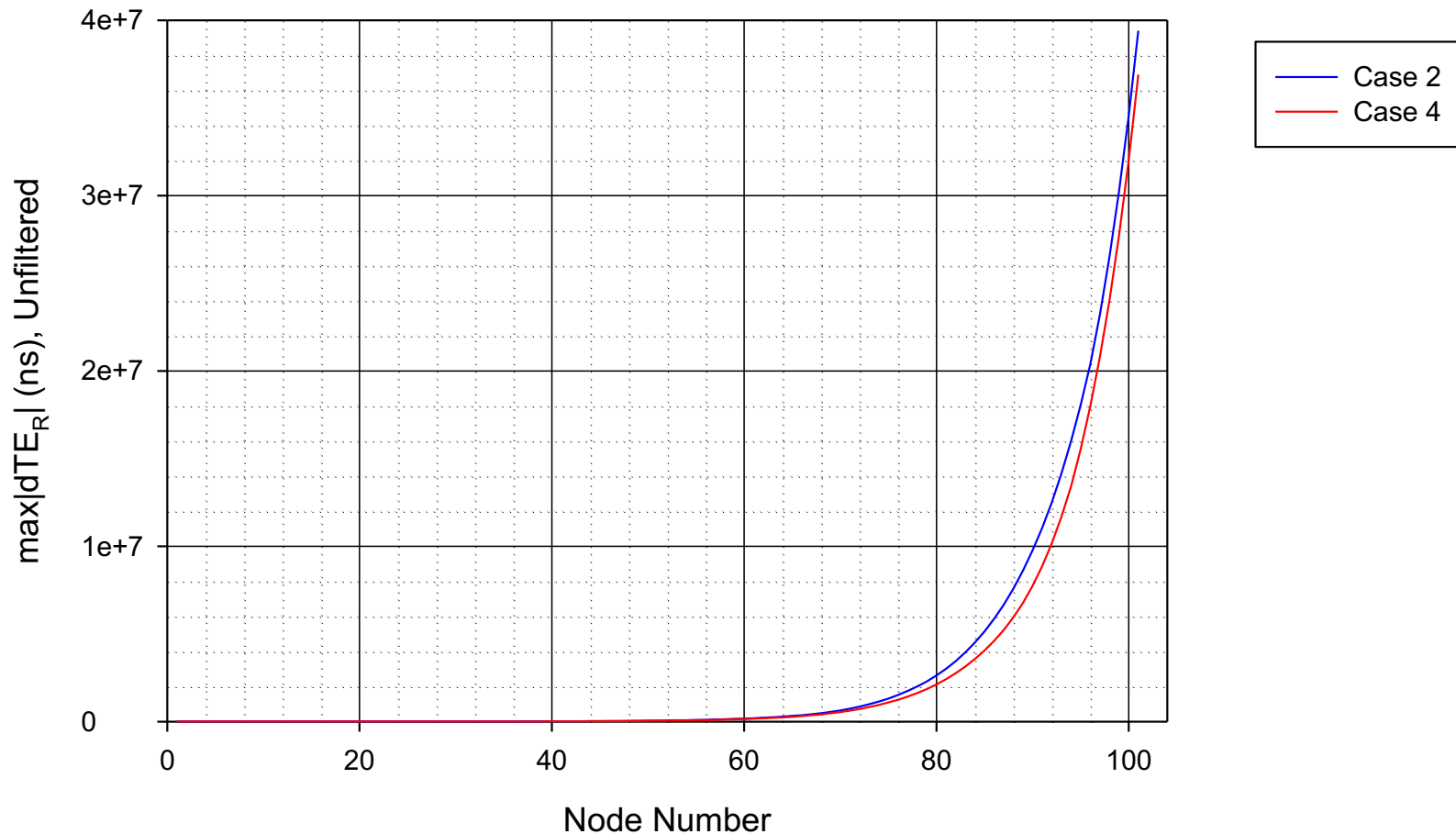
Residence time = 10 ms

+/-10% Sync Interval variation, 0-30% Pdelay Interval variation

GM time error modeled in Cases 1 and 1a; GM time error is zero in Case 3

GM labeled node 1

KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



Simulation Results - 6

Cases 2 and 4; assumptions of [2], except as noted in previous slides

Single replication of simulations

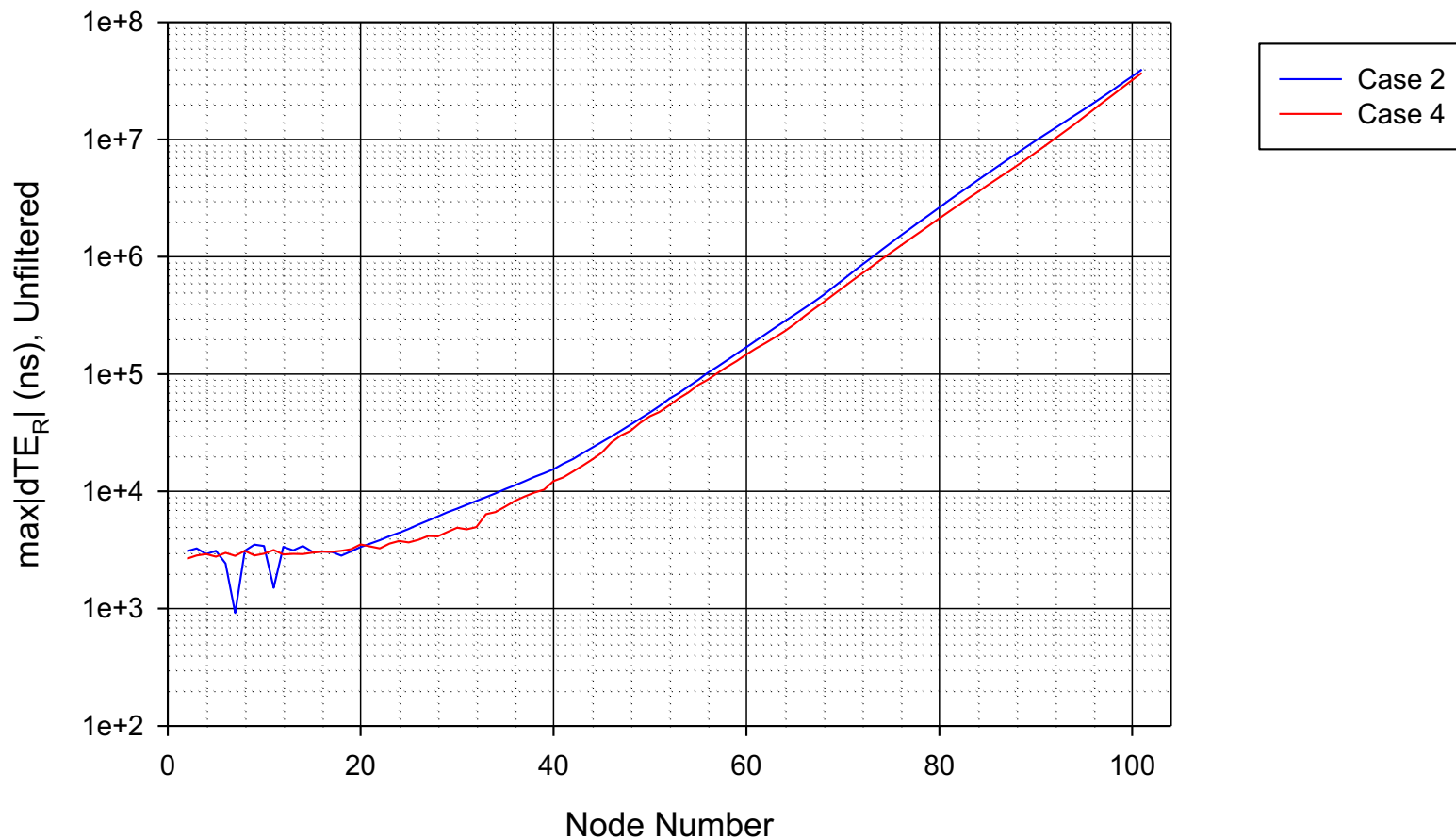
Residence time = 10 ms

+/-10% Sync Interval variation, 0-30% Pdelay Interval variation

GM time error modeled in Cases 1 and 1a; GM time error is zero in Case 3

GM labeled node 1

KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



Conclusion

- The scheme of slide 15 of [1], at least as interpreted here, results in large gain peaking and large accumulation of $\max|dTE_R|$ for 1 ms and 10 ms residence time and controller parameters as indicated here (i.e., $K_oK_p = 11$, $K_iK_o = 65$)
 - The effect is particularly pronounced for 10 ms residence time, where the frequency response for one hop is approximately 1.11 for one hop above the 10 rad/s corner frequency, and increases to 800 for 64 hops, and 3.4×10^4 for 100 hops above the corner frequency
 - The effect is smaller, but still significant, for 1 ms residence time, where the frequency response for one hop is approximately 1.011 for one hop above the 10 rad/s corner frequency, and increases to approximately 2.02 for 64 hops and 3 for 100 hops
 - As a result, $\max|dTE_R|$ reaches approximately 2400 ns for 1 ms residence time and 30 ms for 10 ms residence time, after 100 hops

Conclusion

- High-frequency (i.e., above the corner frequency, which is approximately the undamped natural frequency) gain goes like $(1+2\zeta\omega_n T_{res})^N$, where N is the number of hops
 - This means that gain peaking can be controlled (i.e., decreased) by reducing the residence time
- The above conclusions are based on the scheme of [1] and the interpretation of the scheme given here
 - A White Paper [4] was provided after the work of the current presentation was completed
 - The White Paper gives additional details, which might result in modification of the models described in the current presentation
 - If so, subsequent analysis will be provided in a future presentation

Thank you

References - 1

[1] Dragan Obradovic, *Controller Design for Clock Slaves*, IEC/IEEE 60802 Presentation, June 27, 2022 (available at <https://www.ieee802.org/1/files/public/docs2022/60802-Obradovic-Controller-for-ClockSlaves-0622.pdf>)

[2] Geoffrey M. Garner, *New Simulation results for Base Case and Case 1 of the Time Sync Breakout Held during the IEC/IEEE 60802 Ad Hoc Session, Revision 2*, IEC/IEEE 60802 Presentation, September 12, 2022, available at <https://www.ieee802.org/1/files/public/docs2020/60802-garner-new-simul-results-base-case-case-1-of-June2022-ad-hoc-0722-v02.pdf>

[3] Geoffrey M. Garner, *Improved Analysis of Component of dTER for Synchronization Transport over an IEC/IEEE 60802 Network due to GM Time Error*, IEC/IEEE 60802 Presentation, November 2, 2020 (available at <https://www.ieee802.org/1/files/public/docs2020/60802-garner-improved-analysis-component-dTER-due-to-GM-time-error-1120-v00.pdf>)

References - 2

- [4] Dragan Obradovic, *Input for simulating ClockSync according to IEC/IEEE 60802*, IEC/IEEE 60802 White Paper, September 2, 2022 (available <https://www.ieee802.org/1/files/public/docs2022/60802-Obradovic-Simulation-WhitePaper-0922.pdf>)
- [5] ITU-T Rec. G.8251, *The control of jitter and wander within the optical transport network*, ITU-T, Geneva, November 2018 (available at <https://www.itu.int/rec/T-REC-G.8251-201811-I>)
- [6] Eve Varma and J. Wu, *Analysis of Jitter Accumulation in a Chain of Digital Regenerators*, Proceedings of IEEE Globecom, Vol. 2, 1982, pp. 653 – 657.