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Title	**Packet Error Rate of an IEEE 802.11 WLAN in the Presence of Bluetooth**
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Re	[IEEE 802.15-99/110r0 Call for Submission on the Coexistence Model]
Abstract	[This paper develops the theoretical foundation for calculating the probability of a WLAN packet error in a Bluetooth environment]
Purpose	[The purpose is to start development of a theoretical coexistence model]
Notice	
Release	

1 Introduction

In this paper I develop formula for the probability of an 802.11 packet error in the presence of a Bluetooth Piconet. This is also referred to as the *Packet Error Rate.* In this paper, I am interested in the impact of Bluetooth on a IEEE 802.11 Wireless Local Area Network (WLAN). This paper does not investigate the impact of a WLAN on Bluetooth.

This theoretical analysis is intended to complement the MAC layer simulation that is being developed by Nada Golmie [1].

There are a number of parameters that effect the probability of a packet error. All those parameters are included in this theoretical model. Many of those parameters must be derived based on the Physical (PHY) Layer model. The development of that Physical Layer model in not covered in this paper.

2 Summary of Previous Work

Several people have looked at this issue previously, with a specific focus on the 11 Mbps WLAN. In this section I would like to summarize that previous work. In September 1998 Greg Ennis presented a paper at an IEEE 802.11 meeting entitled *Impact of Bluetooth on 802.11 Direct Sequence* [2]. In this paper Mr Ennis looked at the problem of calculating the probability of an overlap, in both time and frequency, of a continuous sequence of Bluetooth packets and an IEEE 802.11b 11 Mbps packet. That paper made no attempt to address the issue of relative power levels between the desired 802.11 packet and the interfering Bluetooth packet. That paper also assumed that the Bluetooth node is transmitting over the entire 625 μsec slot. The paper did not fully take into account the fact that the time offset between the beginning of the WLAN packet an the first Bluetooth packet is a random variable.

Jim Zyren responded to the paper by Mr Ennis in as subsequent IEEE 802.11 meeting in November 1998. Mr Zyren's paper was entitled *Extension of Bluetooth and 802.11 Direct Sequence Interference Model* [3]. Mr Zyren kept the basic model that had been introduced by Ennis but made some modifications to that model. He reduced the probability of Bluetooth hopping into the WLAN channel from $\frac{1}{3}$ to $\frac{1}{4}$ based on a 20 MHz wide channel. This assumption is based on the effect of the IF filter and the symbol correlator in the WLAN receiver. He also changed from a long preamble and header (192 μsec) to a short preamble and header (92 μsec) He changed the Bluetooth transmission time from 625 μsec to 366 μsec. He also increased the interframe spacing of fragments which results in a longer time between retransmissions.

In June 1999 Mr Zyren presented a more complete paper at the *Bluetooth '99* conference entitled *Reliability of IEEE 802.11 Hi Rate DSSS WLANs in a High Density Bluetooth Environment* [4]. In September he presented a summary of that paper at an IEEE 802.15 meeting [5]. In this paper Mr Zyren includes some more detailed Physical layer assumptions. He has a formula for the RF propagation signal levels and also describes the signal-to-interferer ratio (SIR) at which Bluetooth causes symbol errors in the WLAN packet, assuming co-channel interference. In his model at a SIR of more than 10 dB there is no impact on the WLAN and at a SIR below that level the symbol error rate is high. This is what might be considered a Symbol Error Rate (SER) curve which is very sharp. The SER is zero for a SIR greater than 10 dB, and is close to one for a SIR less than 10 dB. The value of this threshold is due to Kamerman [6].

One of the differences in this current paper from these previous papers is to allow for the incorporation of a more accurate Physical Layer model. In this paper the Symbol Error Rate curve does not have to be discontinuous. Also, in this model it is possible to incorporate a Physical level model that includes the effect of interference that is not necessarily co-channel interference. This paper also takes into consideration that the offset between the WLAN packet and the Bluetooth packet is a random variable.

3 Description of Packet Timing

In this analysis the WLAN packet is assumed to be sent asynchronously with respect to the Bluetooth packets. Figure 1 illustrates the timing of the WLAN packet and the Bluetooth packets. The WLAN packet is assumed to be transmitted on frequency f_W. The WLAN packet duration is T_W seconds and the power level of that packet, at the WLAN receiver, is P_W. Since we are interested in the impact of Bluetooth on the WLAN all the power levels are referenced at the WLAN receiver.

The Bluetooth packets occur on a periodic basis. The period of that communication, called the packet interval, is T_{BI} seconds. The actual transmission time is a fraction of that period. The Bluetooth transmission time is referred to as T_{BP}. Here I have shown a sequence of Bluetooth packets overlapping the WLAN packet. The first packet whose interval overlaps the WLAN packet is packet number 1. Notice that it is possible for the first packet to complete transmission before the WLAN packet, even though the packet interval overlaps the WLAN packet. The last Bluetooth packet to overlap the WLAN packet is packet number N. Later in this paper I will show how to calculate the actual value of N.

By considering only periodic Bluetooth packet transmission I am only considering the case when we are using the same length packet each time. If we are using single-slot packet then $T_{BI} = 625\mu$sec. If we are using three-slot packets then $T_{BI} = 1.875$ msec. And if we are using five-slot packets then $T_{BI} = 3.125$ msec. The time from the beginning of the first Bluetooth packet to the beginning of the WLAN packet is the random variable x. This random variable is uniformly distributed between zero and T_{BI}. We write

$$x \sim U(0, T_{BI}) \tag{1}$$

The time offset is a continuous random variable, however, in our analysis we will quantize x to the resolution of the period of a WLAN symbol. There is no need to know x at

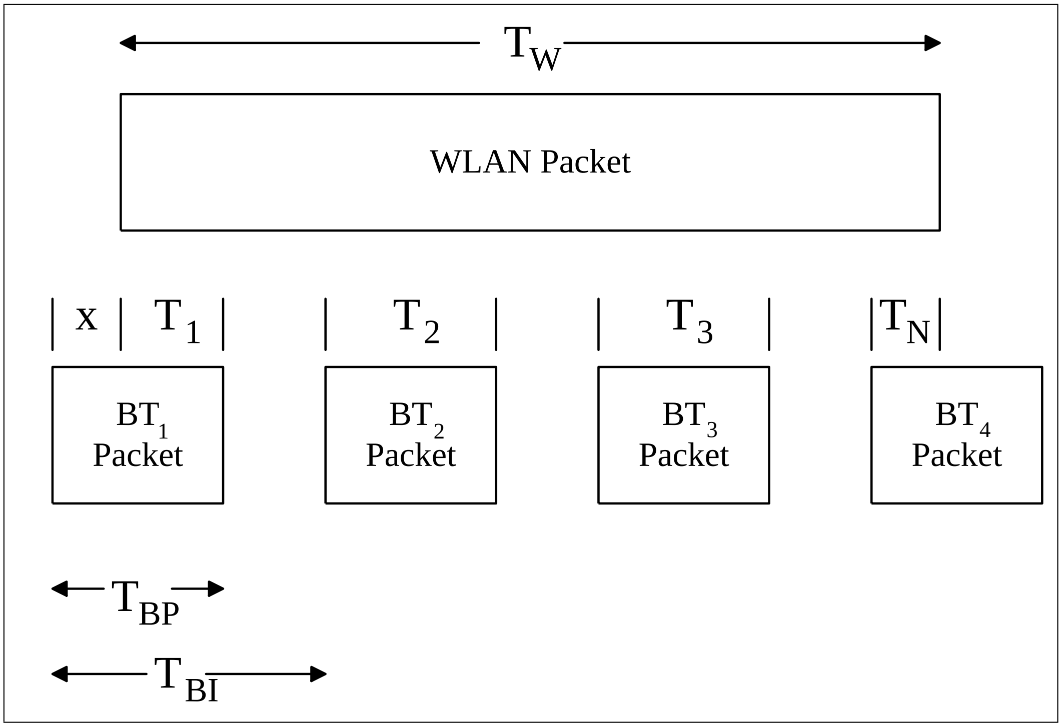

Figure 1: Timing of the WLAN and Bluetooth packets

any accuracy higher than that since in our analysis we will need to know effect of the interference on the WLAN symbol error rate.

The i-th Bluetooth packet is transmitted on frequency f_i and the power level of the i-th Bluetooth packet, measured at the WLAN receiver, is P_i.

4 Calculation of the Packet Overlap Time

In order to determine the probability of a WLAN packet error we need to determine the packet overlap times, T_i. This is the time that Bluetooth packet overlaps the WLAN packet. In order to address this issue we also need to address the actual number of overlapping packets. Previously I stated that there are N overlapping packets but I did not tell you how to calculate N. Clearly N depends on the duration of the WLAN packet and the Bluetooth interval T_{BI}. I will give the formula for the largest number of overlapping packets. Depending on the values of T_W, T_{BI}, T_{BP} and x some of those Bluetooth packets may not overlap with the WLAN packet. So when we calculate the overlap times for those packets we get zero. So there is no problem selecting N this way. The formula for N is,

$$N = \left\lceil \frac{T_W}{T_{BI}} \right\rceil + 1 \tag{2}$$

where $\lceil a \rceil$ is the smallest integer greater than or equal to a.

Now let us determine the values of the packet overlap time, T_i. Figure 1 illustrates the timing of the WLAN and Bluetooth packets. We have identified the first Bluetooth packet as the first packet whose interval overlaps with the WLAN packet. Notice, that it is possible that if $T_{BP} < T_{BI}$ that the first packet might not overlap with the WLAN packet, even if the interval of that packet overlaps with the WLAN packet.

Let us start by writing the formula for the overlap time for the first packet. From the figure you can see that the overlap time is packet duration, T_{BP} minus the random variable x, except it cannot become negative. The the overlap of the first packet is,

$$T_1 = \max(T_{BP} - x, 0) \tag{3}$$

The next packets are simpler until we get close to the end of the WLAN packet. It turns out that until we get to packet number $(N-1)$ the Bluetooth packets entirely overlap with the WLAN packet. So we have,

$$T_i = T_{BP} \qquad i = 2, 3, \ldots N-2. \tag{4}$$

Moving along, we look a the next to the last packet. There is a formula for the length of this packet's overlap with the limitation that it does not exceed T_{BP}, which leads to the following formula,

$$T_{(N-1)} = \min(x + T_W - (N-2)T_{BI}, T_{BP}) \tag{5}$$

Finally, we look at the last packet. There is a formula for this packet overlap with two constraints: it can not be negative, and it can not be larger than T_{BP}. This leads to the rather involved equation,

$$T_N = \min(\max(x + T_w - (N-1)T_{BI}, 0), T_{BP}) \tag{6}$$

Given the value of these overlap times we now need to calculate the probability of a WLAN packet error.

5 Probability of a WLAN Packet Error

I first need to define what I mean by a packet error. In an IEEE 802.11 WLAN if any of the bits are in error the Cyclic Redundancy Check (CRC) will detect the error and flag the packet as being bad. This is what is referred to here as a packet error. The receiver must then request that the packet be retransmitted. This results in a decrease in the WLAN throughput and an increase in network latency. The probability of a packet error is also often referred to as the *packet error rate.*

In order to proceed with the analysis of the probability of a packet error it is convenient to define some random events.

The WLAN Packet Error event, PE, means that the WLAN packet has at least one bit error. I will often refer to a symbol error since not all WLANs use binary modulation. For example, the 2 Mbps DSSS WLAN uses QPSK modulation, and hence there are two bits per symbol. Of course, if there is a symbol error then there is at least one bit error.

The Good WLAN Packet event, GP, means that the WLAN packet has no bit errors. So the GP event is the complement of the PE event.

Now we need to look at the overlap of each of the individual Bluetooth packets and the WLAN packet.

Call the segment of the WLAN packet overlapping with the i-th Bluetooth packet S_i. That segment of the WLAN packet consists of a sequence of contiguous symbols. Let us now define an event that all the symbols in the WLAN segment are correct. We call this event GS_i, meaning that WLAN segment S_i is good. Which means it has no errors.

Given the definition of these events we can now proceed with deriving a formula for the probability of a WLAN packet error. First, the probability of a WLAN packet error is one minus the probability of a good WLAN packet,

$$P(PE) = 1 - P(GP). \tag{7}$$

To proceed further we need to condition on the offset, x, between the WLAN and Bluetooth packets. In the previous section I showed that x is a continuous uniform random variable. For our purposes we do not need to specify the resolution of x that finely. It is sufficient for our purposes to quantize x to the resolution of the WLAN symbol length. Let T_s be the duration of a WLAN symbol, which is the inverse of the symbol rate, R_s.

If binary modulation is used, for example in the 1 Mbps DS and FH versions then the symbol rate is the same as the bit rate. If for example the a higher-level modulation scheme is used, like in the 11 Mbps WLAN, the symbol rate is bit rate divided by the number of bits per symbol. For example, in the 11 Mbps WLAN the data rate is 11 Mbps, and there are 8 bit per symbol. So the symbol rate is 11/8 million symbols per second. To be more precise, the 11 Mbps WLAN has a header that is at 1 Mbps. To keep the analysis tractable I will ignore this fact, and assume that the symbol rate does not change within a WLAN packet. This is an approximation.

We can quantize x by letting $x' = \lceil x/T_s \rceil$ and then relabeling x' as x so we can stick with x as our random variable. So if we quantize x to a discrete uniform random variable its probability mass function turns out to be,

$$p_x(k) = \frac{1}{K} \qquad k = 1, 2, \ldots K \tag{8}$$

where,

$$K = \left\lceil \frac{T_{BI}}{T_s} \right\rceil \tag{9}$$

Given this probability mass function for x we can now write the probability of a good packet in terms of the conditional probabilities,

$$P(GP) = \sum P(GP|x = k)p_x(k) \tag{10}$$

which when we fill in the uniform probability mass function we get,

$$P(GP) = \frac{1}{K} \sum_{k=1}^{K} P(GP|x = k). \tag{11}$$

The next step is to find the probability of good packet conditioned on a specific value of x. We assume that any sections of the WLAN packet that are not overlapping in time with a Bluetooth packet have no bit errors. This assumption means that without Bluetooth there would be no WLAN bit errors. This is equivalent to assuming that the WLAN signal at the receiver is significantly high, so the WLAN is not at its edge of operation. With this assumption the conditional probability of a good packet is,

$$P(GP|x = k) = P(GS_1, GS_2, \ldots GS_N|x = k). \tag{12}$$

Since these segments do not overlap we can assume that the events are independent. The probability of the joint event is the product of the probabilities of the individual events [7]. Therefore we can write,

$$P(GP|x = k) = \prod_{i=1}^{N} P(GS_i|x = k). \tag{13}$$

Now we can look at each individual segment and use the previous equation to combine our results. Each of those WLAN segments overlaps with a Bluetooth packet that can be any of 79 frequencies. Which frequency the Bluetooth packet is on has a major impact on the probability of that WLAN segment being either good or bad. So we need to condition the distribution of the Bluetooth frequency. The Bluetooth transmission frequency is a discrete uniform random variable, with the following probability mass function,

$$p_{f_i}(j) = \frac{1}{79} \qquad j = 1, 2, \ldots 79. \tag{14}$$

Using this probability mass function for f_i we can write,

$$P(GS_i|x = k) = \frac{1}{79} \sum_{j=1}^{79} P(GS_i|x = k, f_i = j) \tag{15}$$

By conditioning on x and the Bluetooth frequency we can start to solve for the probability of each WLAN segment being good. That comes down to knowing the symbol error rate for each segment and the number of symbols in each segment. The number of elements in each segment depends on the length of the segment and the symbol rate.

Since we have quantized x we need to rewrite the formulas for the packet lengths which we derived in the previous section. This involves replacing x with $T_s x$. Here are the equations for the segment lengths T_i.

$$T_1 = \max(T_{BP} - T_s x, 0) \tag{16}$$

$$T_i = T_{BP} \qquad i = 2, 3, \ldots N-2. \tag{17}$$

$$T_{(N-1)} = \min(T_s x + T_W - (N-2)T_{BI}, T_{BP}) \tag{18}$$

$$T_N = \min(\max(T_s x + T_w - (N-1)T_{BI}, 0), T_{BP}) \tag{19}$$

The number of symbols in each segment is just the length of the segment divided by the symbol period,

$$m_i = \left\lceil \frac{T_i}{T_s} \right\rceil \qquad i = 1, 2, 3, \ldots N. \tag{20}$$

The symbol error rate is a function of the signal-to-interference ratio (SIR) and the Bluetooth packet frequency. The formula for this conditional probability of a symbol error is assumed to be a function $g()$ that is supplied by the Physical Layer model.

$$p_{e|\rho_i,j} = P(e|\rho_i, f_i = j) = g(\rho_i, j) \tag{21}$$

where e is a symbol error. The signal-to-interference ratio, measured at the WLAN receiver, is given by,

$$\rho = \frac{P_w}{P} \tag{22}$$

Where I have assumed that each Bluetooth packet is at the same power level. The formula for $p_{e|\rho,j}$ is determined by the WLAN receiver's sensitivity to the Bluetooth interferer. This function $g()$ needs to come from the Physical Layer Model. In this paper it is assumed to be know.

For a fixed SIR and conditioned on the number of symbols that are in the segment and the Bluetooth frequency we can write down the equation for the conditional probability of a good segment,

$$P(GS_i|x = k, f_i = j) = (1 - p_{e|\rho,j})^{m_i} \tag{23}$$

Since the derivation of all these equations was quite long and involved I will attempt to summarize them in the next section.

6 Summary of Probability of Packet Error Equations

The first step is to select all the input parameters: T_w, T_{BI}, T_{BP} and $p_{e|\rho,j}$ for each Bluetooth frequency. The symbol error rate must come from the Physical Layer model. The next step is to calculate the value of N,

$$N = \left\lceil \frac{T_W}{T_{BI}} \right\rceil + 1. \tag{24}$$

Next we calculate the range of the quantized random variable x,

$$K = \left\lceil \frac{T_{BI}}{T_s} \right\rceil. \tag{25}$$

He have to do the next few steps for each value of $x = 1, 2, \ldots K$. We calculate the number of symbols in each WLAN segment.

$$m_1 = \max\left(\frac{T_{BP}}{T_s} - x, 0\right) \tag{26}$$

$$m_i = \frac{T_{BP}}{T_s} \qquad i = 2, 3, \ldots N-2. \tag{27}$$

$$m_{(N-1)} = \min\left(x + \frac{T_W - (N-2)T_{BI}}{T_s}, \frac{T_{BP}}{T_s}\right) \tag{28}$$

$$m_N = \min\left(\max\left(x + \frac{T_w - (N-1)T_{BI}}{T_s}, 0\right), \frac{T_{BP}}{T_s}\right) \tag{29}$$

With these values we can write the equation for the probability of a good segment, conditioned on x,

$$P(GP|x = k) = \prod_{i=1}^{N} \frac{1}{79} \sum_{j=1}^{79} (1 - p_{e|\rho,j})^{m_i} \tag{30}$$

You repeat this procedure for each value of x and then you combine those intermediate results and use the fact that the probability of a packet error is one minus the probability of a good packet and we get the final formula for the *packet error rate*,

$$P(PE) = 1 - \frac{1}{K} \sum_{k=1}^{K} P(GP|x = k). \tag{31}$$

7 Several Example using a Simplified Physical Layer Model

In this section I apply the formulas derived earlier in this paper to several examples. In these examples I am using a very simplified Physical Layer model. The plan is to update this paper with the application of more accurate Physical Layer models as they become available.

7.1 Simplified Frequency Hopping Spread Spectrum Example

In this section I consider an example of 1 Mbps Frequency Hopping Spread Spectrum (FHSS) WLAN in the presence of a Bluetooth piconet. I considered three different WLAN payload sizes: 200, 500, 1000 bytes. I added to the packet 48 bits to account for the FHSS header [8]. I left out the synchronization word which is tolerant to a few errors.

The Bluetooth Piconet is assumed to be transmitting single-slot DH1 packets every slot. The total number of bits in a DH1 packet including Access Code, Header, Payload Header, Payload, and CRS is 366 bits [9]. So for this case we have $T_{BI} = 625\mu$sec and $T_{BP} = 366\mu$sec.

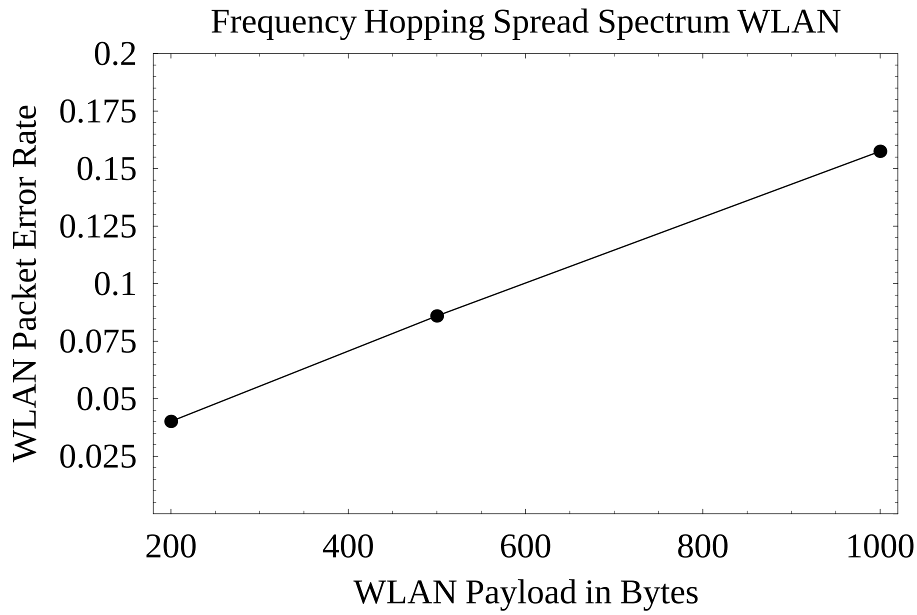

Figure 2: FHSS WLAN Packet Error Rate in the Presence of Bluetooth

The simplified Physical Layer Model that I use here assumes that if the Bluetooth packet is in the same channel as the WLAN packet that the bit error rate is as high as possible, which means the BER is 1/2. I also assumed that if the Bluetooth packet is in a different channel then the WLAN then the BER is zero. This is clearly a very simplified model but it is a reasonable starting point.

Under these conditions the WLAN Packet Error Rate was calculated for the three WLAN payload lengths and the results are plotted in Figure 2

7.2 Simplified Direct Sequence Spread Spectrum Example

In this section I consider an example of a 1 Mbps Direct Sequence Spread Spectrum (DSSS) WLAN. The payloads are the same size as in the previous example and the Bluetooth packets are once again single-slot DH1 packets. The major difference is the WLAN Physical Layer model. Once again I am going to assume that the bit error rate in the different channels is either 1/2 or zero. Which I repeat is a very simplified Physical Layer model. In this case I will set the BER to 1/2 in 22 channels since that is the width of the DSSS signal. This of course has a significant impact on the WLAN Packet Error Rate.

Under these conditions the WLAN Packet Error Rate was calculated for the three WLAN payload lengths and the results are plotted in Figure 3

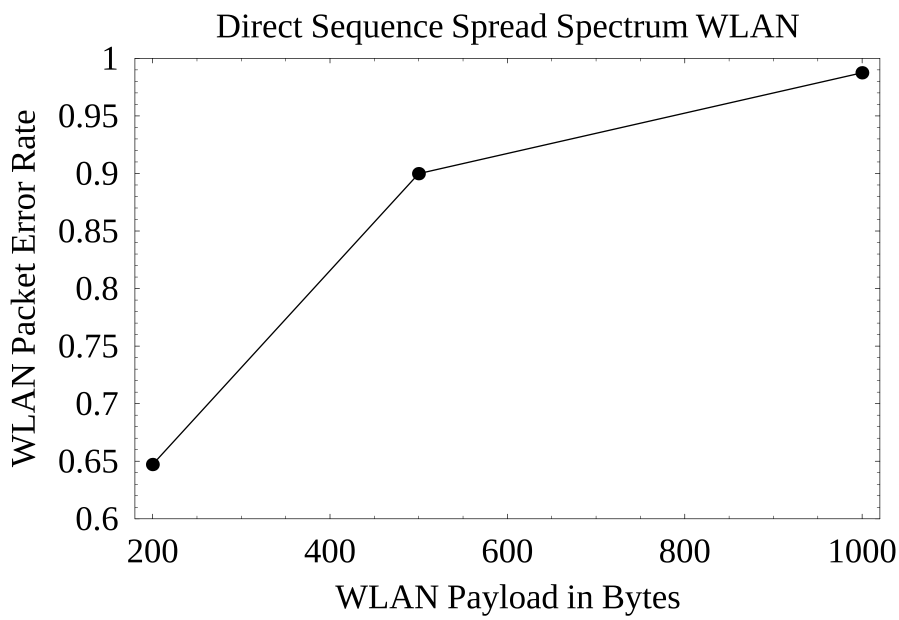

Figure 3: DSSS WLAN Packet Error Rate in the Presence of Bluetooth

8 Conclusions

In this paper I have derived the theoretical formulas for the WLAN Packet Error Rate in the presence of a Bluetooth Piconet. The formula have been applied to simplified examples of both the 1 Mbps FHSS and the 1 Mbps DSSS WLANs. My intent is apply more accurate Physical Layer models in this analysis, as they become available, and then incorporate those results into an revised copy of this paper. I also plan to compare my results to those of the simulation model that is currently being developed.

References

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