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Re:	Email and conference call discussion of OQPSK vs. QPSK and at fading channels
Abstract	A simple analysis of at fading on I/Q modulated signals
Purpose	Provide some mathematical analysis to support discussion of the merits/problems with OQPSK modulation vs. QPSK modulation
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Impact of Flat fading on I/Q modulated signals

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1 Introduction

This short article provides a very simple analysis of flat fading on I/Q modulated signals. The goal is to show that flat fading and other factors can cause cross-coupling of I and Q channels, regardless of the form of the baseband signals.

2 Analysis

For I/Q modulations, the baseband inputs to the I/Q modulator are $I(t)$ and $Q(t)$. After upconversion to RF (by an ideal direct I/Q upmixer with harmonic filtering), the modulated signal at the transmitter is:

$$s(t) = I(t)\cos(\omega_{RF}t) + Q(t)\sin(\omega_{RF}t) \tag{1}$$

At the receiver, the incoming RF signal is mixed with the LO in an I/Q downmixer. The LO provides both I and Q signals to the downmixer as:

$$s_I(t) = A_I \sin\left(\omega_{LO}t + \phi_I\right) \tag{2}$$

$$s_Q(t) = A_Q \cos\left(\omega_{LO}t + \phi_Q\right) \tag{3}$$

where $\omega_{LO} = \omega_{RF}$ for a direct conversion receiver.

Ideally, $A_I = A_Q = A_{LO}$ and $\phi_I = \phi_Q = \phi_{LO}$ (more on that later). Assuming for now the ideal case, the downmixed desired signal (after low pass filtering) is simply

$$I_{BB}(t) = \frac{1}{2}I(t) \tag{4}$$

$$Q_{BB}(t) = \frac{1}{2}Q(t). \tag{5}$$

Now suppose that a copy of the desired signal with relative amplitude $A_d < 1$ and time delay τ, is also present at the ideal downconverter. This signal is:

$$s_d(t) = A_d I(t-\tau)\cos\left[\omega_{RF}(t-\tau)\right] + A_d Q(t-\tau)\sin\left[\omega_{RF}(t-\tau)\right] \tag{6}$$

In the I channel of the receiver, $s_d(t)$ is multiplied (in the time domain) by $s_I(t)$, with the result

$$\begin{aligned} I_{BBd}(t) &= s_d(t)\cdot s_I(t) & (7)\\ &= A_d I(t-\tau)\cos\left[\omega_{RF}(t-\tau)\right]\cos(\omega_{RF}t)\\ &\quad + A_d Q(t-\tau)\sin\left[\omega_{RF}(t-\tau)\right]\cos(\omega_{RF}t) & (8)\end{aligned}$$

Using equations 20 and 22 gives

$$\begin{aligned} I_{BBd}(t) &= \frac{1}{2}A_d I(t-\tau)\left[\cos(2\omega_{RF}t-\omega_{RF}\tau)+\cos(\omega_{RF}\tau)\right]\\ &\quad + \frac{1}{2}A_d Q(t-\tau)\left[\sin(2\omega_{RF}t-\omega_{RF}\tau)+\sin(-\omega_{RF}\tau)\right] & (9)\end{aligned}$$

After low pass filtering the baseband signal, the result on the I channels is

$$I_{BBd}(t) = \frac{1}{2}A_d\left[I(t-\tau)\cos(\omega_{RF}\tau) - Q(t-\tau)\sin(\omega_{RF}\tau)\right] \tag{10}$$

Similarly, in the Q channel of the receiver, the delayed signal, $s_d(t)$ is multiplied, in time, with the quadrature LO signal, $s_{LOQ}(t)$. This results in:

$$\begin{aligned} Q_{BBd}(t) &= s_d(t)\cdot s_Q(t) & (11)\\ &= A_d I(t-\tau)\cos\left[\omega_{RF}(t-\tau)\right]\sin(\omega_{RF}t)\\ &\quad + A_d Q(t-\tau)\sin\left[\omega_{RF}(t-\tau)\right]\sin(\omega_{RF}t) & (12)\\ &= \frac{1}{2}A_d I(t-\tau)\left[\sin(2\omega_{RF}t-\omega_{RF}\tau)-\sin(-\omega_{RF}\tau)\right]\\ &\quad \frac{1}{2}A_d Q(t-\tau)\left[\cos(\omega_{RF}\tau)-\cos(2\omega_{RF}t-\omega_{RF}\tau)\right] & (13)\end{aligned}$$

After low-pass filtering, the Q channel baseband signal is

$$Q_{BBd}(t) = \frac{1}{2}A_d\left[I(t-\tau)\sin(\omega_{RF}\tau) + Q(t-\tau)\cos(\omega_{RF}\tau)\right] \tag{14}$$

The total received signal, from the desired and the delayed signal, is then

$$\begin{aligned} I_{BB}(t) &= \frac{1}{2}\left[I(t) + A_d I(t-\tau)\cos(\omega_{RF}\tau) - A_d Q(t-\tau)\sin(\omega_{RF}\tau)\right] & (15)\\ Q_{BB}(t) &= \frac{1}{2}\left[Q(t) + A_d Q(t-\tau)\cos(\omega_{RF}\tau) + A_d I(t-\tau)\sin(\omega_{RF}\tau)\right] & (16)\end{aligned}$$

Thus, the presence of a time delayed copy of the RF signal causes cross-coupling of the I and Q information. Note that $\omega_{RF}\tau$ is a constant with respect to time and can be thought of as a phase angle, $\Delta\phi$. It can be shown that the non-ideality of the LO phase split (i.e. $\phi_I - \phi_Q \neq 0$) gives the same result of mixing the I signal into the Q path and vice versa. This can occur not only in the received path, but also in the transmit path. Since all radios are non-ideal, there is a finite amount of I/Q cross coupling even when there is no flat fading.

3 Hand Waving Regarding QPSK and OQPSK

Up to this point, the nature of I and Q have been left undefined. In the "book" version of QPSK, the I and Q channels are bipolar NRZ square pulses. However, this pulse shape is almost never used in practice (the GPS positioning signal is a noteable exception of this). Generally, a shaped pulse is used to reduce the power spectral density of the transmitted signal.

For a raised cosine pulse, the maximum of the pulse, for both I and Q, occur mid-symbol. If the phase delay, $\Delta\phi$, of the delayed signal is 1/2 of a symbol, then the maximum cross coupling will occur between symbol intervals, which should have a reduced effect.

In OQPSK, the Q channel is delayed by 1/2 of a symbol to start with, so the cross mixing of I and Q by LO phase errors is suppressed. However, a 1/2 symbol phase delay would align the maximum of the Q pulse with the I pulse, which would give the largest amount of cross-coupling. For OQPSK, the baseband signals at the reciever (before removing the T/2 delay) are:

$$I_{BB}(t) = \frac{1}{2}\left[I(t) + A_d I(t-\tau)\cos(\omega_{RF}\tau) - A_d Q(t-\tau-T/2)\sin(\omega_{RF}\tau)\right] \quad (17)$$

$$\begin{aligned} Q_{BB}(t) &= \frac{1}{2}\left[Q(t-T/2) + A_d Q(t-\tau-T/2)\cos(\omega_{RF}\tau)\right. \\ &\quad \left. + A_d I(t-\tau)\sin(\omega_{RF}\tau)\right] \end{aligned} \quad (18)$$

For the proposed system, the symbol period is 91 ns, the Q channel for OQPSK is delayed by 45 ns and the delay spread in a home environment is likely to be less than 25 ns. When the delay is 1/4 of a symbol, the effect on OQPSK and QPSK are roughly the same, the peaks of the delayed signal occurs half-way between the sampling periods of the desired signal. The actual environment is a superposition of many different reflected paths, each with a different amplitude and phase delay rather than the simple 2-ray environment used in this discussion. The effects of the more complex multipath environment are left as an exercise for the motivated reader.

4 Trignometric Identities

This is here so you don't have to look them up in Schaum's outline on Trignometry.

$$\cos(\phi)\sin(\theta) = \frac{1}{2}\left[\sin(\phi+\theta) - \sin(\phi-\theta)\right] \tag{19}$$

$$\sin(\phi)\cos(\theta) = \frac{1}{2}\left[\sin(\phi+\theta) + \sin(\phi-\theta)\right] \tag{20}$$

$$\sin(\phi)\sin(\theta) = \frac{1}{2}\left[\cos(\phi-\theta) - \cos(\phi+\theta)\right] \tag{21}$$

$$\cos(\phi)\cos(\theta) = \frac{1}{2}\left[\cos(\phi-\theta) + \cos(\phi+\theta)\right] \tag{22}$$