IEEE P802.15
Wireless Personal Area Networks

Project	IEEE P802.15 Working Group for Wireless Personal Area Networks (WPANs)
Title	**CONVERGENCE OF THE INSTANT CHANNEL REPLACEMENT ALGORITHM (ACL + SCO – HV2 LINK)**
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Source	Vitaliy Sapozhnykov, HongBing Gan, Bijan Treister, Efstratios (Stan) Skafidas, Bandspeed Inc., 500 Collins Street, Level 9, Melbourne, Victoria, Australia 3000 — Voice: 613-9614-6299; Fax: 613-9614-6699; E-mail: v.sapozhnykov@ h.gan@ b.treister@ e.skafidas@ bandspeed.com.au
Re:	[]
Abstract	Convergence of the adaptive frequency hopping algorithm for ACL + SCO HV2 traffic is analyzed in the Bandspeed Adaptive Frequency Hopping – Instant Channel Replacement proposal.
Purpose	Clarification for TG2 members.
Notice	
Release	

DEFINITIONS

Transaction	a pair of Tx – Rx slots
N	total number of hop channels
N_G	number of ‘Good’ channels
N_{BK}	number of ‘Bad-To-Keep’ channels
N_{BN}	number of ‘Bad-To-Replace’ channels

1. PROBABILITIES

1.1. The probability of generating a ‘Good’ channel

$$P_G = \frac{N_G}{N} \qquad (1)$$

1.2. The probability of generating a ‘Bad-To-Keep’ channel

$$P_{BK} = \frac{N_{BK}}{N} \qquad (2)$$

1.2. The probability of generating a ‘Bad-To-Replace’ channel

$$P_{BN} = \frac{N_{BN}}{N} \qquad (3)$$

1.3. The probability of a ‘Bad-To-Replace’ channel is the sum of the probability that a ‘Bad-To-Replace’ channel will be replaced by ‘Good’ channel and the probability that ‘Bad-To-Replace’ channel will be replaced by ‘Bad-To-Keep’ channel.

$$P_{BN} = P_{BN \to G} + P_{BN \to BK} \qquad (4)$$

1.4. The probability that 'Bad-To-Replace' channel will be replaced by 'Good' channel (see App. 1)

$$P_{BN \to G} = \frac{1}{1+K} \cdot P_{BN} \tag{5}$$

1.5. The probability that 'Bad-To-Replace' channel will be replaced by 'Bad-To-Keep' channel (see App. 1)

$$P_{BN \to BK} = \frac{K}{1+K} \cdot P_{BN} \ , \tag{6}$$

$$K = \frac{N_{BK}}{N_G}$$

1.6. Total probabilities of appearance of 'Good' and 'Bad-To-Keep' channels

$$P_{\sum G} = P_G + P_{BN \to G} \tag{7}$$

$$P_{\sum BK} = P_{BK} + P_{BN \to BK} \tag{8}$$

1.7. Transaction probabilities: 'GG', 'GBK', 'BKG', 'BKBK'

$$P_{GG} = P_{\sum G} \cdot P_{\sum G} \tag{9}$$

$$P_{GBK} = P_{\sum G} \cdot P_{\sum BK} \tag{10}$$

$$P_{BKG} = P_{\sum BK} \cdot P_{\sum G} \tag{11}$$

$$P_{BKBK} = P_{\sum BK} \cdot P_{\sum BK} \tag{12}$$

Note: $P_{GBK} = P_{BKG}$

1.8. Probabilities in ACL + SCO (HV2) link.

In an ACL + SCO (HV2) link, half of the Tx – Rx pairs (transactions) are allocated for voice and another half of transactions are allocated for data.

HV2 transaction probabilities:

$P_{GG\_HV2} = 0.5 P_{GG}$;
$P_{GBK\_HV2} = 0.5 P_{GBK}$;
$P_{BKG\_HV2} = 0.5 P_{BKG}$;
$P_{BKBK\_HV2} = 0.5 P_{BKBK}$

ACL transaction probabilities:

$P_{GG\_ACL} = 0.5 P_{GG}$;
$P_{GBK\_ACL} = 0.5 P_{GBK}$;
$P_{BKG\_ACL} = 0.5 P_{BKG}$;
$P_{BKBK\_ACL} = 0.5 P_{BKBK}$

1.9. The probability of appearance of 'Bad-To-Keep' channels ($B_K \to G$ replacements) in HV2 slots

$$\begin{aligned} P_{R\_HV2} &= P_{GBK\_HV2} + P_{BKG\_HV2} + P_{BKBK\_HV2} \\ &= 0.5(P_{GBK} + P_{BKG} + P_{BKBK}) \end{aligned} \qquad (13)$$

1.10. The probability of appearance of 'Good' channels ($G \to B_K$ replacements) in ACL slots

$$\begin{aligned} P_{A\_ACL} &= P_{GBK\_ACL} + P_{BKG\_ACL} + P_{GG\_ACL} \\ &= 0.5(P_{GBK} + P_{BKG} + P_{GG}) \end{aligned} \qquad (14)$$

2. CONVERGENCE WHEN $N_G \ge N_{BK}$

For the ICR algorithm to converge (the GUD does not grow indefinitely large) we require the probability of appearance of 'Bad-To-Keep' channels in HV2 slots to be smaller than or equal to the probability of appearance of 'Good' channels in ACL slots

$$P_{R\_HV2} \leq P_{A\_ACL} \tag{15}$$

or, equivalently, (by substitution of (13) and (14) into (15))

$$P_{GG} \geq P_{BKBK} \tag{16}$$

Condition (16) is satisfied if

$$N_G \geq N_{BK} \tag{17}$$

(See App. 2 for proof).

3. CONVERGENCE WHEN $N_G \leq N_{BK}$

To provide convergence when $N_G \leq N_{BK}$, do the following first in the low priority timeslots:

- Replace 'Good Good' channel pair to 'Bad Bad' channel pair, to save 2 good channel usage, *i.e.*, decrease Good Channel Usage Debt Counter (GUD) by 2.
- Replace 'Good Bad' channel pair to 'Bad Bad' channel pair, to save 1 good channel usage, *i.e.*, decrease GUD by 1.

In the high priority timeslots, do the following:

- Keep 'Good Good' channel pair untouched
- Replace 'Good Bad' pair to 'Good Good' channel pair as usual
- For 'Bad Good' channel pair, if GUD < -1, then replace to 'Good Good' channel pair, and increment GUD by 1. If GUD > -1, replace to 'Bad Bad' as usual.
- For 'Bad Bad' channel pair, if GUD < -1, then replace to 'Good Good' channel pair, and increment GUD by 2. If GUD > -1, keep it untouched as usual.

By this way, GUD is always converged towards to zero.

APPENDIX 1

$P_{BN \to G}$ AND $P_{BN \to BK}$ PROBABILITIES

$P_{BN \to G}$ and $P_{BN \to BK}$ may be obtained from the following system of equations

$$\begin{cases} P_{BN} = P_{BN \to G} + P_{BN \to BK} \\ P_{BN \to BK} = K \cdot P_{BN \to G} \end{cases} \quad (1A)$$

Where K is the proportionality coefficient

$$K = \frac{N_{BK}}{N_G}$$

Solving the system (1A) subject to $P_{BN \to G}$ we get

$$P_{BN \to G} = \frac{1}{1 + K} \cdot P_{BN}$$

By analogy

$$P_{BN \to BK} = \frac{K}{1 + K} \cdot P_{BN}$$

APPENDIX 2

THE PROOF

$P_{GG} \geq P_{BKBK}$ is true if $N_G \geq N_{BK}$

$$P_{GG} = P_{\sum G} \cdot P_{\sum G} = (P_G + P_{BN \to G})^2$$

$$P_{BKBK} = P_{\sum BK} \cdot P_{\sum BK} = (P_{BK} + P_{BN \to BK})^2$$

So the condition is

$$(P_G + P_{BN \to G})^2 \geq (P_{BK} + P_{BN \to BK})^2$$

$$P_G^2 + 2P_G P_{BN \to G} + P_{BN \to G}{}^2 \geq P_{BK}^2 + 2P_G P_{BN \to BK} + P_{BN \to BK}{}^2;$$

$$\frac{N_G^2}{N^2} + \frac{2N_G^2 N_{BN}}{N^2(N_G + N_{BK})} + \frac{N_{BN}^2 N_G^2}{N^2(N_G + N_{BK})^2} \geq$$

$$\geq \frac{N_{BK}^2}{N^2} + \frac{2N_{BK}^2 N_{BN}}{N^2(N_G + N_{BK})} + \frac{N_{BN}^2 N_{BK}^2}{N^2(N_G + N_{BK})^2};$$

$$N_G^2\left[\frac{1}{N^2}+\frac{2N_{BN}}{N^2(N_G+N_{BK})}+\frac{N_{BN}^2}{N^2(N_G+N_{BK})^2}\right]\geq$$

$$N_{BK}^2\left[\frac{1}{N^2}+\frac{2N_{BN}}{N^2(N_G+N_{BK})}+\frac{N_{BN}^2}{N^2(N_G+N_{BK})^2}\right];$$

$$N_G^2 \geq N_{BK}^2$$

N_G , N_{BK} are always positive, so

$$\boxed{N_G \geq N_{BK}}$$