Calculating ES1 and ES2 using Least Squares Algorithm

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*Background: 802.3bs Draft 1.1 refers to "**94.3.12.5.1 Transmitter linearity**" to calculate the inner PAM4 levels of the transmitter using a DC pattern. This is a proposal to use PRBS13Q and least squares fitting instead.*

The proposal borrows from the method to calculate SNDR in **94.3.12.5.2 Linear fit to the measured waveform** except for this change "For aligned symbol values x(n) use –1, -1/3, +1/3, and 1 to represent symbol values of 0, 1, 2, and 3."

> Compute the linear fit pulse response $p(k)$ from the captured waveform per 85.8.3.3.5 using $N_p = 16$ and $D_p = 2$. For aligned symbol values x(n) use -1, $-ES_1$, ES_2, and 1 to represent symbol values of 0, 1, 2, and 3, respectively, and where ES_1 and ES_2 are the effective symbol levels determined in 94.3.12.5.1.

Continue the calculations referenced to "**85.8.3.3.5 Linear fit to the waveform measurement at TP2**" until equation 85-7

> **85.8.3.3.5 Linear fit to the waveform measurement at TP2**
>
> Given the captured waveform $y(k)$ and corresponding aligned symbols $x(n)$ derived from the procedure defined in 85.8.3.3.4, define the M-by-N waveform matrix Y as shown in Equation (85–4).

$$Y = \begin{bmatrix} y(1) & y(M+1) & \dots & y(M(N-1)+1) \\ y(2) & y(M+2) & \dots & y(M(N-1)+2) \\ \dots & \dots & \dots & \dots \\ y(M) & y(2M) & \dots & y(MN) \end{bmatrix} \quad (85\text{–}4)$$

Rotate the symbols vector x by the specified pulse delay D_p to yield x_r as shown in Equation (85–5).

$$x_r = \begin{bmatrix} x(D_p+1) & x(D_p+2) \dots & x(N) & x(1) & \dots & x(D_p) \end{bmatrix} \quad (85\text{–}5)$$

Define the matrix X to be an N-by-N matrix derived from x_r as shown in Equation (85–6).

$$X = \begin{bmatrix} x_r(1) & x_r(2) & \dots & x_r(N) \\ x_r(N) & x_r(1) & \dots & x_r(N-1) \\ \dots & \dots & \dots & \dots \\ x_r(2) & x_r(3) & \dots & x_r(1) \end{bmatrix} \quad (85\text{–}6)$$

Define the matrix X_1 to be the first N_p rows of X concatenated with a row vector of ones of length N. The M-by-(N_p + 1) coefficient matrix, P, corresponding to the linear fit is then defined by Equation (85–7). The superscript "T" denotes the matrix transpose operator.

$$P = YX_1^T(X_1X_1^T)^{-1} \quad (85\text{–}7)$$

The following section describes how to create a 4-by-MN matrix, W where each row contains the linear component contributed by each of the symbol values [-1, -1/3, 1/3, +1]

Start with the symbol value, S = -1. Define the Np-by-N matrix X_A to be the result of an element wise comparison operator on the first Np rows of X against symbol value S. The comparison yields a value of 1 when the element matches the symbol value S and is 0 otherwise. Define the matrix X_{A1} to be X_A concatenated with a row of vector of ones of length N. Define the M-by-N matrix $W_A = PX_A$. Its elements are read out column wise to yield the row vector $w_a(k)$.

Repeat these steps for symbol values S = -1/3, +1/3 and +1, to yield the row vectors $w_b(k)$, $w_c(k)$ and $w_d(k)$ respectively.

Define the matrix W to be the concatenation of the the four row vectors $w_a(k)$, $w_b(k)$, $w_c(k)$, $w_d(k)$

Define the row vector Y_R to be the elements of y(k).

Define the 4-by-1 row vector L as

$L = Y_R W^T (WW^T)^{-1}$

The levels corresponding to the four symbol values are read row wise from L as shown

$L = [L_A \; L_B \; L_C \; L_D]$

Define $L_{mid} = (L_D+L_A)/2$

Calculate $ES1 = (L_B - L_{mid})/(L_A - L_{mid})$

Calculate $ES2 = (L_C - L_{mid})/(L_D - L_{mid})$

R_{LM} can now be defined as

R_{LM} = Minimum(3*ES1, 3*ES2, 2-3*ES1, 2-3*ES2)

This equation captures the maximum deviation from ideal level of 1/3, allowing +/-5% error