A New Method to Accurately Estimate (ADD, σ_{RJ}) of Dual Dirac Jitter Model from (J3u, JRMS)

(A presentation related to draft 2.0 comments #134, 135)

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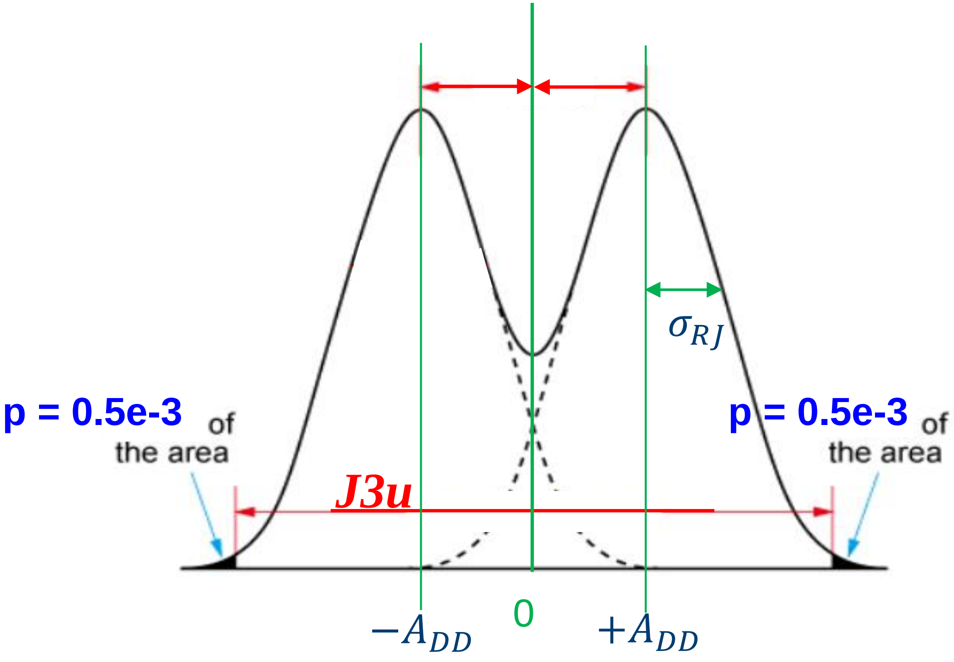
Problem Statement (1/2)

- The method to estimate (A_{DD},σ_{RJ}) from (J3u,J_{RMS}) specified by (802.3: 163-2&3) can be derived from Dual-Dirac model (DD-Model Eq.1&2).
- However, solving the set of equations (DD-Model Eq.1&2) for (A_{DD},σ_{RJ}) for a given (J3u,J_{RMS}) is not mathematically straightforward because of the 3rd unknown Q3.
  - **Three unknowns/variables with two equations, an under-determined non-linear system !!!**



$$\left\{ \begin{aligned} A_{DD} &= \frac{\frac{J3u}{2} + Q3\sqrt{(Q3^2+1)J_{RMS}^2 - \left(\frac{J3u}{2}\right)^2}}{Q3^2+1} && \text{(802.3: 163-2)} \\ \sigma_{RJ} &= \frac{\frac{J3u}{2} - A_{DD}}{Q3} && \text{(802.3: 163-3)} \end{aligned} \right.$$

$$\left\{ \begin{aligned} \frac{J3u}{2} &= A_{DD} + Q3 \cdot \sigma_{RJ} && \text{(DD-Model Eq.1)} \\ J_{RMS}^2 &= A_{DD}^2 + \sigma_{RJ}^2 && \text{(DD-Model Eq.2)} \end{aligned} \right.$$

Problem Statement (2/2)

- To overcome this issue, 802.3 D2p0 assumes Q3 = 3.2905 ≈ norminv(1-0.5*10^{-3})
  - The accuracy issue with this was pointed out by Yasuo, and he proposed revised equations to improve the estimation accuracy.
- Yasuo's proposal assumes Q3 = 3.0902 ≈ norminv(1-1*10^{-3}) as default, and conditionally changes to $Q3 = \sqrt{\left(\frac{J3u}{2J_{RMS}}\right)^2 - 1}$
- Yasuo's proposal is an improvement compared with the method in D2.0, however still suffers from accuracy as it still treats Q3 variable with two constants.
- Thus, we show a new method to accurately estimate (A_{DD},σ_{RJ},Q3) from (J3u,J_{RMS}) and Dual-Dirac model look up table (LUT) without any assumption on Q3
  - Use numerical examples of σ_{RJ} = 0.01UI and A_{DD} = 0 to 0.005 UI
  - Their ratio is essential, or jitter may be considered as normalized with σ_{RJ}

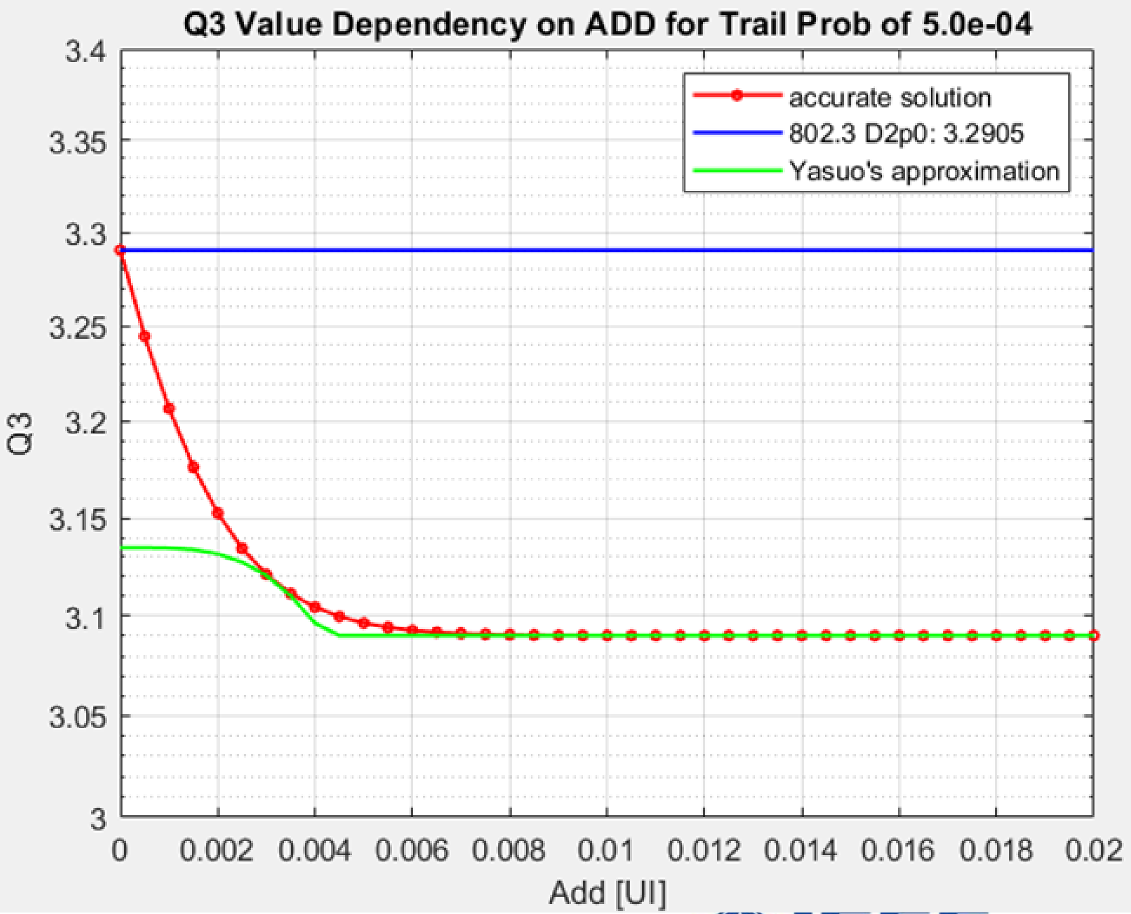
What is True Q3?

- Directly solve the non-linear Dual-Dirac equation for x (=J3u/2) with given A_{DD} and σ_{RJ}

$$\frac{normcdf(x, -A_{DD}, \sigma_{RJ}) + normcdf(x, +A_{DD}, \sigma_{RJ})}{2} = 1 - 0.5 \times 10^{-3}$$

- Calculate true Q3 and J3u, A_{DD} and σ_{RJ}

$$\frac{J3u}{2} = A_{DD} + Q3 \cdot \sigma_{RJ}$$

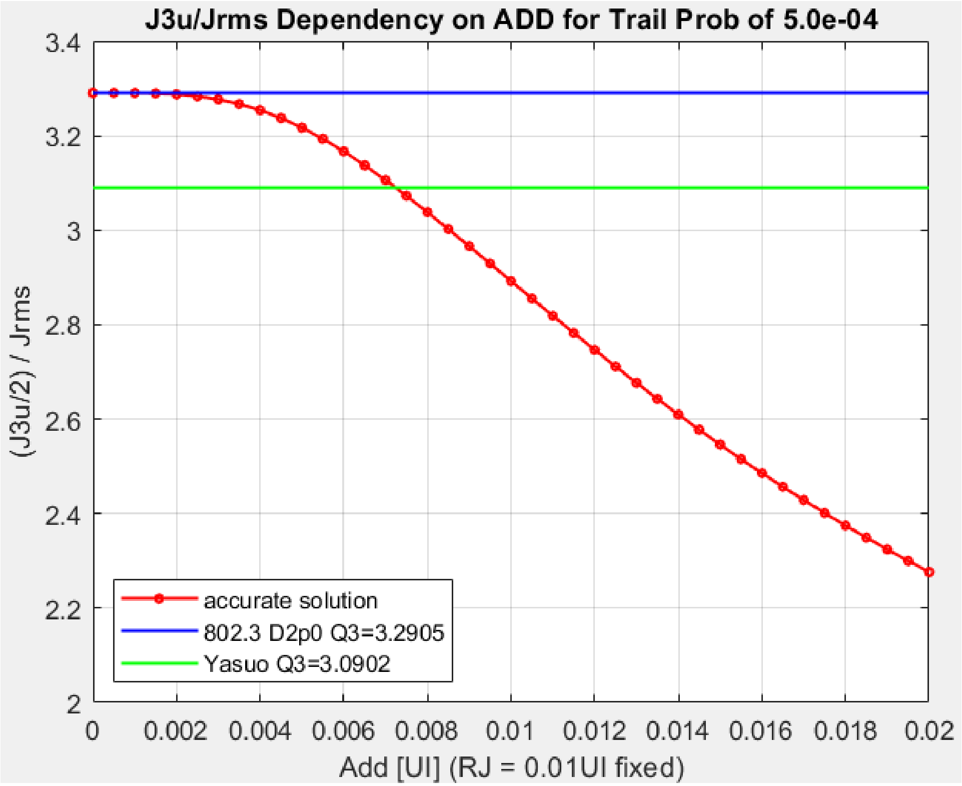


Results on the right for Add (= 0 to 0.02) and σ_{RJ}=0.01

- Q~3.2905 when Add is much smaller than σ_{RJ}
- Q is smaller than 3.2905 when Add is relatively small compared with σ_{RJ}
- Q~3.0902 when Add is relatively large compared with σ_{RJ}

New Q3 Estimation Method

- When A_{DD} is very small compared with σ_{RJ}, $(J3u/2)/J_{RMS}$ must be close to true Q3
- What's the relation between true $(J3u/2)/J_{RMS}$ and A_{DD} or (A_{DD}/σ_{RJ})?



Results on the left for Add (= 0 to 0.02) and σ_{RJ}=0.01

- $(J3u/2)/J_{RMS}$ may not be so bad estimate for Q when Add is smaller 0.005~0.006
- When Add is larger than 0.005~0.006, it is known from the previous slide that Q3 = 3.0902 is good estimate
  - This is the region where (J3u/2)/Jrms is smaller than about 3.2

Solution:

- Create Look up table (LUT): $(J3u/2)/J_{RMS}$ vs. A_{DD}/σ_{RJ}

- Interpolate if not exact in the table

Look Up Table (LUT): $(J3u/2)/J_{RMS}$ vs. A_{DD}/σ_{RJ}

$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}
3.290526731	0.00	3.289505919	0.15	3.276836881	0.30	2.966027424	0.90
3.29052671	0.01	3.289216389	0.16	3.275153197	0.31	2.892268544	1.00
3.290526388	0.02	3.288871709	0.17	3.273340465	0.32	2.818667475	1.10
3.290524997	0.03	3.288466178	0.18	3.271395702	0.33	2.746543609	1.20
3.290521261	0.04	3.287994056	0.19	3.269316254	0.34	2.676773028	1.30
3.29051341	0.05	3.287449601	0.20	3.267099806	0.35	2.609894848	1.40
3.290499194	0.06	3.286827116	0.21	3.264744379	0.36	2.546202854	1.50
3.290475901	0.07	3.286120976	0.22	3.262248328	0.37	2.485818176	1.60
3.290440379	0.08	3.285325672	0.23	3.259610342	0.38	2.428744197	1.70
3.290389066	0.09	3.284435838	0.24	3.256829437	0.39	2.374906753	1.80
3.290318015	0.10	3.283446286	0.25	3.253904949	0.40	2.324182896	1.90
3.290222927	0.11	3.282352032	0.26	3.216826237	0.50	2.276421092	2.00
3.290099186	0.12	3.281148325	0.27	3.16660322	0.60		
3.289941896	0.13	3.279830665	0.28	3.10594193	0.70		
3.289745921	0.14	3.278394828	0.29	3.03807708	0.80		

New Method to Estimate (A_{DD},$\sigma_{RJ,}$Q3)

For a given / measured data set: (J3u,J_{RMS})

- Create Look up table (LUT): (J3u/2)/J_{RMS} vs. A_{DD}/σ_{RJ}
- Find g_α for a given / measured (J3u,J_{RMS}) using LUT

$$\frac{\frac{J3u}{2}}{J_{RMS}} \equiv \alpha \equiv f\left(\frac{A_{DD}}{\sigma_{RJ}}\right) \qquad \therefore \frac{A_{DD}}{\sigma_{RJ}} = f^{-1}\left(\frac{\frac{J3u}{2}}{J_{RMS}}\right) \equiv g(\alpha) \equiv g_\alpha$$

(DD-Model Eq.1&2) can be written as follows

$$\begin{cases} \frac{J3u}{2} = A_{DD} + Q3 \cdot \sigma_{RJ} & \text{(DD-Model Eq.1)} \\ J_{RMS}^2 = A_{DD}^2 + \sigma_{RJ}^2 & \text{(DD-Model Eq.2)} \end{cases} \Rightarrow \begin{cases} \frac{J3u}{2} = A_{DD} + Q3 \cdot \sigma_{RJ} = (g_\alpha + Q3)\sigma_{RJ} & \text{(New Eq.1)} \\ J_{RMS}^2 = A_{DD}^2 + \sigma_{RJ}^2 = (g_\alpha^2 + 1)\sigma_{RJ}^2 & \text{(New Eq.2)} \end{cases}$$

From (DD-Model Eq.3&4)

$$\frac{\left(\frac{J3u}{2}\right)^2}{J_{RMS}^2} = \alpha^2 = \frac{(g_\alpha + Q3)^2}{g_\alpha^2 + 1}$$

$$\therefore Q3 = -g_\alpha + \alpha\sqrt{g_\alpha^2 + 1} \qquad \text{(New Eq.3)}$$

- Estimated Q3 accurately with α = (J3u/2/J_{RMS}) and g_α obtained with LUT

Then,

- Estimated s_{RJ} and $_{ADD}$ as follows

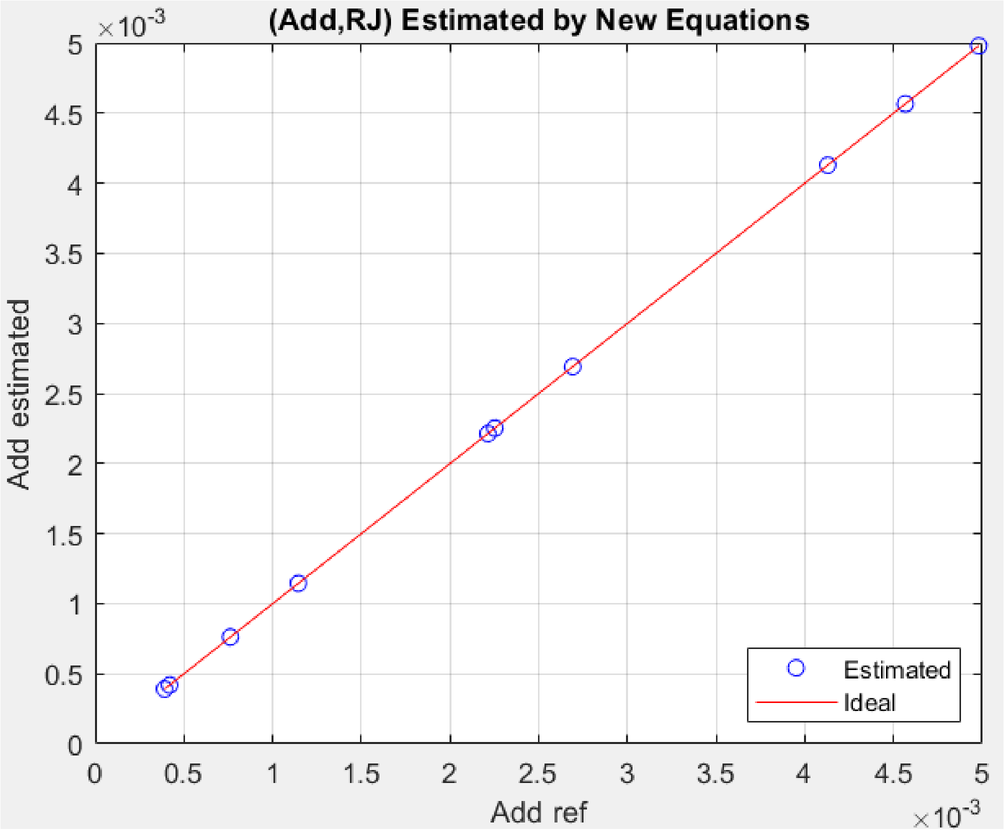
$$\begin{cases} \sigma_{RJ} = \frac{\frac{J3u}{2}}{Q3 + g_\alpha} & \text{(New Eq.4)} \\ A_{DD} = g_\alpha \sigma_{RJ} & \text{(New Eq.5)} \end{cases}$$

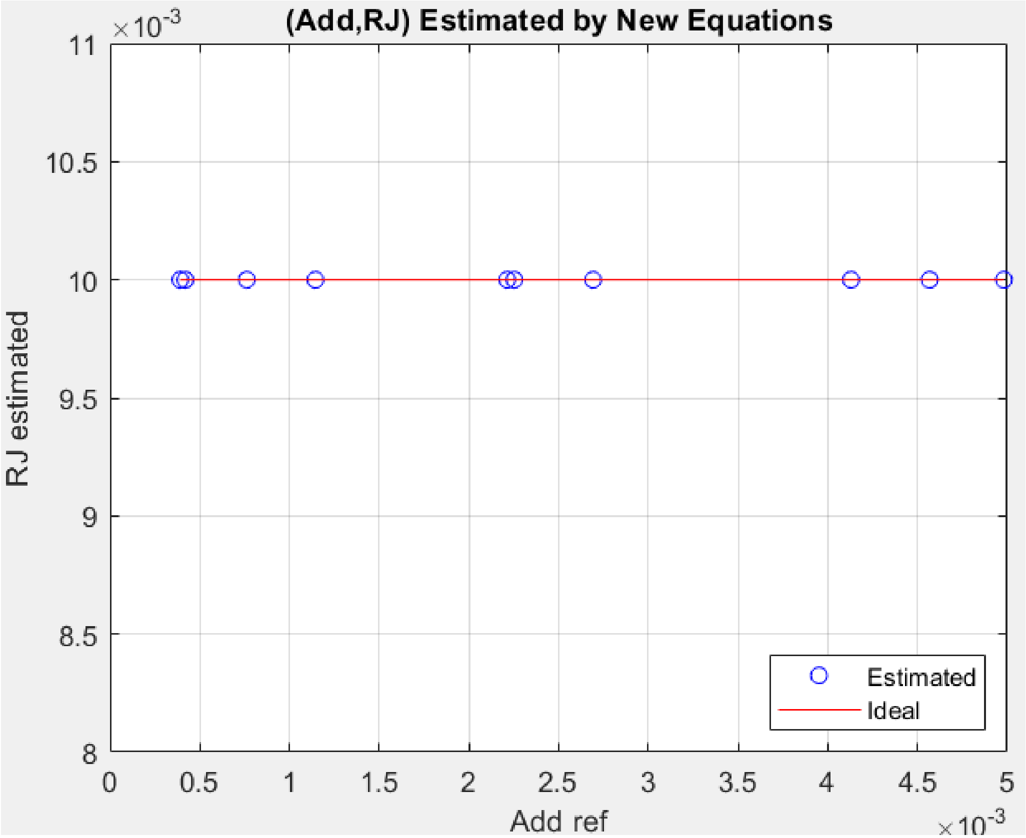
Accuracy Evaluation of Estimated (A_{DD},σ_{RJ}) (1/3)

- Estimation accuracy when A_{DD} is small compared with σ_{RJ} is of our concern
- Several A_{DD} values were randomly regenerated between 0 and 0.005 UI, and accurate ($J3u$,J_{RMS}) were generated with these A_{DD} values and σ_{RJ} = 0.001 UI.
- (A_{DD}, σ_{RJ}) were estimated from these ($J3u$,J_{RMS}) using the method, and the estimation accuracy was evaluated.

Accuracy Evaluation of Estimated (A_{DD},σ_{RJ}) (2/3)

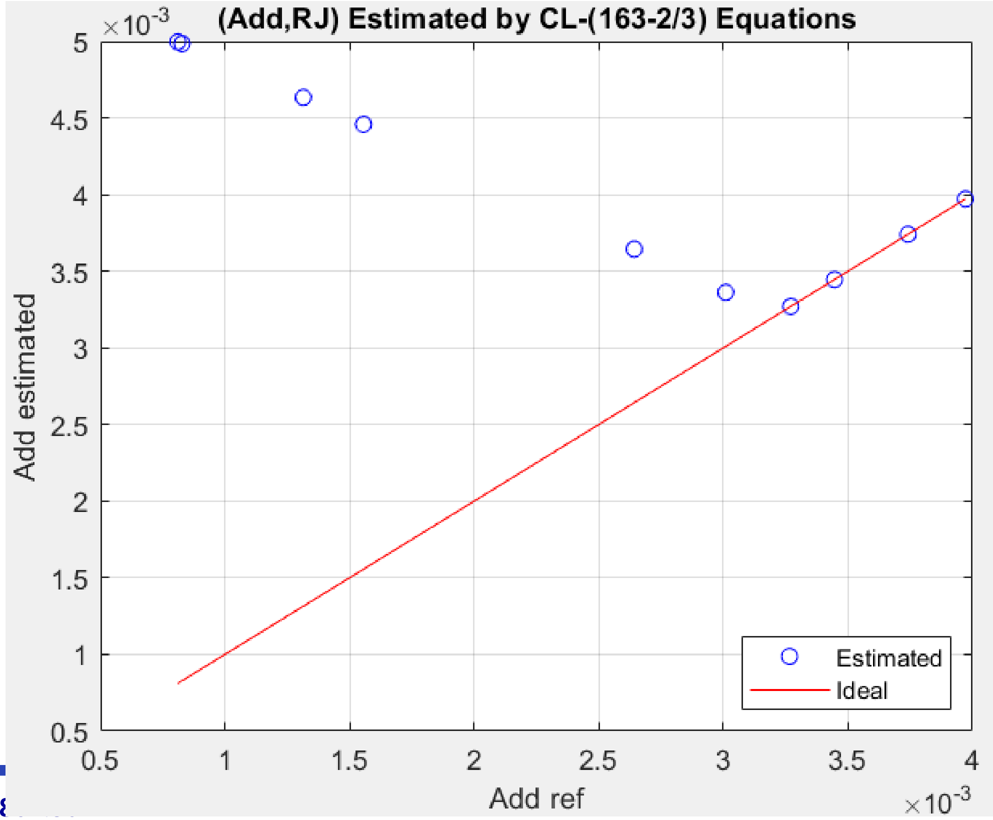
- As shown below, (A_{DD}, σ_{RJ}) were very accurately estimated from these ($J3u$,J_{RMS}) using the new method even when A_{DD} is almost 0

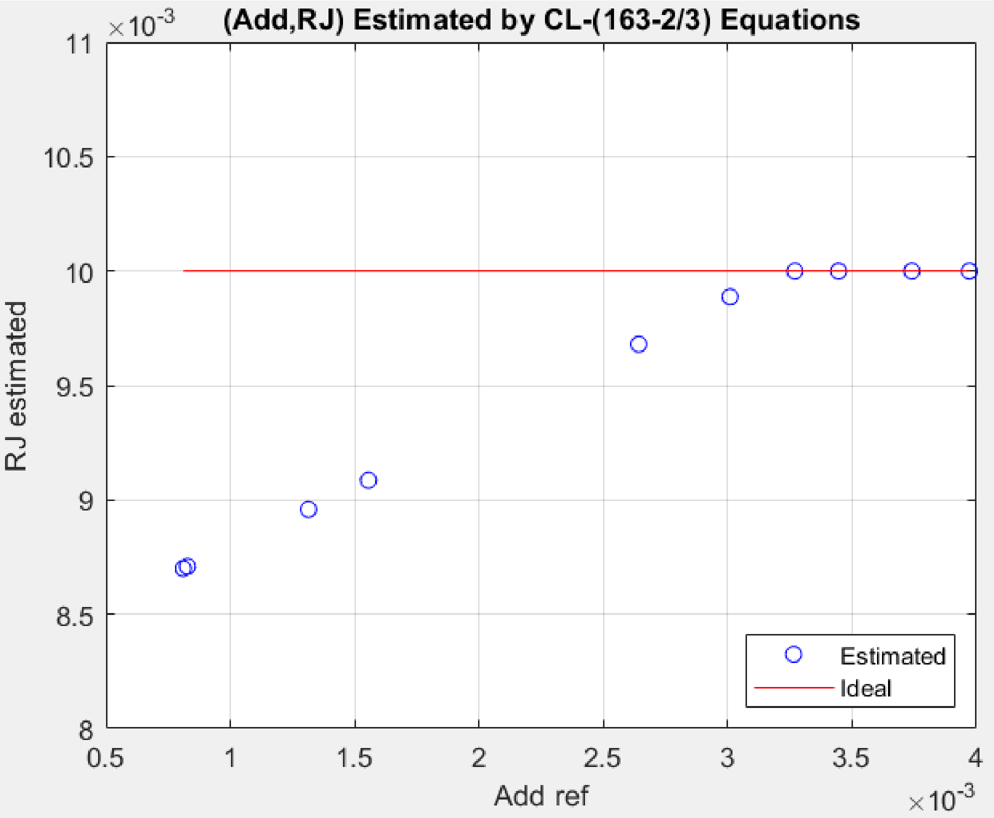




Accuracy Evaluation of Estimated (A_{DD},σ_{RJ}) (3/3)

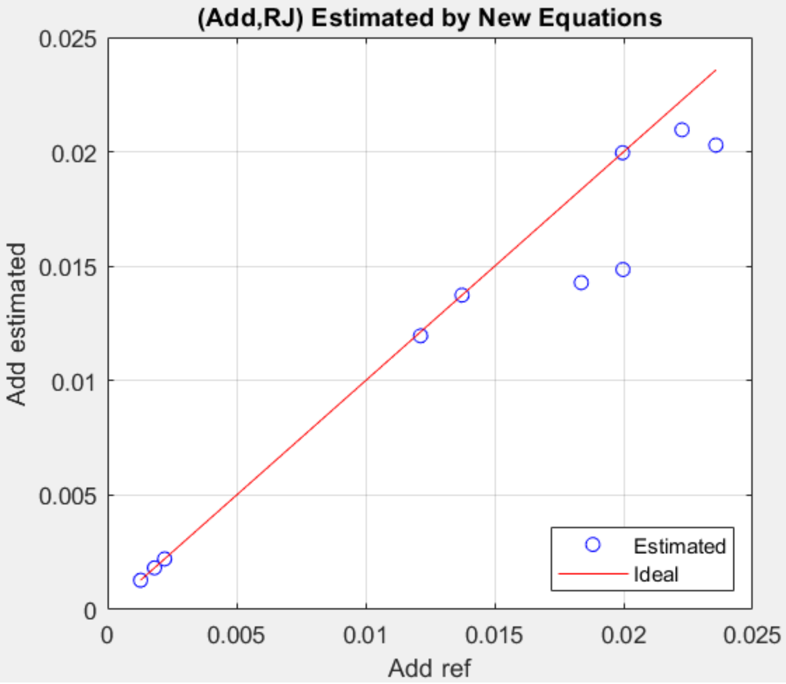
- Why equations (802.3: 163-2&3) are not used to estimate (A_{DD}, σ_{RJ}) once Q3 is accurately estimated by the new method
  - As shown below, equations (802.3: 163-2&3) do not very accurately estimate (A_{DD}, σ_{RJ}) even with very accurately estimated Q3 value when A_{DD} is very small
  - Many uses of Q3 in these equations, especially in (802.3: 163-2), seems to amplify very small error in Q3 resulting in "relatively" large error in (A_{DD}, σ_{RJ})

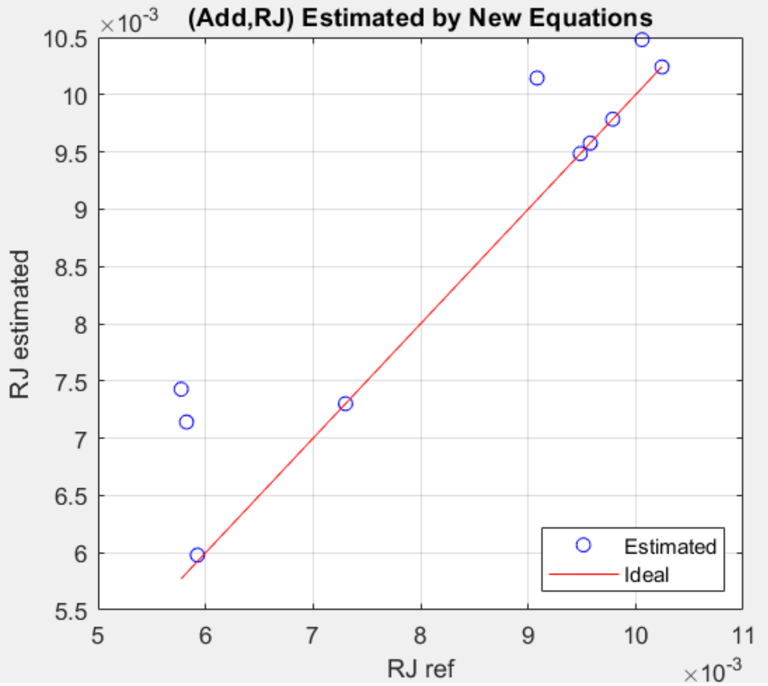


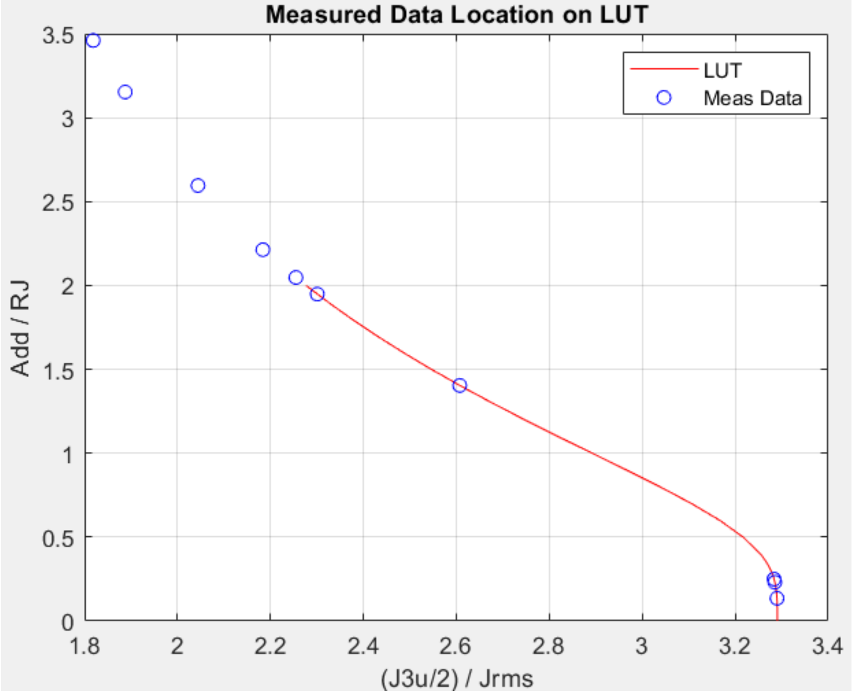


Inaccuracy due to Limited LUT Range

- Test data set ($J3u, J_{RMS}$) generated from (A_{DD}, σ_{RJ})
  - A_{DD} ~ unifrnd(0,0.025), σ_{RJ} ~ unifrnd(0.005,0.012)
- LUT A_{DD}/σ_{RJ} range: 0 ~ 2.0





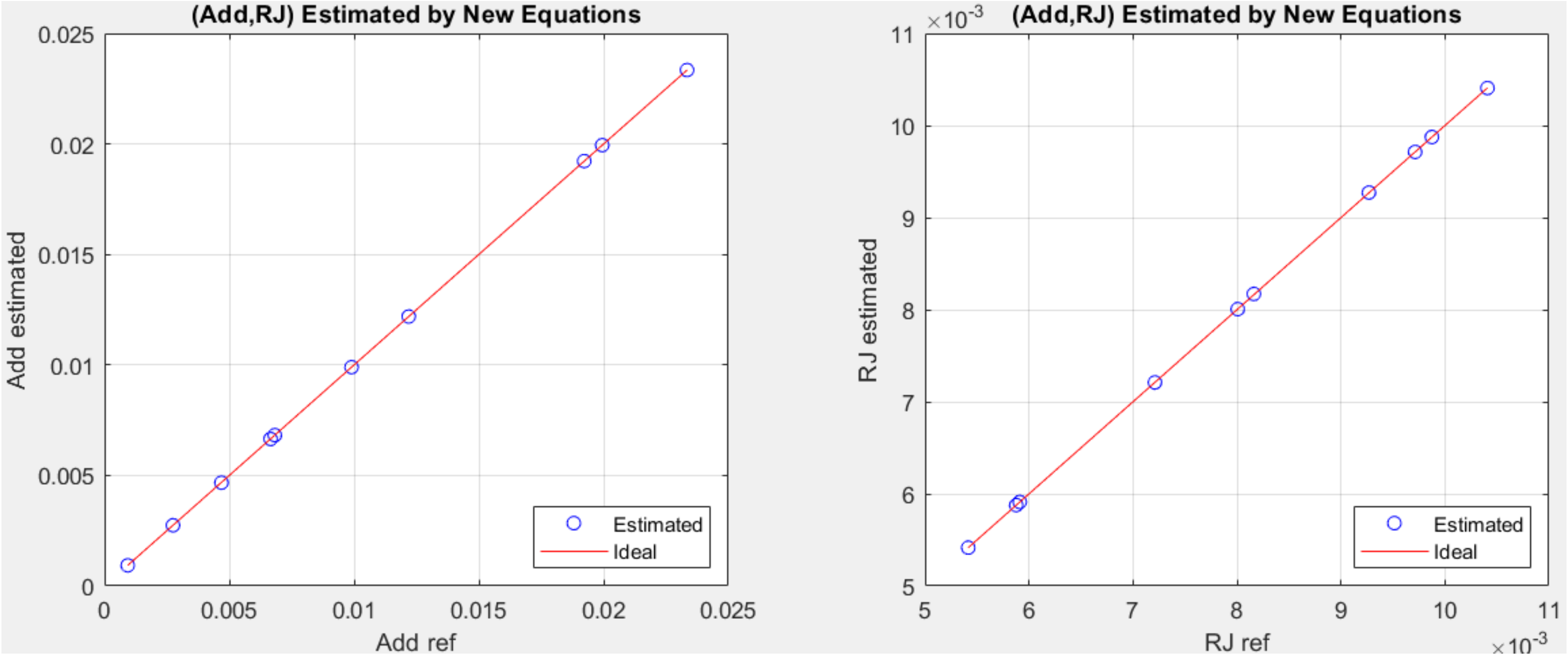


(A_{DD}, σ_{RJ}) estimation accuracy degrades when LUT does not cover

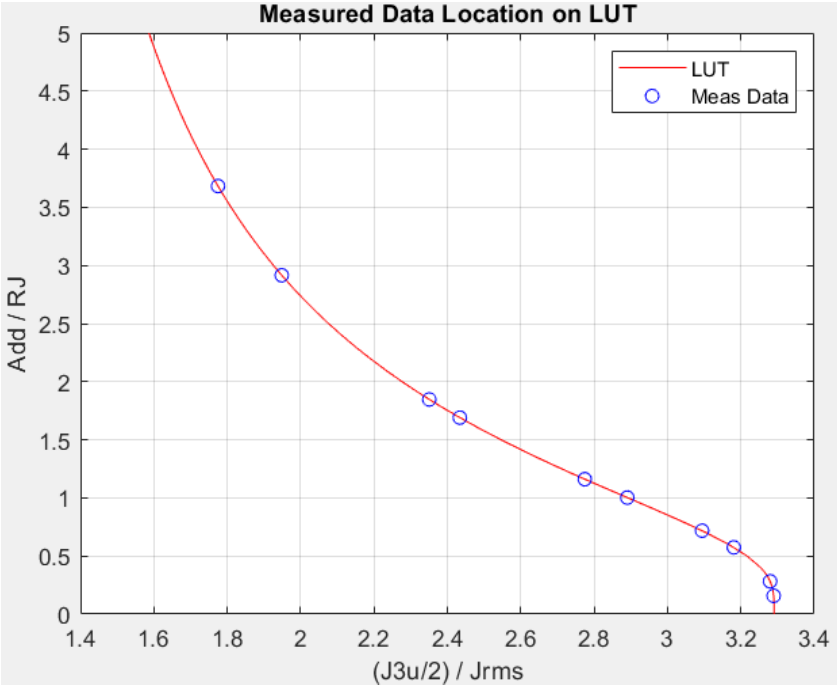
High Accuracy Maintained by Wider-Range LUT

- Should not reply on “extrapolation” of LUT

Good (A_{DD}, σ_{RJ}) estimation accuracy for large data range



LUT extended to cover large A_{DD}/σ_{RJ}



What LUT Range A_{DD}/σ_{RJ} is Enough?

- COM Spec. $(A_{DD,}\sigma_{RJ})$ = (0.02, 0.01) UI
- A_{DD} max = 0.02 UI seems large enough considering EOJ max spec = 0.025 UI
- Since RJ can be smaller than 0.01UI, let assume RJ min = 0.005 UI
- Then, max $(A_{DD,}\sigma_{RJ})$ can be 0.02/0.005 = 4

Look Up Table (LUT): $(J3u/2)/J_{RMS}$ vs. A_{DD}/σ_{RJ}

$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}	$J3u/2/J_{RMS}$	A_{DD}/σ_{RJ}
3.29052673	0.00	3.28612098	0.22	3.03807708	0.80	1.92590056	3.00
3.29052671	0.01	3.28532567	0.23	2.96602742	0.90	1.90041853	3.10
3.29052639	0.02	3.28443584	0.24	2.89226854	1.00	1.87621900	3.20
3.29052500	0.03	3.28344629	0.25	2.81866747	1.10	1.85321463	3.30
3.29052126	0.04	3.28235203	0.26	2.74654361	1.20	1.83132499	3.40
3.29051341	0.05	3.28114832	0.27	2.67677303	1.30	1.81047605	3.50
3.29049919	0.06	3.27983066	0.28	2.60989485	1.40	1.79059962	3.60
3.29047590	0.07	3.27839483	0.29	2.54620285	1.50	1.77163289	3.70
3.29044038	0.08	3.27683688	0.30	2.48581818	1.60	1.75351795	3.80
3.29038907	0.09	3.27515320	0.31	2.42874420	1.70	1.73620140	3.90
3.29031802	0.10	3.27334047	0.32	2.37490675	1.80	1.71963392	4.00
3.29022293	0.11	3.27139570	0.33	2.32418290	1.90	1.70376998	4.10
3.29009919	0.12	3.26931625	0.34	2.27642109	2.00	1.68856745	4.20
3.28994190	0.13	3.26709981	0.35	2.23145516	2.10	1.67398735	4.30
3.28974592	0.14	3.26474438	0.36	2.18911370	2.20	1.65999355	4.40
3.28950592	0.15	3.26224833	0.37	2.14922637	2.30	1.64655257	4.50
3.28921639	0.16	3.25961034	0.38	2.11162781	2.40	1.63363331	4.60
3.28887171	0.17	3.25682944	0.39	2.07616016	2.50	1.62120690	4.70
3.28846618	0.18	3.25390495	0.40	2.04267435	2.60	1.60924644	4.80
3.28799406	0.19	3.21682624	0.50	2.01103080	2.70	1.59772695	4.90
3.28744960	0.20	3.16660322	0.60	1.98109951	2.80	1.58662509	5.00
3.28682712	0.21	3.10594193	0.70	1.95275992	2.90		

Summary

- Estimating (A_{DD},$\sigma_{RJ,}$Q3) for a given (J3u,J_{RMS}) assuming Dual Dirac jitter model is not mathematically straightforward
  - **Three unknowns with two equations problem, an underdetermined non-linear system !!!**
  - 802.3ck D2p0 method of linear approximation is too simple and has room to improve
- We have shown a new method to accurately estimate (A_{DD},$\sigma_{RJ,}$Q3) from (J3u,J_{RMS}) and Dual-Dirac model look up table (LUT) without any assumption on Q3