

Comparison of ACT and TDD Theoretical DFE Performance

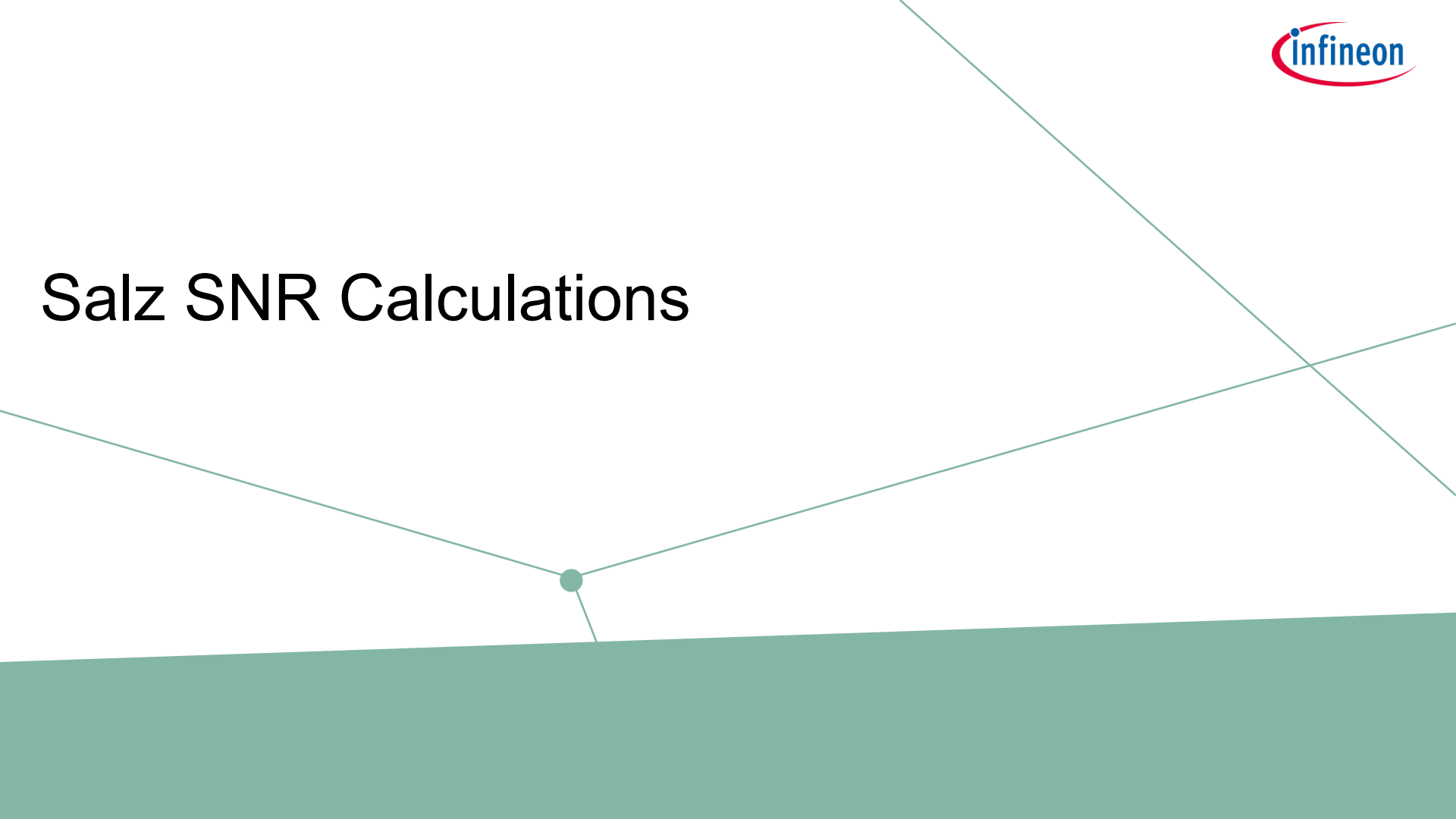
Ragnar Jonsson – Infineon
October 30, 2025



Introduction

- › This presentation evaluates the theoretical performance of ACT and TDD for a Gaussian channel
- › This calculation requires considerable **attention to detail**, both in the theoretical derivations and in the assumptions in the calculations
- › In this presentation every attempt is made to provide an unbiased evaluation of the ACT vs TDD performance

Salz SNR Calculations



Background

- › Salz SNR analysis assumes Gaussian channel model

$$r(t) = \int g(\tau) s(t - \tau) d\tau + v(t)$$

Signal Notations

In this text the following notations are used:

- $x(t)$ is a continuous time signal at time t
- $x[n]$ is the discrete signal at (time) index n
- $X(f)$ is the frequency domain representation of the continuous time signal $x(t)$
- $X(Z)$ is the Z-transform of the discrete signal $x[n]$

Signals can be converted between continuous time and discrete time using

$$x[n] = x(nT)$$

and

$$x(t) = \sum_n \delta(nT - t) \cdot x[n],$$

where $\delta(t)$ is the continuous time Dirac delta function.

Gaussian Channel Model

The channel model for (colored) additive Gaussian noise is given in Figure GMC-1. The input signals $s[n]$ and $w(t)$ are the transmitted signal and a white Gaussian noise signal, respectively. The output signal $r(t)$ has been shaped by the signal transfer function $G(f)$ and the noise transfer function $\Gamma(f)$. The signal $v(t)$ represents the colored additive Gaussian noise.

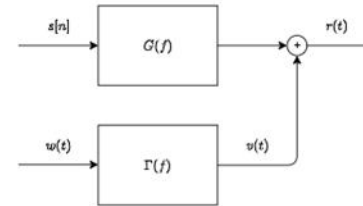


Figure GMC-1 - Simplified signal path for Gaussian channel

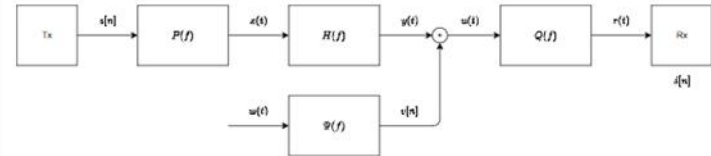


Figure GCM-2 - Signal path for Gaussian channel communication link

Figure GCM-2 shows a break down of the Gaussian channel model for end-to-end Ethernet link, where

- $P(f)$ is the transmit filter
- $H(f)$ is the insertion loss of the link segment
- $Q(f)$ is the receive filter (before equalization)
- $\Psi(f)$ is the coloring filter that generates the colored noise $v(t)$ from the white noise $w(t)$

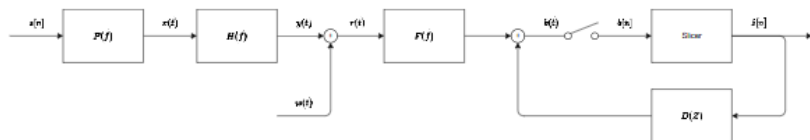
The relationship between the Ethernet channel model transfer functions and the Gaussian channel model transfer functions is given by

$$G(f) = P(f) \cdot H(f) \cdot Q(f) \quad (\text{gcm-1})$$

and

$$\Gamma(f) = \Psi(f) \cdot Q(f). \quad (\text{gcm-2})$$

Salz Classic 1973 Paper



In his classic 1973 paper, "Optimum Mean-Square Decision Feedback Equalization", J. Salz analyzed the theoretical performance of the communication link in the figure above. The $P(f)$ block is the transmit filter that converts discrete transmit symbols $s[n]$ into continuous transmit signal $x(t)$. The $H(f)$ block is the physical medium (the cable) and $w(t)$ is white additive Gaussian noise (WAGN). The $F(f)$ and $D(f)$ blocks are the feed forward and feed back equalizer filters, respectively. In Salz analysis the feed forward filter was continuous, but it is trivial to change the block diagram such that the sampling happens before the feed forward filter, and it becomes a discrete filter $F'(Z)$.

The slicer input signal $b[n]$ can be expressed as

$$b[n] = \sum_{k=-\infty}^{\infty} g(kT)s[n-k] - \sum_{k=-\infty}^{\infty} d[n]s[n-k] + (w(t) * f(t))|_{t=nT}$$

where

$$c(t) = g(t) * f(t) = p(t) * h(t) * f(t).$$

The slicer error mean square error (MSE) is therefore

$$\begin{aligned} E\{e^2[n]\} &= E\{(b[n] - s[n])^2\} \\ &= E\left\{\left(\sum_{k=-\infty}^{-1} c(kT)s[n-k] + (c(0) - 1)s[n] + \sum_{k=1}^{\infty} (c(kT) - d[k])s[n-k] \right. \right. \\ &\quad \left. \left. + (w(t) * f(t))|_{t=nT}\right)^2\right\} \\ &= \sigma_s^2 \sum_{k=-\infty}^{-1} c^2(kT) + \sigma_s^2(g(0) - 1)^2 + \sigma_s^2 \sum_{k=1}^{\infty} (c(kT) - d[k])^2 + \sigma_w^2 \int_{-\infty}^{\infty} f^2(t)dt. \end{aligned}$$

Where σ_s^2 and σ_w^2 are the variance of $s[n]$ and $w(t)$, respectively. The optimal decision feedback filter is obviously

$$d[n] = c(nT), \quad (\text{salz-1})$$

because then the third term in the error will disappear. This simplifies the optimization problem, because $f(t)$ can now be optimized independent of $d[n]$.

Salz states that standard calculus-of-variation optimization for $f(t)$ gives

$$g(-t) = f(t) \cdot \sigma_w^2 + \int_{-\infty}^{\infty} f(\tau) \left\{ \sum_{k=-\infty}^0 g(kT - \tau)g(kT - t) \right\} d\tau \quad (\text{salz-2})$$

This leads Salz to the result

$$E\{e^2[n]\} = \frac{\sigma_w^2}{\gamma_0^2}, \quad (\text{salz-3})$$

where

$$\gamma_0^2 = \exp\left\{\frac{1}{2\pi} \int_{-\pi}^{\pi} \ln\left\{\sum_n G\left(\omega - \frac{2\pi n}{T}\right) + \sigma_w^2/\sigma_s^2\right\} d\omega\right\}. \quad (\text{salz-4})$$

This in turn leads to the famous Salz SNR formula:

$$\text{SNR}_{\text{salz}} = \left\{ \exp\left\{\frac{-1}{2\pi} \int_{-\pi/2}^{\pi/2} \ln\left(\sum_n \text{SNR}\left(\omega - \frac{2\pi n}{T}\right) + 1\right) d\omega\right\} \right\}. \quad (\text{salz-5})$$

It can be shown that while Salz derived the above formula for AWGN, it also applies to colored Gaussian noise.

Salz SNR Approximation

- › The Salz SNR calculation can be simplified to

$$SNR_{Salz} = \exp \left(\frac{1}{F_{BW}} \int_{F_{BW}} \log_e (SNR(f) + 1) df \right)$$

- › This can be further simplified to

$$SNR_{Salz} \approx \frac{1}{F_{BW}} \int_{F_{BW}} 10 \log_{10} (S_r(f)) df - \frac{1}{F_{BW}} \int_{F_{BW}} 10 \log_{10} (S_n(f)) df.$$

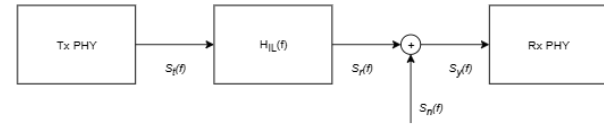
Salz SNR Approximation

Salz SNR is a theoretical estimate of an optimal Decision Feedback Equalizer (DFE) of infinite complexity. The full Salz SNR calculation includes folding of frequencies out of band into frequency band of interest. This is the summation in (sals-5). If there is reasonably good filtering at the receiver AFE, then the Salz SNR can be approximated with only the in-band term as

$$SNR_{Salz} = \exp \left(\frac{1}{F_{BW}} \int_{F_{BW}} \log_e (SNR(f) + 1) df \right), \quad (\text{snr-1})$$

where

- $SNR(f)$ is the Signal-to-Noise ratio at any give frequency, f
- F_{BW} is the bandwidth of the transmitted signal



For high SNR case, where $SNR(f) \gg 1$ for all f , this can further be approximated with

$$\begin{aligned} SNR_{Salz} &\approx \exp \left(\frac{1}{F_{BW}} \int_{F_{BW}} \log_e (SNR(f)) df \right) \\ &= \exp \left(\frac{1}{F_{BW}} \int_{F_{BW}} \log_e (S_r(f)) df - \frac{1}{F_{BW}} \int_{F_{BW}} \log_e (S_n(f)) df \right) \quad (\text{snr-2}) \\ &= 10^{\left(\frac{1}{F_{BW}} \int_{F_{BW}} 10 \log_{10} (S_r(f)) df - \frac{1}{F_{BW}} \int_{F_{BW}} 10 \log_{10} (S_n(f)) df \right)}. \end{aligned}$$

When the SNR is expressed in dB, this becomes

$$SNR_{Salz} \approx \frac{1}{F_{BW}} \int_{F_{BW}} 10 \log_{10} (S_r(f)) df - \frac{1}{F_{BW}} \int_{F_{BW}} 10 \log_{10} (S_n(f)) df, \quad (\text{snr-3})$$

where

- $S_r(f)$ is the transmit signal power after channel attenuation, but without noise
- $S_n(f)$ is the receiver noise power

Insertion Loss and Return Loss

Salz SNR calculations requires a model for the insertion loss. The IL (in dB) for most twisted pair and coax cables can be approximated with

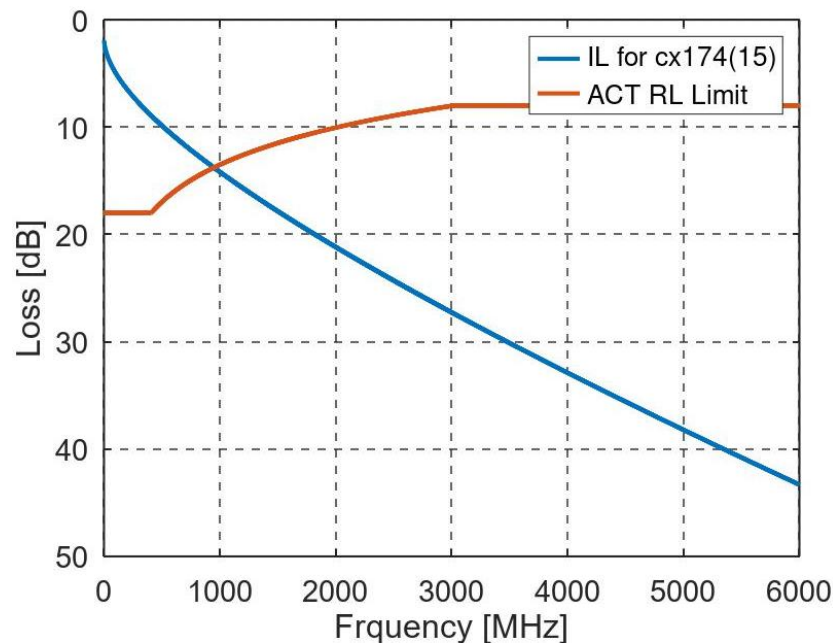
$$\hat{H}_{IL} = 20 \log_{10} (|H_{IL}(f)|) = (b_0 + b_1 \cdot f^{1/2} + b_2 \cdot f) \quad (\text{il-1})$$

where b_0 , b_1 , and b_2 are constants that characterize the IL for the cable. For example, this model is used to specify the IL characteristics for CX31, CX44, and CX174 in ISO19642-11.

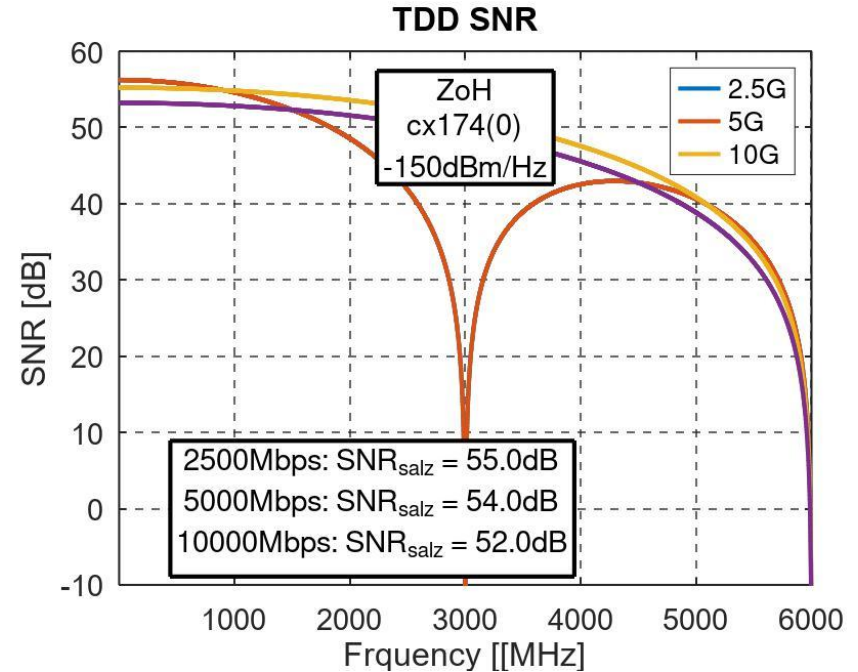
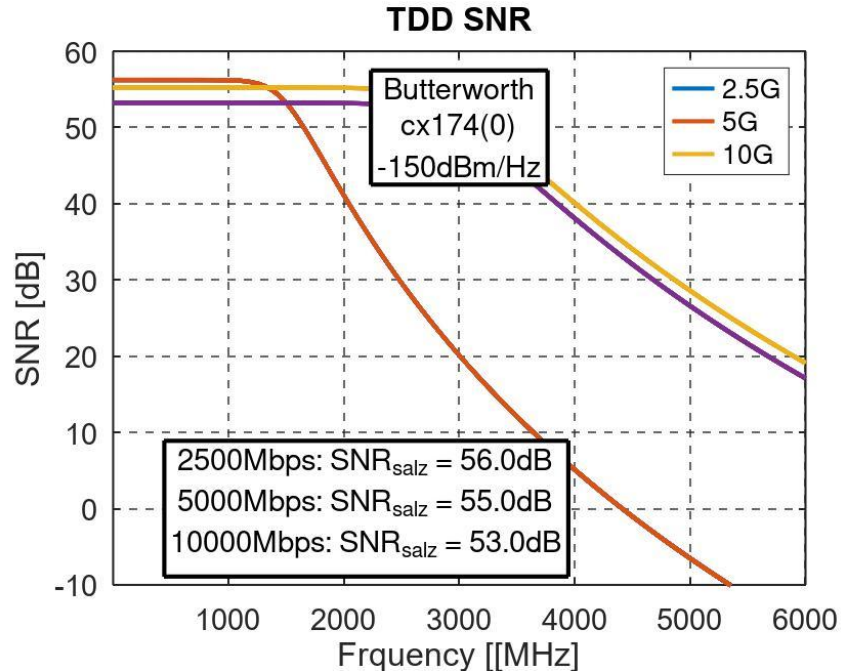
Draft Proposal for
IEEE P802.3dm Asymmetrical Electrical Automotive Ethernet Task Force

0.7a proposal
9th September 2025

$$\text{Return Loss}(f) \geq \left\{ \begin{array}{ll} 18 & 1 \leq f < 400 \\ 16.5 - 11.5 \log_{10} \left(\frac{f}{550} \right) & 400 \leq f < 3000 \\ 8 & 3000 \leq f < 4000 \end{array} \right\} (\text{dB})$$

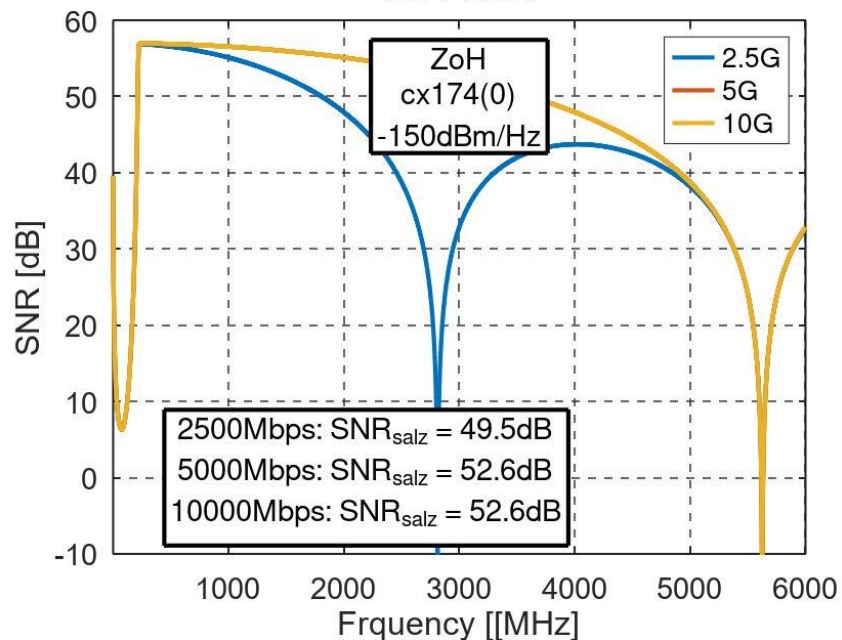


Transmit Filters – Butterworth vs. Zero-Order-Hold

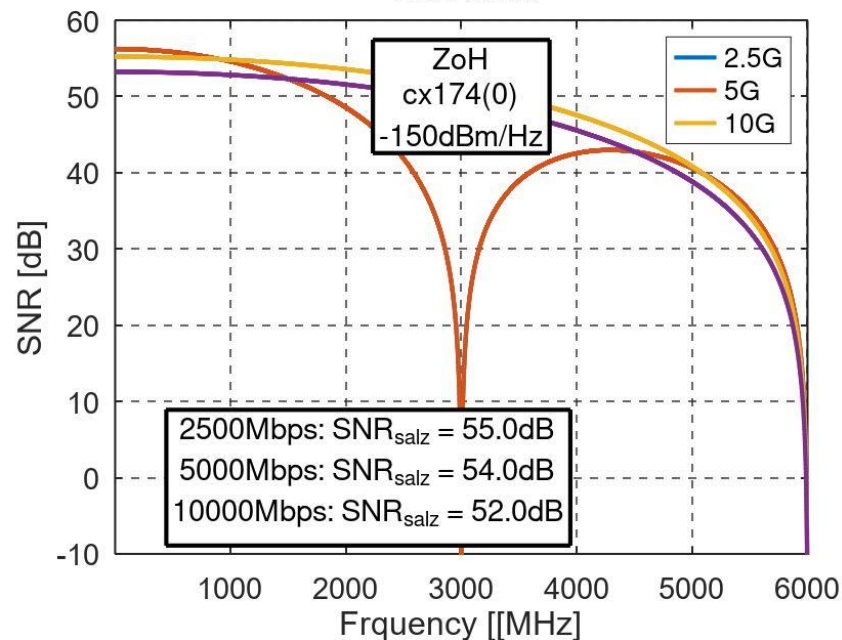


Salz SNR Calculation Results

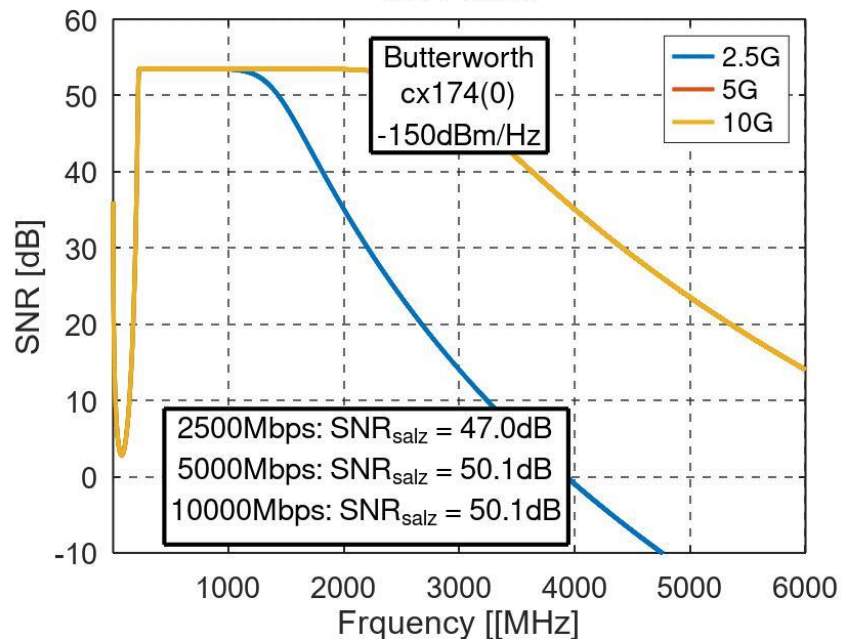
ACT SNR



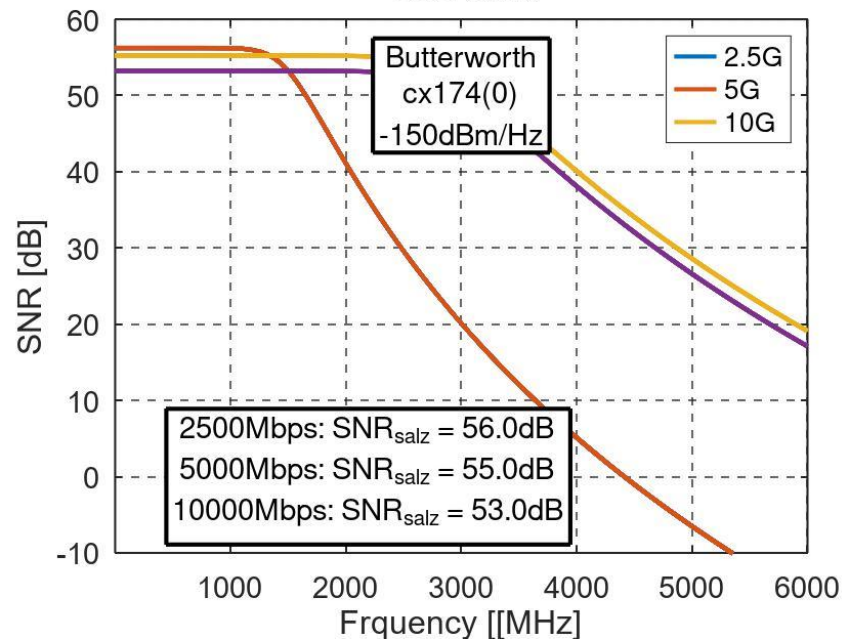
TDD SNR



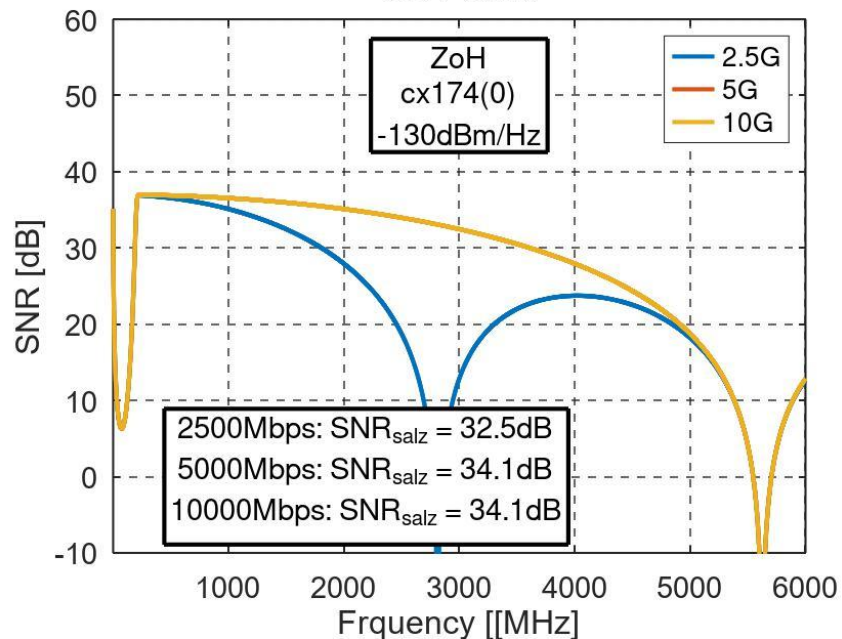
ACT SNR



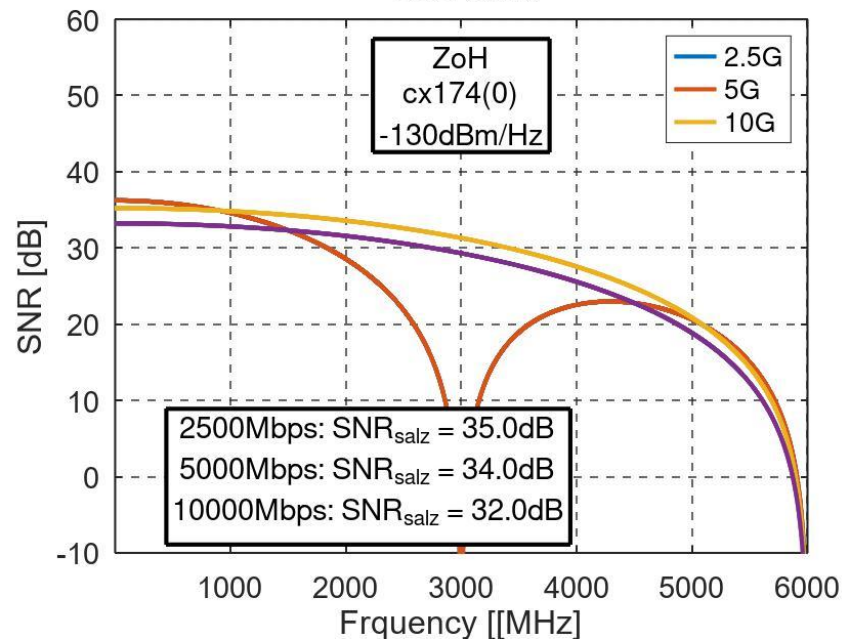
TDD SNR



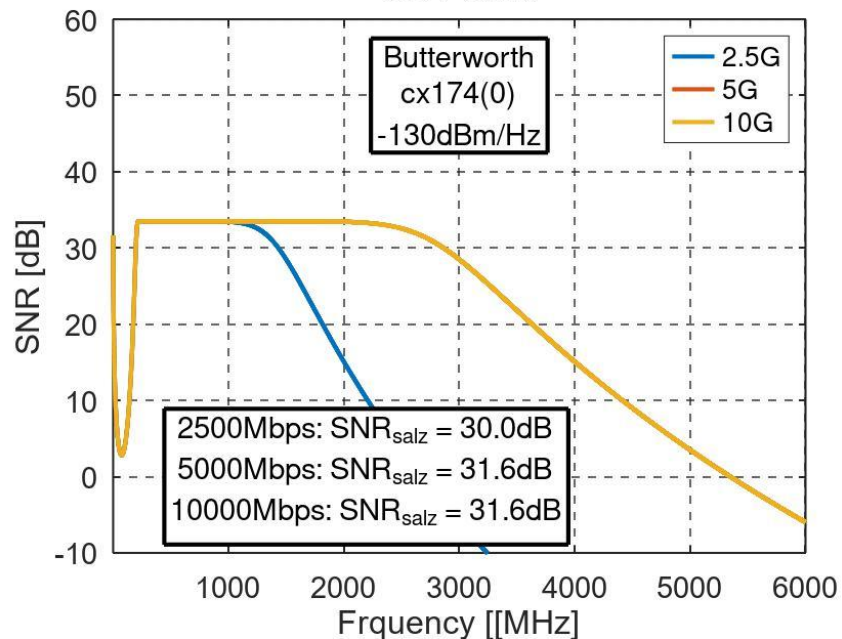
ACT SNR



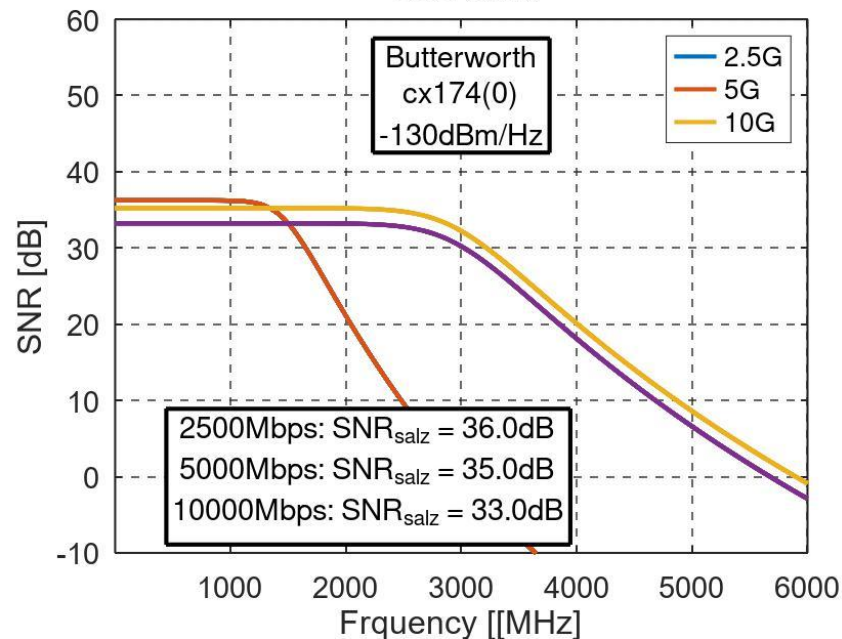
TDD SNR



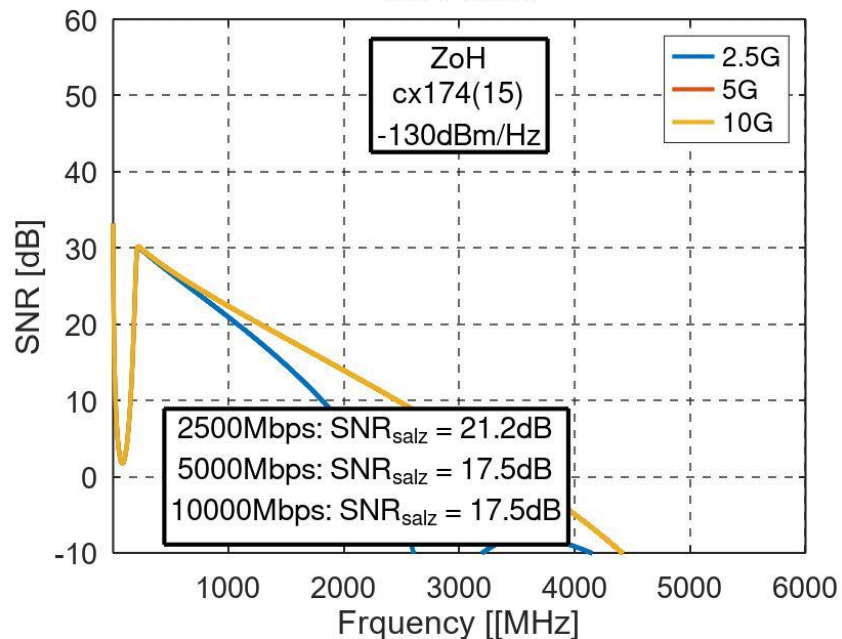
ACT SNR



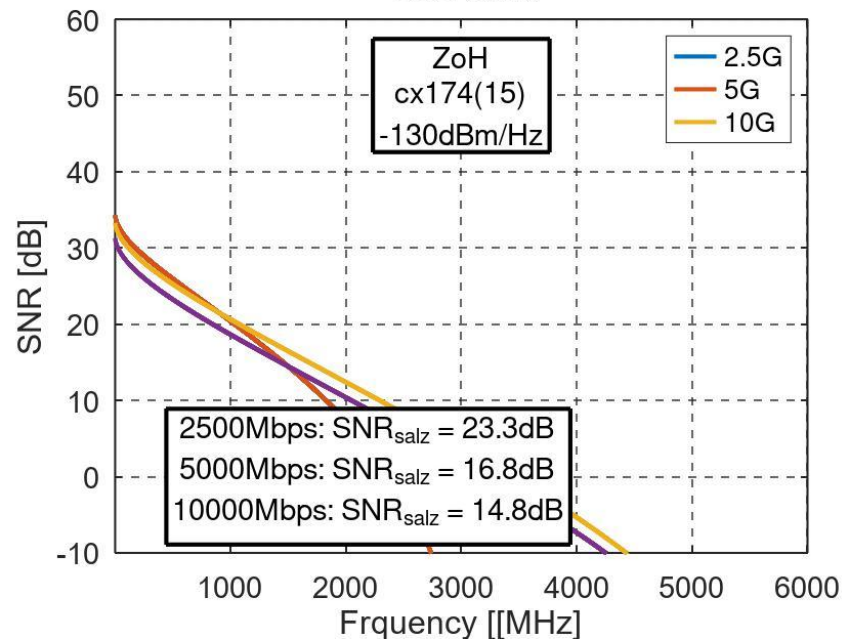
TDD SNR



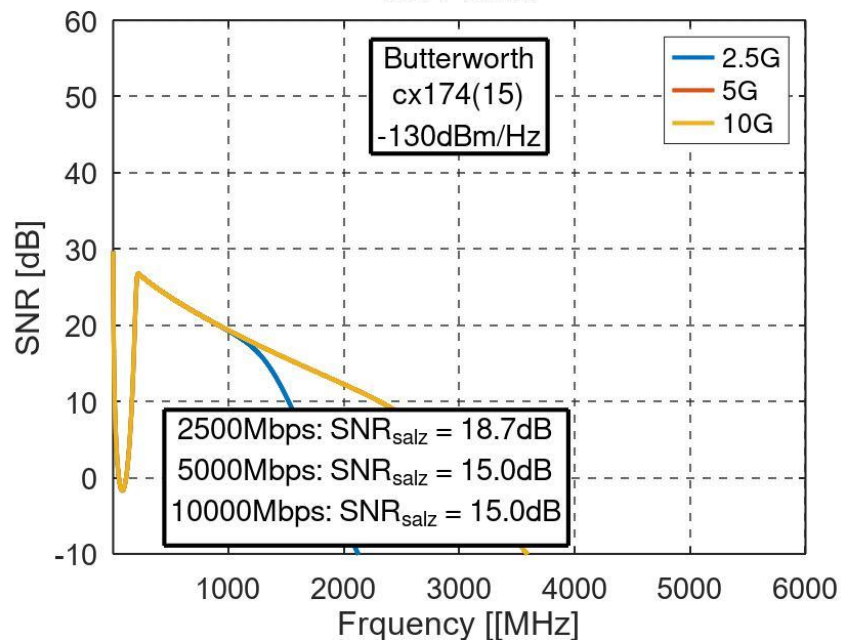
ACT SNR



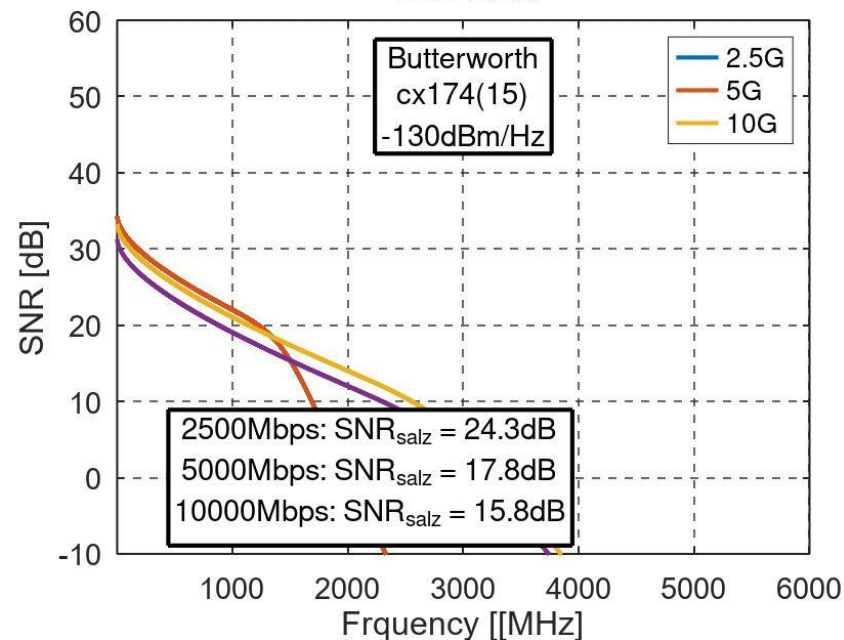
TDD SNR



ACT SNR



TDD SNR



Summary

- › Theoretical Salz SNR calculations can be used to compare ACT vs TDD performance for a Gaussian channel
- › The outcome of that calculation depends heavily on the assumptions used in the calculations
- › The few examples in this presentation shows that there is no significant difference in theoretical performance of ACT vs TDD
- › The main difference is that TDD shows better SNR on short cables with low noise, but ACT has better performance on longer cables with more noise
- › Any calculations showing significant difference in Gaussian channel performance should be taken with a “grain of salt”, because it is all about the assumptions



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