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Title	**Soft Iterative Decoding for Mobile Wireless Communications**
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Re:	Mobility Enabling Technologies and Capabilities
Abstract	This submission discusses the soft iterative decoding of certain types of error-control codes, and their applications to mobile wireless. Clarifying comments have been added.
Purpose	For informational use only.
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Revised

Soft Iterative Decoding for Mobile Wireless Communications

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Contents

- Error-Control Codes
- Understanding Soft Iterative Decoding
- Application to Mobile Wireless

Error-Control Codes (ECC)

- ECCs introduce redundancy into a data sequence
- Allows for correction of errors resulting from the noisy, imperfect channel

- In past decade, new paradigm: ECCs with structure that allows soft iterative decoding
- Soft = probabilistic messages
- Iterative = repeated passing of messages
- Significant coding gain

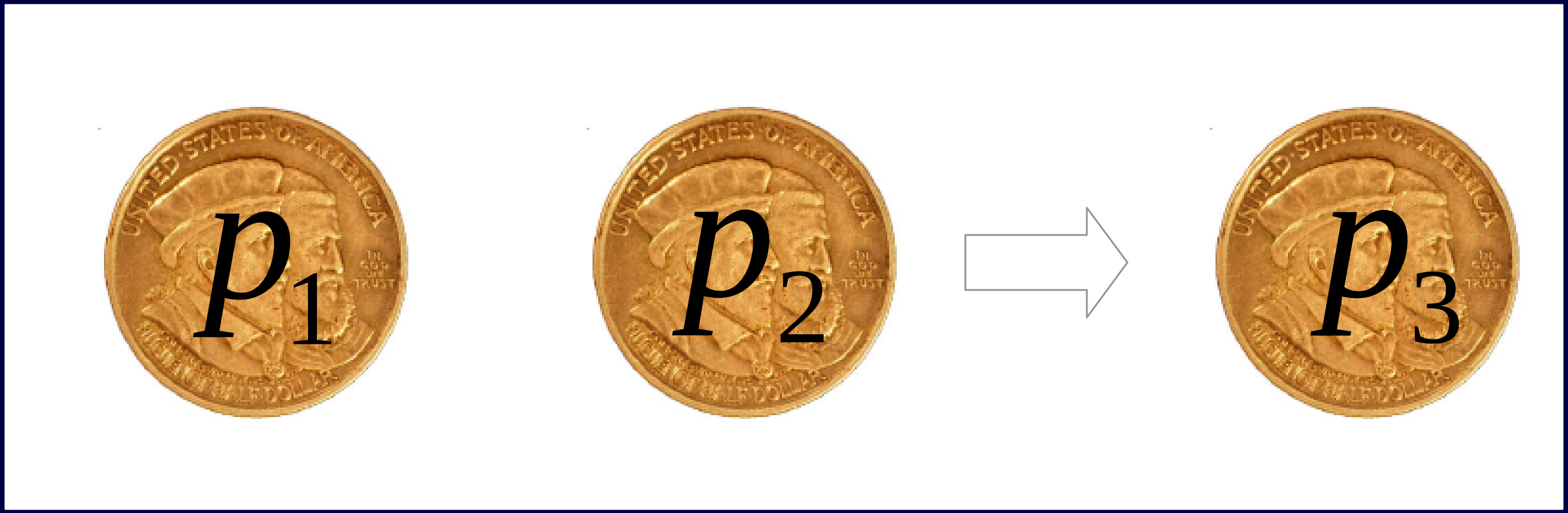
Examples of ECCs

“Traditional ECCs”	“ECCs that allow soft iterative decoding”
• Repetition • Single parity check • Hamming codes • Convolutional codes • BCH codes • Reed-Solomon codes	• Turbo decoding of concatenated convolutional codes • Turbo decoding of product codes • Low-density parity-check (LDPC) codes

Comparison

Traditional ECCs	Soft iterative ECCs
• Algebraic decoding methods	• Probabilistic decoding methods
• Lower complexity	• Higher complexity
• Lower coding gain	• Higher coding gain
• Good for short codewords	• Good for long codewords

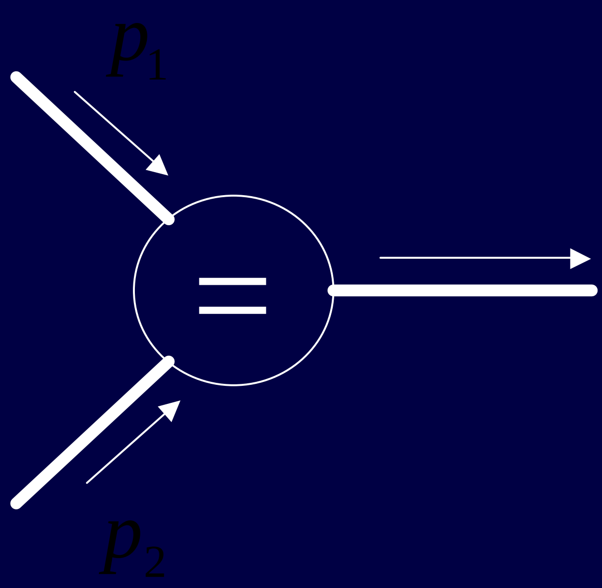
Coin Puzzle: Equality



Suppose the probability that the first coin is a head is p_1=2/3 and the probability that the second coin is a head is p_2=2/3.
If all three coins are all heads or all tails,
what is the probability p_3 that the third coin is a head?

$x_1 = x_2 = x_3$

Equality node



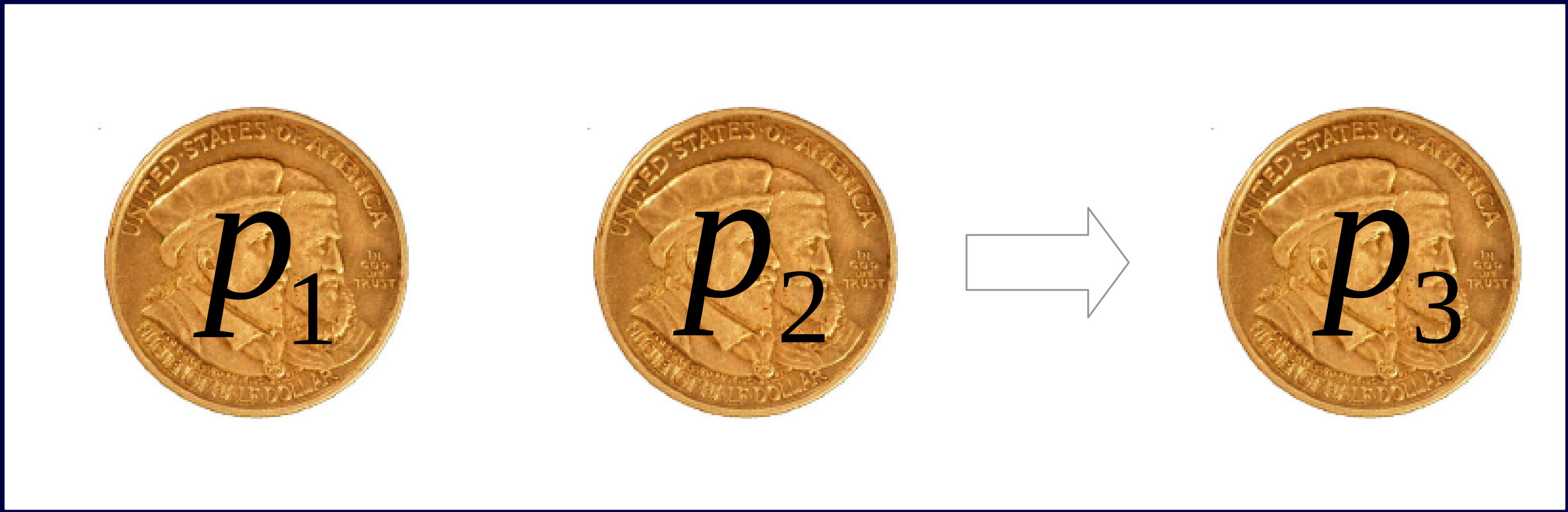
Coin 1	Coin 2	Coin 3	Probability
Head	Head	Head	$p_1 p_2$
Head	Head	Tail	
Head	Tail	Head	
Head	Tail	Tail	
Tail	Head	Head	
Tail	Head	Tail	
Tail	Tail	Head	
Tail	Tail	Tail	$(1-p_1)(1-p_2)$

Answer:

$$p_3 = \frac{p_1 p_2}{p_1 p_2 + (1-p_1)(1-p_2)}$$

$$\log \frac{p_3}{1-p_3} = \log \frac{p_1}{1-p_1} + \log \frac{p_2}{1-p_2}$$

Coin Puzzle: Parity-check

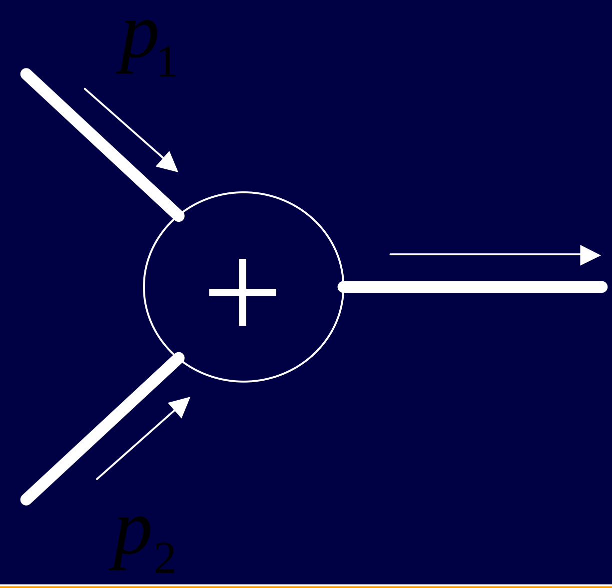


Suppose the probability that the first coin is a head is p_1=2/3
and the probability that the second coin is a head is p_2=2/3.
If exactly two of the coins are heads, or all are tails,
what is the probability p_3 that the third coin is a head?

$x_1 \oplus x_2 \oplus x_3 = 0$

Parity-check node

Coin 1	Coin 2	Coin 3	Probability
Head	Head	Head	
Head	Head	Tail	$p_1 p_2$
Head	Tail	Head	$p_1(1-p_2)$
Head	Tail	Tail	
Tail	Head	Head	$(1-p_1)p_2$
Tail	Head	Tail	
Tail	Tail	Head	
Tail	Tail	Tail	$(1-p_1)(1-p_2)$



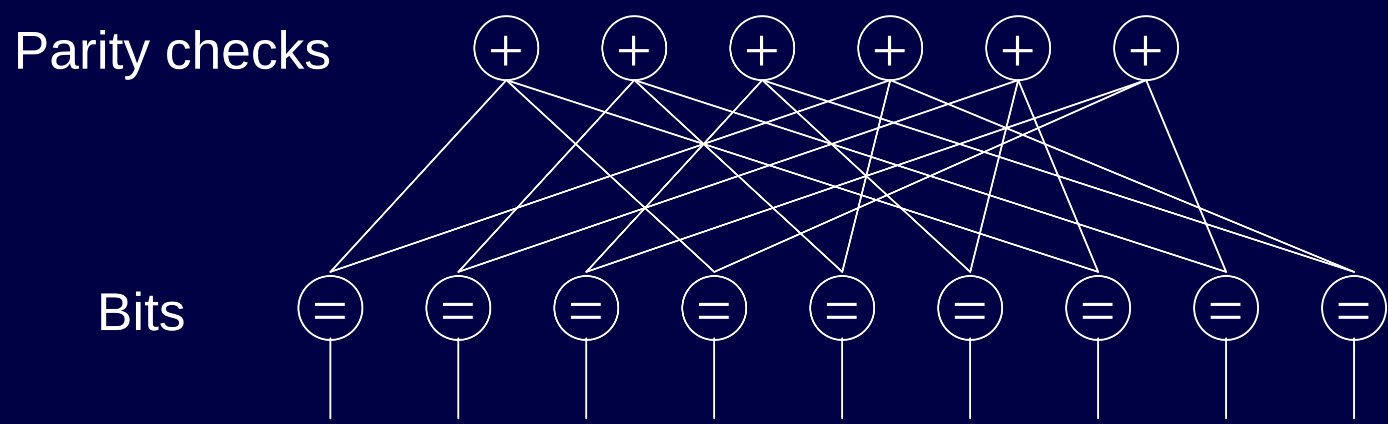
Answer:

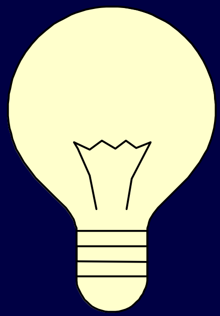
$$p_3 = p_1(1-p_2) + (1-p_1)p_2$$

$$1-2p_3 = (1-2p_1)(1-2p_2)$$

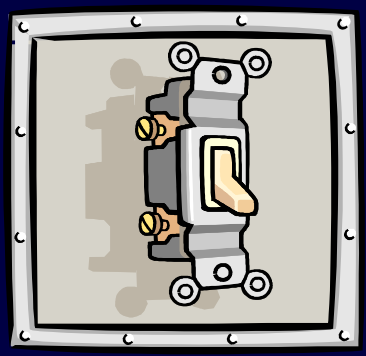
Parity-check matrix as a graph

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



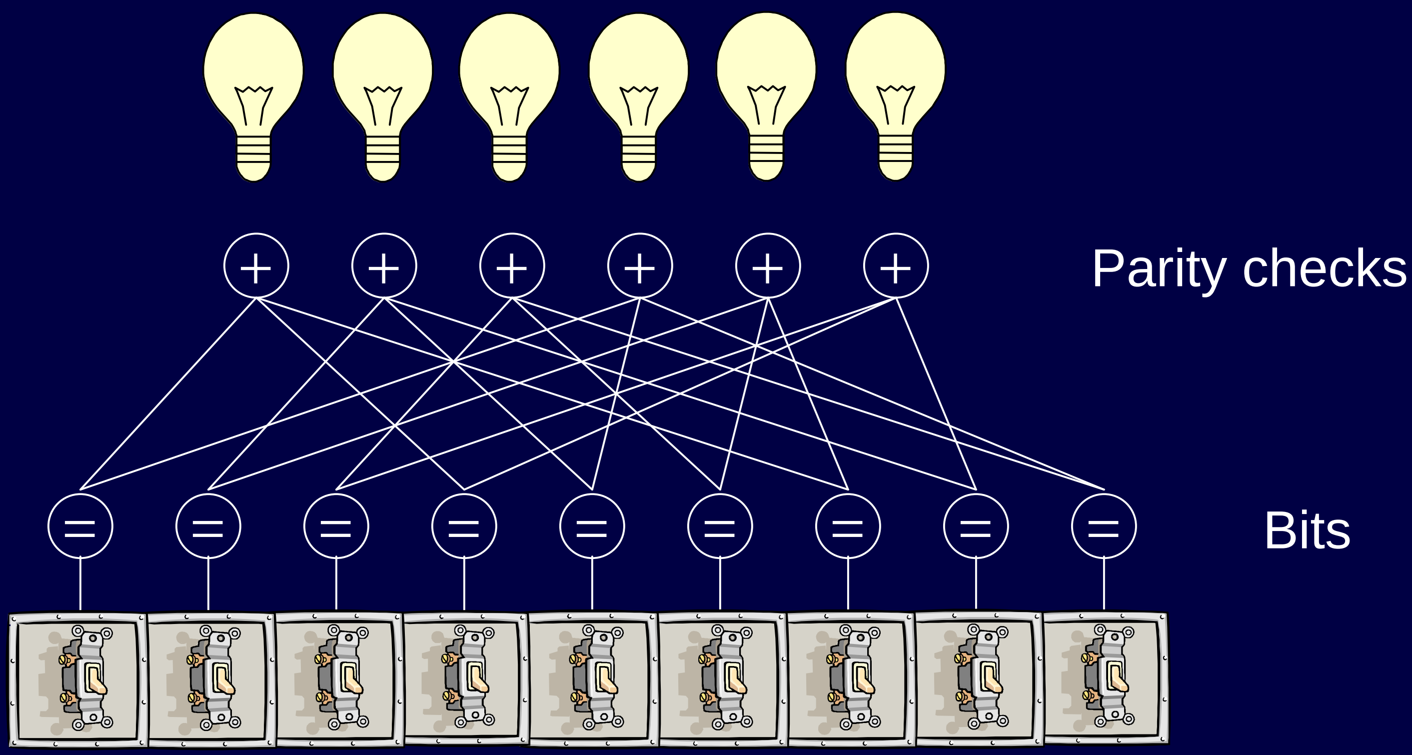


The Case of the Mysterious Light Bulbs



- Someone has broken into the Lightbulb Factory at night and turned on some of the light bulbs. Your job is to flip the light switches so as to turn off all the lights in order to conserve energy.
- The problem is that each light switch affects many light bulbs, and each light bulb is connected to many light switches. Light bulbs go between two states ("on" and "off") whenever a connected switch is flipped.
- Your mission is to turn off all the lights while flipping the fewest number of switches possible.

An analogy for LDPC codes

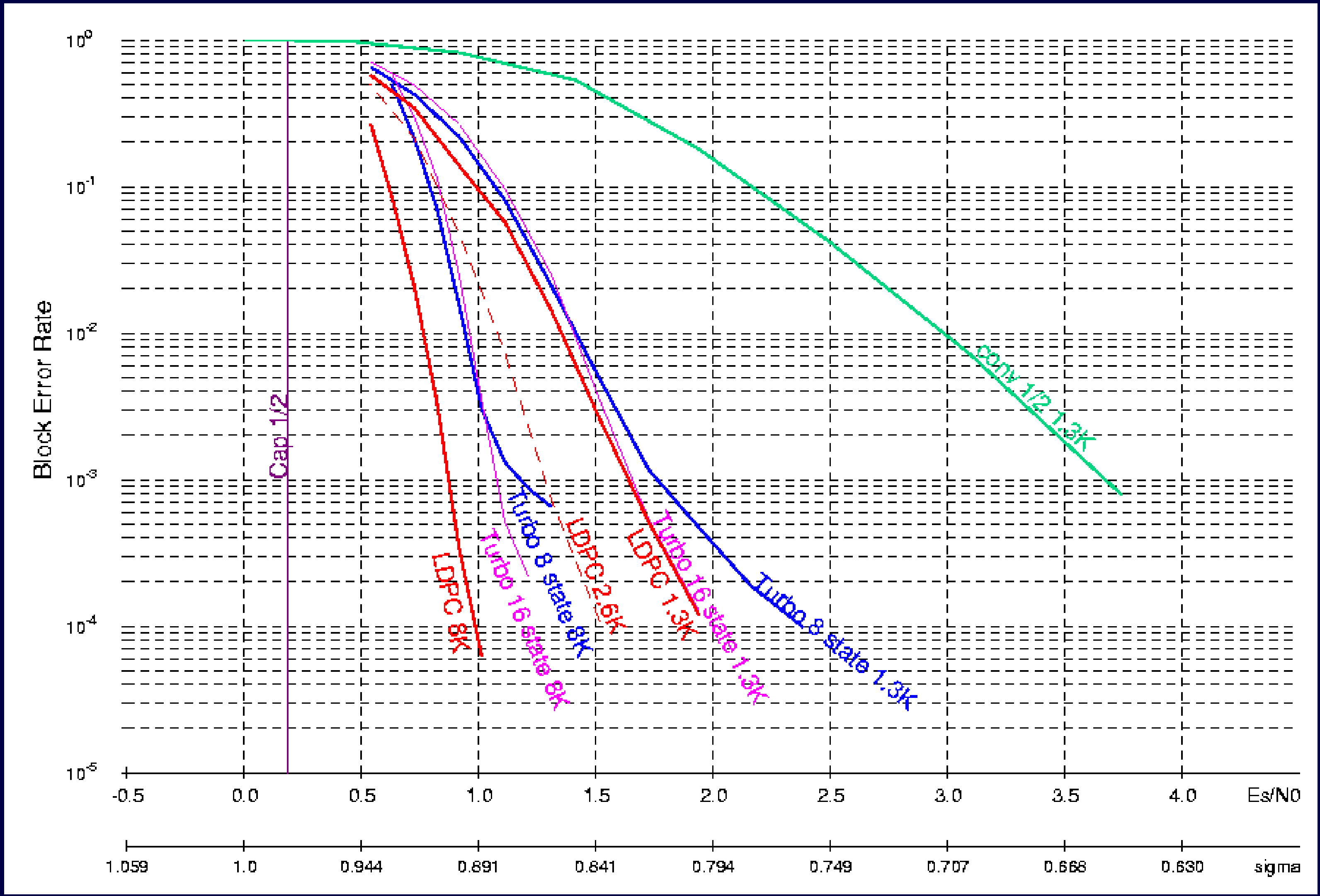


Flipping the minimum set of switches to turn off all the lights. ⟺ Correcting bit errors in the received word so that all the parity checks are satisfied

More analogies

Error-Control Code	Analogy
“Turbo” Product Codes (TPC)	Crossword puzzle *A T E* *G E L* *O A K*
Turbo Convolutional Codes (TCC)	Anagram 

Comparison of various ECCs



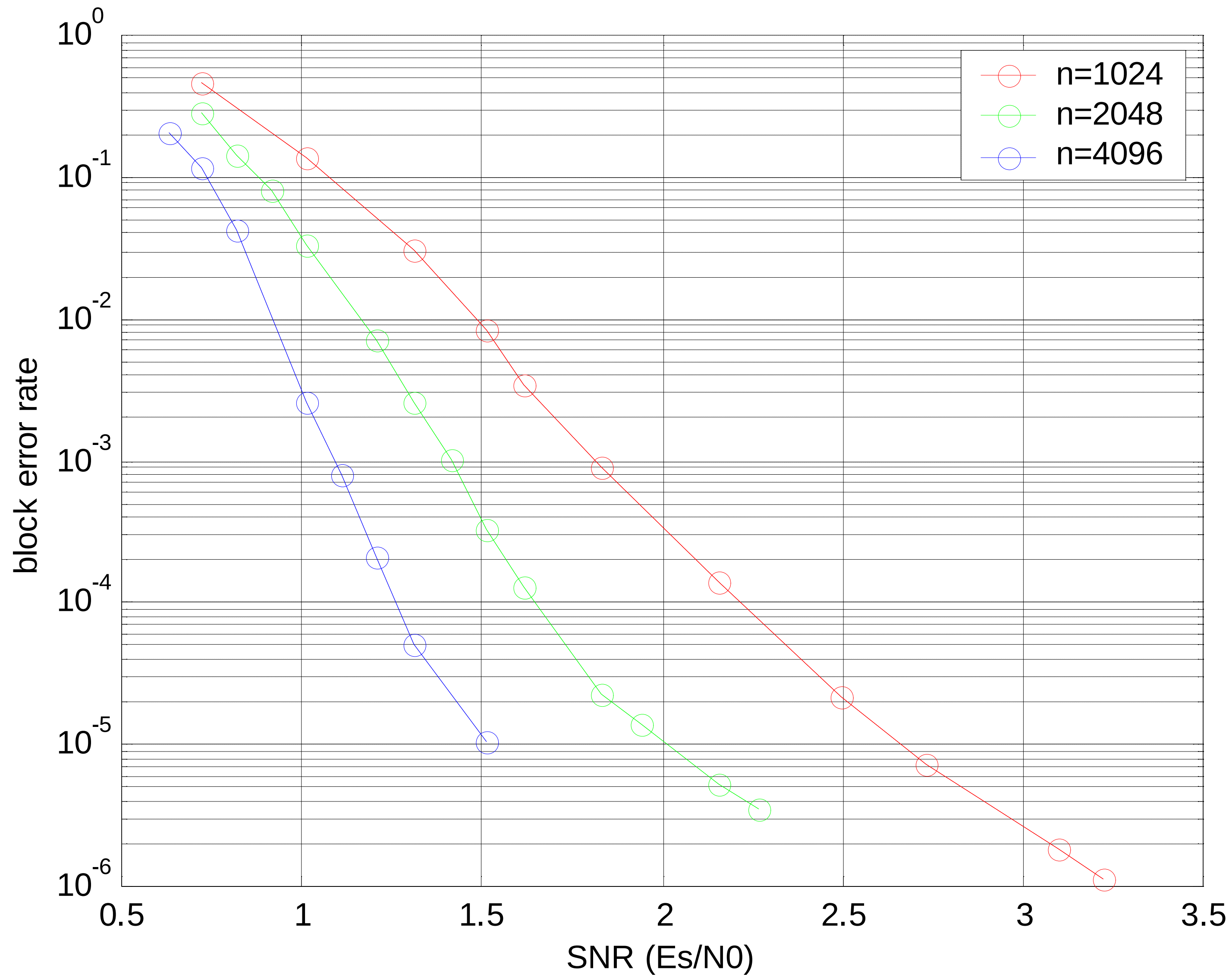
Regarding the figure, “Comparison of various ECCs”:

- In discussing the number of iterations, it should be emphasized that an iteration of the LDPC decoder requires significantly less complexity than a turbo coding iteration. Any comparison of iterations should take into account these differences in complexity and latency between LDPC and turbo decoder. At 10^{-3} frame error rate, the average number of iterations of the LDPC decoder is around 10. For n=1300, a maximum of 20 iterations is sufficient to achieve nearly optimal performance. Also, the number of iterations required decreases rapidly as the SNR increases.
- Clarification: the curves for the turbo and LDPC codes are *all* shown for simulations with floating-point calculations. With 5-bit quantization for the LDPC codes, however, the performance is near-optimal and differs by less than 0.1 dB from the floating point simulations.
- Note that in this figure (as in all subsequent figures), all block lengths are in terms of the codeword length (n), i.e., corresponding to the number of coded bits, as opposed to the number of information bits (k).

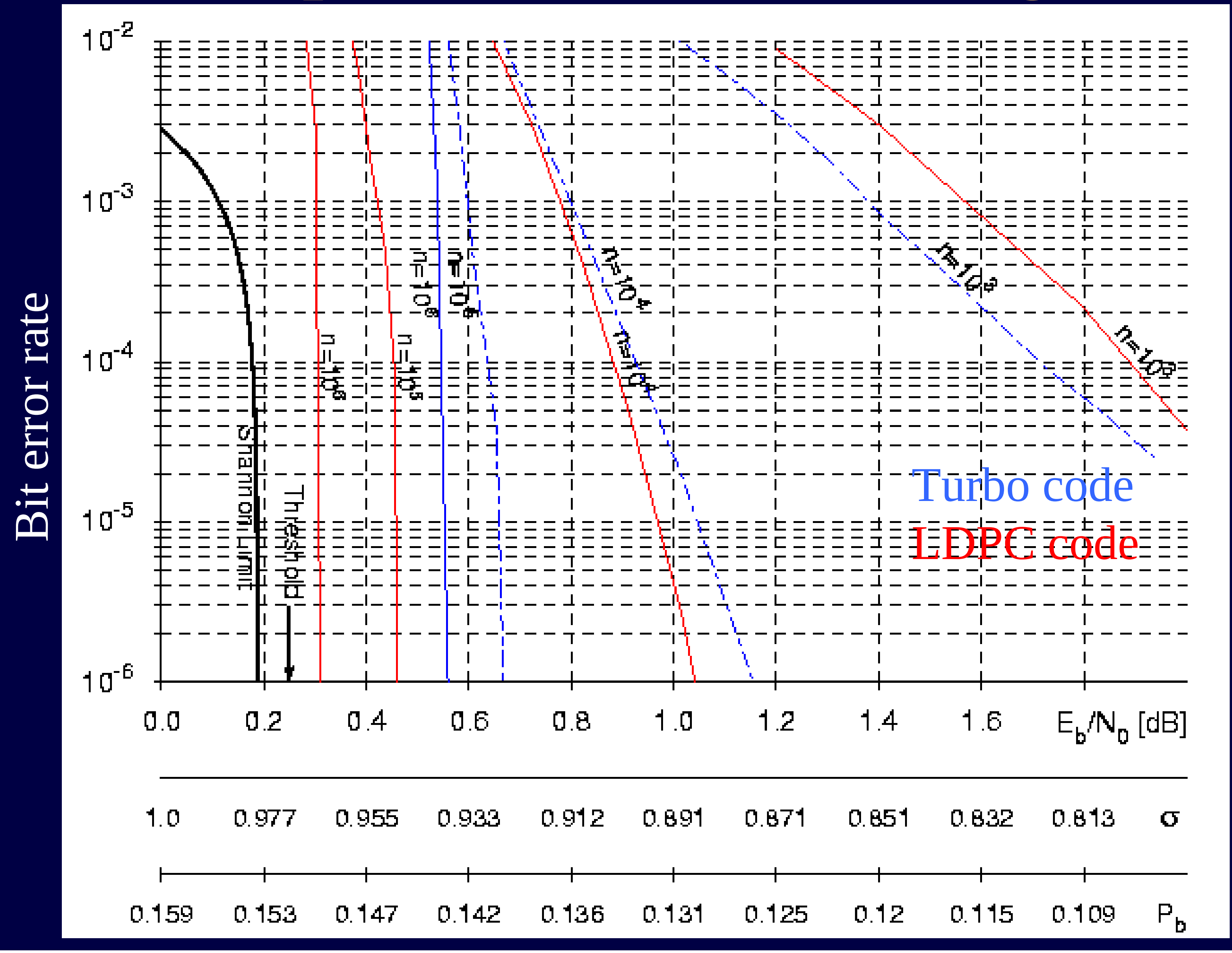
Mobile Wireless Considerations

- A packet-based mobile wireless system needs:
  - Short blocks for frequent control messages
  - Long blocks for data traffic
  - A variety of code rates for link adaptation
- Retransmission / ARQ
  - Maximize coding gain at $\sim 10^{-3}$ packet error rate
  - Error floor not a serious problem
- Multipath fading channels
  - With OFDM, multipath becomes frequency selectivity
  - ECC sees variations in channel gain across codeword

Comparison of block lengths



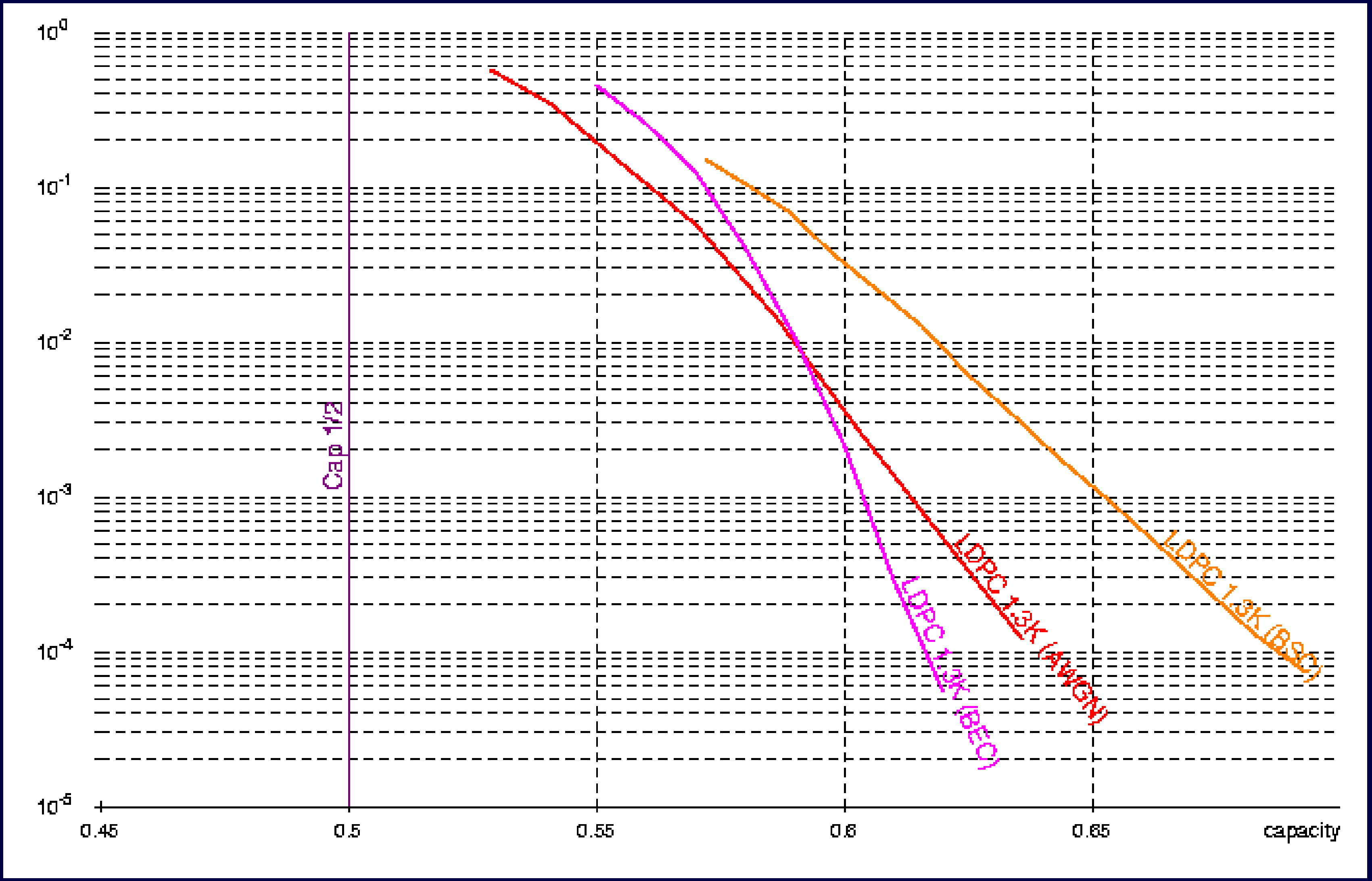
Comparison of block lengths



Regarding the previous two figures:

- The point of these two figures is to show how increased block length leads to better performance, and to give an indication of trade-offs that will need to be made in choosing the block length (e.g., longer block length for more coding gain, but shorter block length for lower latency and physical constraints).
- Note that the LDPC codes used here have not been optimized (e.g., the degrees of the graph have not been properly chosen to improve the performance) so that the Turbo code appears to be 0.2 dB better than the LDPC code of length 1000. With better optimized designs, the LDPC code has equal or better performance (and also less complexity) than comparable Turbo codes, as shown in the figure "Comparison of various ECCs" (slide 14).
- In addition to long blocks used to convey data traffic, in a wireless system it would be desirable to have short codes to convey control messages and signals. We have found that even LDPC codes shorter than 200 bits can be decoded using the message-passing algorithm for usable coding gain, and that LDPC codes of several hundred bits are very competitive with other types of ECC.

Robustness on different channels



Regarding the previous figure:

- This three curves represent the performance of a rate ½ LDPC code of length 1300 bits on the Binary Symmetric Channel (BSC), the Binary Erasure Channel (BEC) and the Binary-Input Additive White Gaussian Noise (AWGN) channel.
- The x-axis represents “channel capacity,” which represents the data rate that could theoretically be supported on the binary-input channel. The curves represent the frame error rate that would be seen if using this LDPC code of rate 0.5 on that channel. At 10^{-3} frame error rate on an AWGN channel, the channel could support a rate of 0.62 for a perfect code (using infinite block length and decoding complexity). (This is roughly 1.6 dB from the Shannon limit.)
- The BSC and the BEC represent extremes in terms of channel conditions, so that the point is to make an information-theoretic analysis to show that LDPC codes are robust under the gamut of channel conditions. That the LDPC performs well on the BEC (where some bits are received perfectly and others are completely lost) indicates that it is well suited to situations where the SNR varies for the different bits of a codeword (e.g., with multipath fading using OFDM). Further simulations with specific fading channel models will be presented at a later date.

Summary

- Turbo and LDPC codes can provide practical methods for achieving high coding gain in communication systems
- Key elements are soft decoding and iterative message-passing.
- These codes meet the needs of wireless communications, e.g., in terms of block lengths, code rates, and robustness in multipath channels.

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